

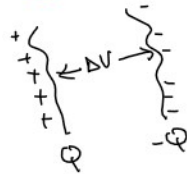
# CH 24

## CAPACITANCE

### CAPACITORS

### DIELECTRIC (isolator)

$$\text{capacitance} \equiv C \equiv \frac{Q}{\Delta V}$$

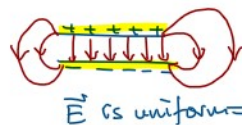


$$\left[ F = \frac{C}{V} \right] ; \left[ C = FV \right]$$

$$\left[ \text{Farad} = \frac{\text{Coulombs}}{\text{Volt}} \right]$$

1 Farad is a HUGE cap. value  
 $1 \text{ nF} ; 1 \mu\text{F} \quad 1 \text{ mF}$

uniform  $\vec{E}$  field two parallel plate



$\sqrt{\text{Area}} \gg \text{distance, } d$

$1 \text{ cm} \gg 1 \text{ mm}$

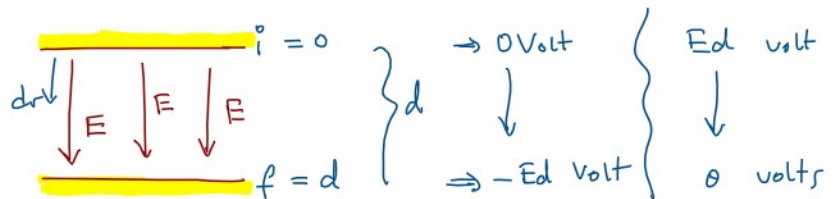
$$C = \frac{Q}{\Delta V}$$

$E$  is constant

$$-\int \vec{E} \cdot d\vec{r} = \Delta V$$

$$-\int E dr = -E \int_{\Delta r} dr = -E(r_f - r_i) = -Ed = \Delta V = V_f - V_i$$

$E = \text{const.}$



$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed}$$

Ch 22  $E$  field of a  $\infty$  large metal surface

$$E = \frac{\sigma}{\epsilon_0} ; \sigma = \frac{Q}{\text{Area}} = \frac{Q}{A}$$

$$= \frac{Q}{\frac{\sigma}{\epsilon_0} d} = \frac{Q}{\frac{Q}{A\epsilon_0} d}$$

$$C = \epsilon_0 \frac{A}{d} \quad \text{parallel plate capacitor}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} = 8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}$$

1)  $C$  is small number

2)  $C$  is purely geometrical

$$C = \frac{Q}{\Delta V} = \epsilon_0 \frac{A}{d} \quad \left\{ \text{This does not have any } Q, V \right\}$$

$$\left[ C = \epsilon_0 \frac{A}{d} ; \Rightarrow \epsilon_0 = 8.85 \times 10^{-12} \frac{\text{F}}{\text{m}} \right] \quad C = \text{the ability to store (hold) charge } +Q, -Q \text{ over potential difference.}$$

$$C = (\sim 10^{-11}) \frac{A}{d}$$

$$\Rightarrow 10^8 \Rightarrow 10^{-3} \text{ F} = 1 \text{ mF}$$

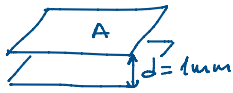
$$\Rightarrow 10^5 \Rightarrow 10^{-6} \text{ F} = 1 \mu\text{F}$$

$$\Rightarrow 10^2 \Rightarrow 10^{-9} \text{ F} = 1 \text{ nF}$$

↓  $\left[ C = \epsilon_0 \frac{A}{d} ; \Rightarrow \epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m} \right]$   $C =$  the ability to store (hold) charge  $+Q, -Q$  per potential difference.

$\left[ F = \frac{C^2}{Nm^2} \frac{m^2}{m} = \frac{C^2}{Nm} = \text{Farad} \right]$

ex) parallel plate capacitor with  $d = 1 \text{ mm}$   
what would be the area of capacitor if  $C = 1 \text{ F}$ ?



$A = ?$

$$C = \epsilon_0 \frac{A}{d}$$

$$1 \text{ F} = (8.85 \times 10^{-12}) \frac{A}{10^{-3}} \Rightarrow A = \frac{10^9}{8.85} \approx 10^8 \text{ m}^2$$

$100,000,000 \text{ m}^2 = 30 \text{ times of campus size}$

According to campus  $3.5 \times 10^6 \text{ m}^2$

x) parallel plate capacitor  $A = 2 \text{ m}^2$   $d = 5 \text{ mm}$   $\Delta V = 10 \text{ kV}$   $Q = ?$

$$C = \epsilon_0 \frac{A}{d} = \frac{Q}{\Delta V}$$

$$Q = C \Delta V = 3.5 \times 10^{-9} \times 10 \times 10^3 \text{ C}$$

$$C = 8.85 \times 10^{-12} \frac{2}{0.005} = 3.5 \times 10^{-9} \text{ F} = 3.5 \text{ nF}$$

$$= 3.5 \times 10^{-5} \text{ C}$$

$$Q = 35 \mu\text{C}$$

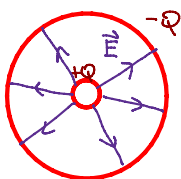
b)  $\vec{E}$  between the capacitor plate?

$$\Delta V = -Ed$$

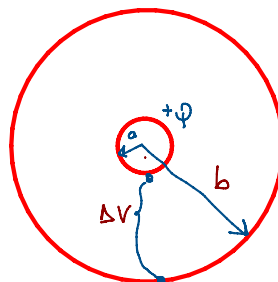
$$C = \frac{Q}{\Delta V} \rightarrow + \quad C \rightarrow +$$

$$E = \frac{\Delta V}{d} = \frac{10 \times 10^3 \text{ V}}{5 \times 10^{-3} \text{ m}} = 2 \times 10^6 \text{ V/m}$$

## SPHERICAL CAPACITORS



$a =$  inner radius  
 $b =$  outer radius



$$C = \frac{Q}{\Delta V}$$

$$V(r_a) = \frac{kQ}{a}$$

$$V(r_b) = \frac{kQ}{b}$$

$$\Rightarrow \Delta V = \frac{kQ}{a} - \frac{kQ}{b} = kQ \left( \frac{b-a}{ab} \right)$$

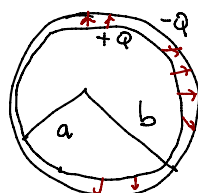
$$C = \frac{Q}{kQ \left( \frac{b-a}{ab} \right)} = \frac{1}{k} \frac{ab}{b-a} =$$

$$4\pi\epsilon_0 \frac{ab}{b-a} = C$$

$$C \sim \epsilon_0 [m]$$

spherical cap.

$$C \uparrow ; ab \uparrow ; b-a \downarrow$$

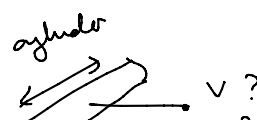


## CYLINDRICAL

## CAPACITOR

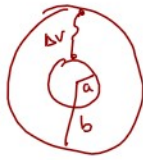
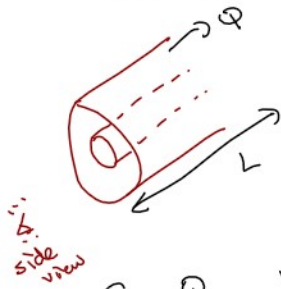


$$r = \frac{Q}{L}$$

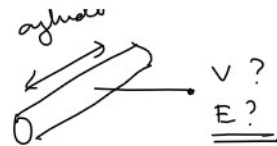


## CYLINDRICAL

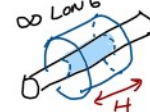
## CAPACITOR



$$C = \frac{Q}{\Delta V} \rightarrow ?$$

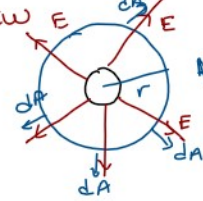


Ch 22 Gauss Law



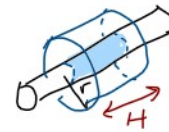
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = \frac{\lambda H}{\epsilon_0}$$

TOP VIEW



$$\oint \vec{E} \cdot d\vec{A} \cos 0$$

$$E \int dA + \underbrace{\int \vec{E} \cdot d\vec{A}}_{\text{TOP BOTTOM}} = 0$$

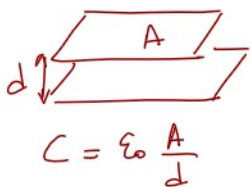


$$2\pi r H = \text{side area}$$

$$E 2\pi r H = \frac{\lambda H}{\epsilon_0} ; E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r} = -\int \frac{\lambda}{2\pi\epsilon_0} \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0} \left[ \ln r \right]_{r_i=a}^{r_f=b}$$

$$\Delta V = \frac{\lambda (\ln b - \ln a)}{2\pi\epsilon_0} = \frac{\lambda \ln(b/a)}{2\pi\epsilon_0}$$



$$C = \epsilon_0 \frac{A}{d}$$



$$C = 4\pi\epsilon_0 \frac{ab}{b-a}$$



$$C = 2\pi\epsilon_0 L / \ln(b/a)$$

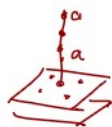
}  $\Phi, V$  ?!

capacitors in series & parallel connections.  $\equiv$  CIRCUIT (device)

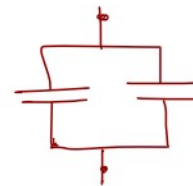
capacitors are circuits elements



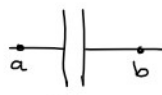
SERIES connection



parallel



$\Leftrightarrow$

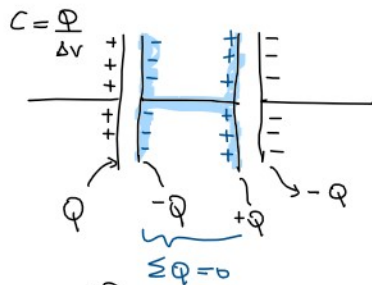
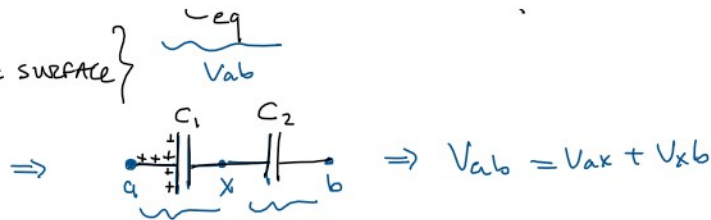
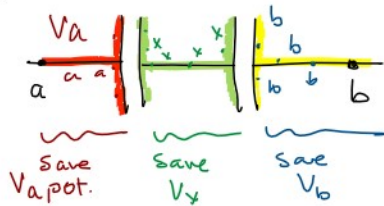


$C_{eq} =$  equivalent capacity value. (esp. deeper)

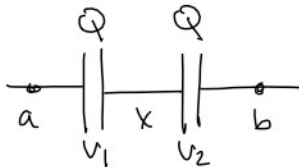
{ METALS HAVE THE SAME POTENTIAL VALUE ON THE SURFACE }



METALS HAVE THE SAME POTENTIAL VALUE ON THE SURFACE



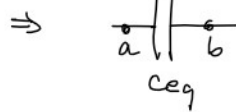
series capacitors will ALWAYS have the same amount of charge Q



$$V_{ax} + V_{xb} = V_{ab}$$

$$V_1 + V_2 = V_{ab}$$

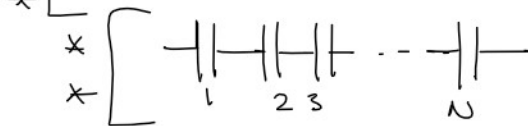
$$\frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{eq}}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

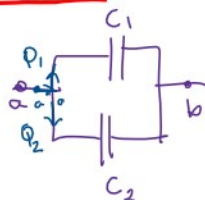
Cap. in series connection



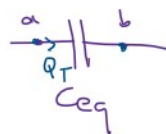
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

**SERIES CONNECTION**

Capacitors in parallel connection



⇒



wire

same pot. on the surface.

\* metals have equipotential surface.

$$V_{ab} = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = V_{ab}$$

$$Q_1 + Q_2 = Q_T$$

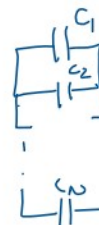
$$C_{eq} = \frac{Q_T}{V_{ab}}$$

$$Q_1 + Q_2 = Q_T$$

$$C_1 V_{ab} + C_2 V_{ab} = C_{eq} V_{ab}$$

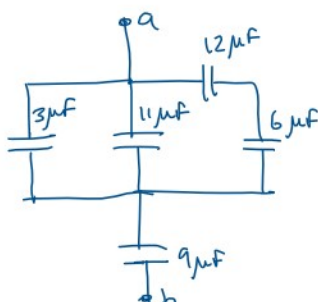
$$C_1 + C_2 = C_{eq}$$

parallel.

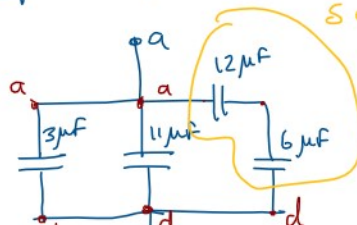


$$C_{eq} = C_1 + C_2 + \dots + C_N$$

**parallel connection**



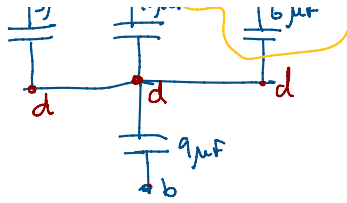
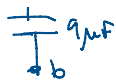
$$C_{eq} = C_{ab} = ?$$



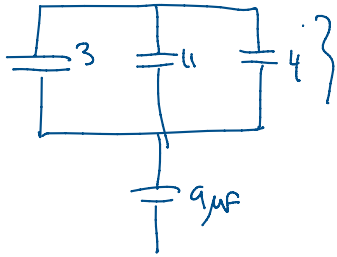
series

$$\left( \frac{1}{12} + \frac{1}{6} \right)^{-1} = \frac{6 \cdot 12}{6 + 12} = 4 \mu F$$

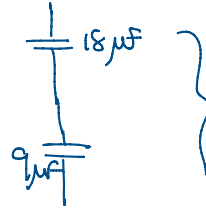
$$\frac{C_1 C_2}{C_1 + C_2} = C_{eq}$$



$$\frac{C_1 C_2}{C_1 + C_2} = C_{eq}$$



$$3 + 11 + 4 = 18 \mu F$$



$$\frac{18(9)}{18+9} = 6 \mu F = C_{ab}$$

If  $V_{ab} = 120 V \Rightarrow$  find charges at each capacitor!

120V { 6 μF } Q

$$Q = C V$$

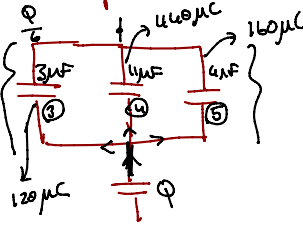
$$= 6(120)$$

$$= 720 \mu C$$

18 μF { Q } 40V  
9 μF { Q } 80V

$$V_1 = \frac{720}{9} = 80 V$$

$$V_2 = \frac{720}{18} = 40 V$$



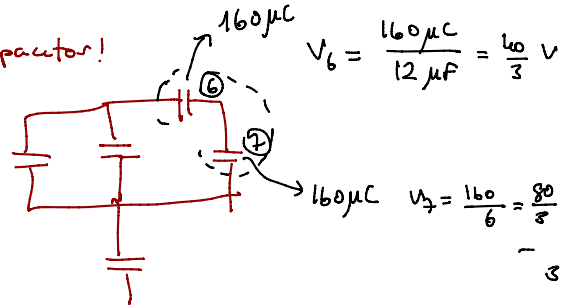
$$Q_3 = (40V) 3 \mu F$$

$$= 120 \mu C$$

$$Q_4 = (40V) 11 \mu F$$

$$= 440 \mu C$$

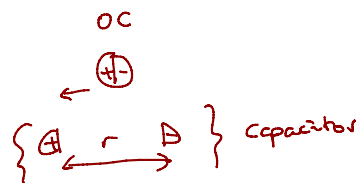
$$Q_5 = (40V) 4 = 160 \mu C$$



$$V_6 = \frac{160 \mu C}{12 \mu F} = \frac{40}{3} V$$

$$V_7 = \frac{160}{6} = \frac{80}{3}$$

capacitors store charge energy

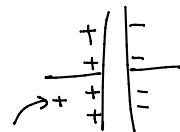


Energy of a capacitor

$$W = -\Delta U = -\Delta(qV) = -q \Delta V = \int \vec{F} \cdot d\vec{r} = q \int E \cdot d\vec{r}$$

$$C = \frac{Q}{\Delta V}$$

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2} (C \Delta V) \Delta V = \frac{1}{2} C (\Delta V)^2$$



$$U = q V$$

related

$$Q = C V$$

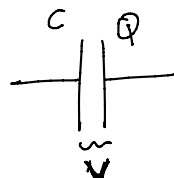
$$\int q dV$$

$$\int q \frac{dq}{C} = \frac{1}{C} \int q dq = \frac{q^2}{2C} \equiv U$$

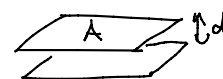
$$V C = q \Rightarrow dV C = dq$$

$$dV = \frac{dq}{C}$$

$$U = \frac{Q^2}{2C} = \frac{C V^2}{2} = \frac{Q V}{2}$$



$$V \equiv \Delta V$$



Energy density of a capacitor =  $\frac{\text{Energy}}{\text{volume}} = \frac{C \frac{V^2}{2}}{A d} = \frac{(\epsilon_0 \frac{A}{d}) \frac{V^2}{2}}{A d} = \epsilon_0 \frac{V^2}{2 d^2}$



$$\text{Energy density of a capacitor} = \frac{\text{Energy}}{\text{volume}} = \frac{C \frac{V^2}{2}}{Ad} = \frac{\left(\epsilon_0 \frac{A}{d}\right) \frac{V^2}{2}}{Ad} = \epsilon_0 \frac{V^2}{2d^2}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{l} \quad d \left\{ \begin{array}{c} \downarrow \downarrow \downarrow E \\ \downarrow \downarrow \downarrow E \end{array} \right\} \Delta V = \epsilon_0 \frac{E^2 d^2}{2d^2} = \frac{\epsilon_0 \left(\frac{\sigma}{\epsilon_0}\right)^2}{2} = \frac{\sigma^2}{2\epsilon_0} = \left[ \frac{J}{m^3} \right]$$

$$V = \Delta V = +Ed$$

$$\sigma = \frac{Q}{A} = \text{charge density}$$

energy density of a parallel plate capacitor

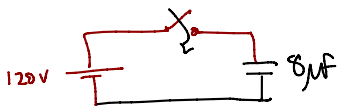
ex)  $C_1 = 8\mu F$  is charged by 120V battery; battery is disconnected

$C_1$  is connected an uncharged  $C_2 = 4\mu F$  capacitor.

what's the final charge distribution of  $C_1$  &  $C_2$ ?

" " final voltage " ?

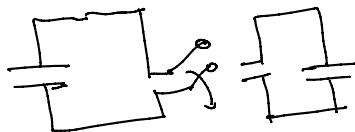
Discuss the total energy before and after charging  $C_2$ .



$$Q = (8\mu F)(120V) = 960\mu C \text{ total charge}$$

$$U = \frac{QV}{2} = \frac{(960)(120)}{2} = 57600 \text{ J}$$

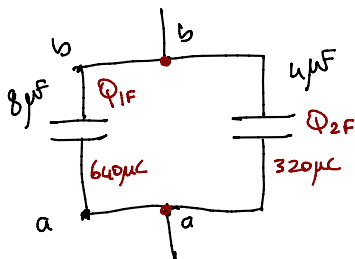
} initial



$$Q_{1i} = 960\mu C \quad Q_{2i} = 0$$

$$V_{1i} = 120V \quad V_{2i} = 0$$

} initial



\* total charge will be conserved.

$$Q_{1F} + Q_{2F} = Q_T = 960\mu C$$

$$V_{ab} = V_{1F} = V_{2F} \Rightarrow Q_{1F} = V_{ab} C_1$$

$$Q_{2F} = V_{ab} C_2$$

$$V_{ab} C_1 + V_{ab} C_2 = 960\mu C \Rightarrow V_{ab} = \frac{960\mu C}{C_1 + C_2} = \frac{960\mu C}{12\mu F}$$

$$Q_{1F} = V_{ab} C_1 = 80(8) = 640\mu C$$

$$\underline{V_{ab} = 80V}$$

$$Q_{2F} = V_{ab} C_2 = 80(4) = 320\mu C$$

|       | initial | final   |
|-------|---------|---------|
| $V_1$ | 120V    | 80V     |
| $V_2$ | 0V      | 80V     |
| $Q_1$ | 960μC   | 640μC   |
| $Q_2$ | 0       | 320μC   |
| $U_1$ | 57600   | 25600   |
| $U_2$ | 0       | 12800   |
|       | 57600 J | 38400 J |

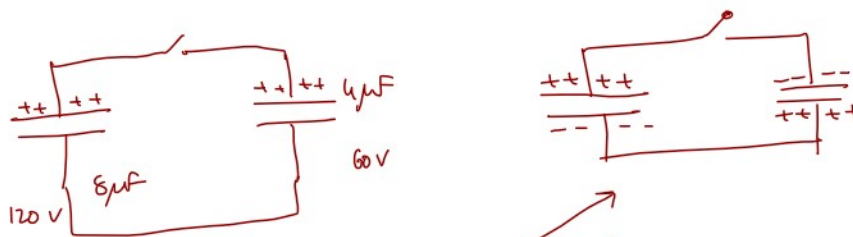
energy is NOT conserved?!

$$U_{1F} = \frac{Q_{1F} V_{ab}}{2} = \frac{640(80)}{2} = 25600 \text{ J}$$

$$U_{2F} = \frac{Q_{2F} V_{ab}}{2} = \frac{320(80)}{2} = 12800 \text{ J}$$

$$\frac{576 - 384}{576} = \% 33 \text{ is lost}$$

$$\underline{\underline{\text{Final}}} \quad \underline{\underline{38400 \text{ J}}}$$

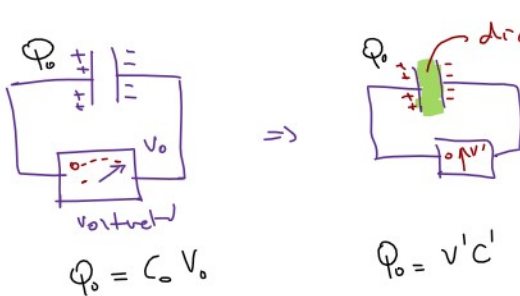


$$Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$$

$$\begin{aligned} Q_{60} &+ Q_{120} \\ Q_{60} &- Q_{120} \end{aligned}$$

## DIELECTRICS (isolators)

Dielectric used to increase the capacity of capacitors



$$V' < V_0$$

$$C' > C_0$$

$$C' = \kappa C_0$$

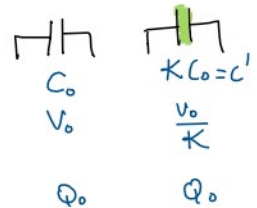
$\rightarrow$  Kappa (dielectric const.)

$$\kappa > 1$$

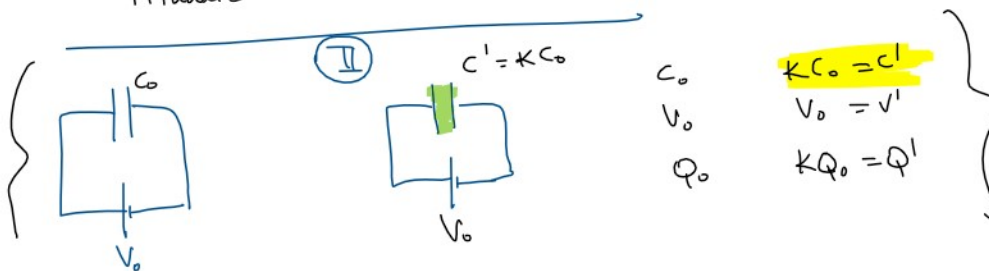
|                    | $\kappa$                    |
|--------------------|-----------------------------|
| vacuum             | $\rightarrow 1.00000 \dots$ |
| air                | $\rightarrow 1.0006$        |
| teflon             | $\rightarrow 2.1$           |
| glass              | $\rightarrow 5 - 10$        |
| glycerin           | $\rightarrow 42.5$          |
| strontium titanate | $\rightarrow 310$           |

$$\textcircled{I} \left\{ \begin{aligned} C' &= C_0 \kappa \\ V' &= V_0 \frac{1}{\kappa} \\ Q_0 &= Q' \end{aligned} \right.$$

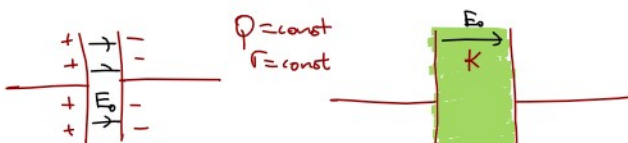
$$V_0 = \kappa V'$$



$$Q = CV$$



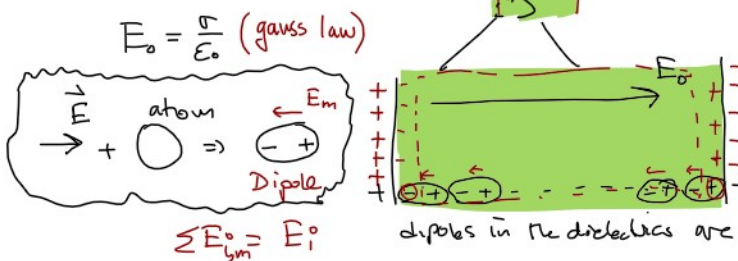
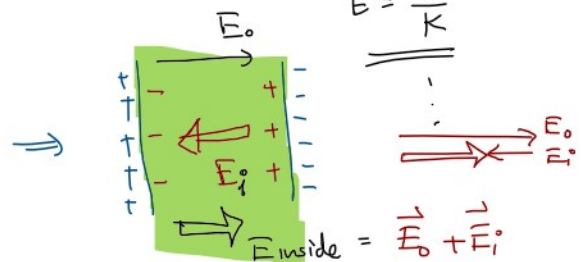
what's happening to  $\vec{E}$  inside of a capacitor when dielectric mat. is filled?



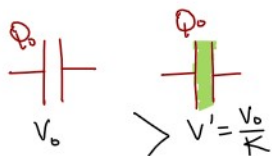
$$\vec{E}' = \vec{E}_0 + \vec{E}_i$$

$$E' < E_0$$

$$E' = \frac{E_0}{\kappa}$$



dipoles  
 $\sum \vec{E}_{i, \text{ind}} = \vec{E}_i$   
 dipoles in the dielectrics are induced by  $E_0$



$$\Delta V = - \int \vec{E} \cdot d\vec{r}$$

$$\downarrow$$

$$\frac{V_0}{K} \Rightarrow \frac{E_0}{K}$$

$$\vec{E}_{\text{inside}} = \vec{E}_0 + \vec{E}_i$$

$$E_{\text{inside}} < E_0$$

$$\vec{E}_{\text{inside}} = \vec{E}' = \vec{E}_0 + \vec{E}_i \quad E_i = |\vec{E}_i|$$

$$E' = E_0 - E_i = \frac{E_0}{K}$$

$$E_0 - E_i = \frac{E_0}{K} \Rightarrow E_i = \left( \frac{K-1}{K} \right) E_0 \Rightarrow \frac{\sigma_i}{\epsilon_0} = \frac{K-1}{K} \frac{\sigma_0}{\epsilon_0}$$

$$\sigma_i = \frac{K-1}{K} \sigma_0$$

$$\sigma_i < \sigma_0$$

$$Q_i < Q_0$$

$$K = 1.5 \Rightarrow \sigma_i = \frac{0.5}{1.5} \sigma_0 \Rightarrow$$

$$\sigma_i \approx (33\%) \sigma_0$$

$$\left[ \sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{A} \right]$$

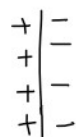
$$Q_i = \frac{K-1}{K} Q_0$$

$$\Rightarrow Q_i \approx Q_0$$

$$K \gg 1$$

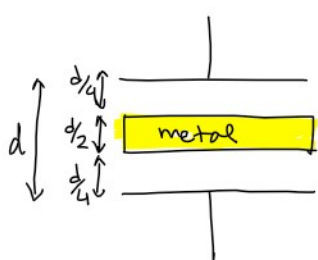
$$\frac{299}{300} \approx 1 = \frac{K-1}{K}$$

$K \rightarrow \infty$  material behaves like metal



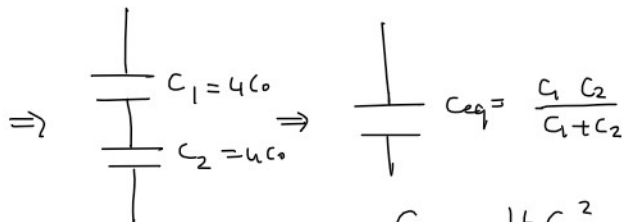
$$\left\{ \begin{array}{l} C' = K C_0 \rightarrow V_0 = \text{const} \quad Q' = K Q_0 \\ \rightarrow Q_0 = \text{const} \quad V' = \frac{V_0}{K} \end{array} \right\}$$

$Q_0 \approx Q_i \Rightarrow \text{metal property}$   
 END of chapter

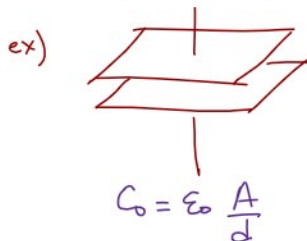


$$C_0 = \epsilon_0 \frac{A}{d}$$

$$C_1 = \epsilon_0 \frac{A}{d/4} = 4 \epsilon_0 \frac{A}{d} = 4 C_0 = C_2$$

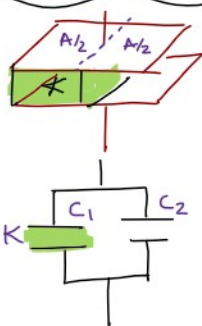


$$C_{eq} = \frac{16 C_0^2}{4 C_0 + 4 C_0} = 2 C_0$$



$$C_0 = \epsilon_0 \frac{A}{d}$$

$\Rightarrow$



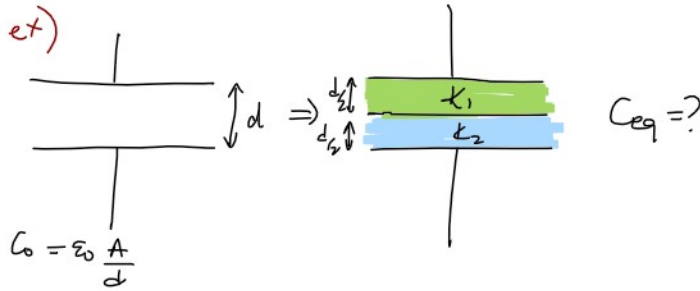
$C_{eq} = ?$

$$C_{eq} = C_1 + C_2$$

$$= \epsilon_0 \frac{A/2}{d} K + \epsilon_0 \frac{A/2}{d}$$



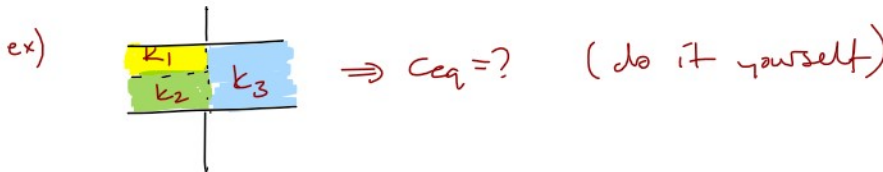
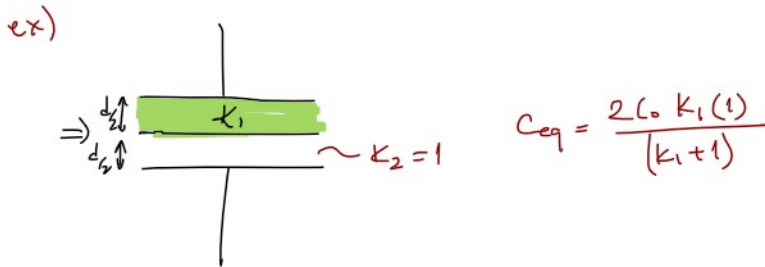
$$C_{eq} = \frac{\epsilon_0 A}{2d} (k+1) = \frac{C_0}{2} (k+1)$$



$$C_1 = k_1 \epsilon_0 \frac{A}{d/2} = 2k_1 C_0$$

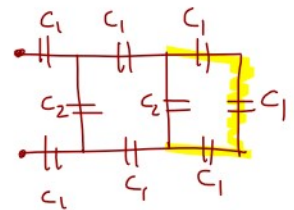
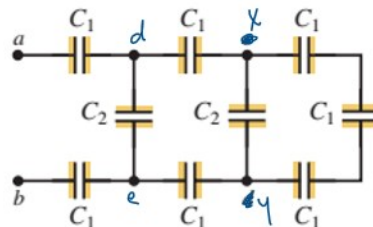
$$C_2 = k_2 \epsilon_0 \frac{A}{d/2} = 2k_2 C_0$$

$$\frac{C_1 C_2}{C_1 + C_2} = C_{eq} = \frac{4k_1 k_2 C_0^2}{2C_0 (k_1 + k_2)} = \frac{2C_0 k_1 k_2}{k_1 + k_2}$$



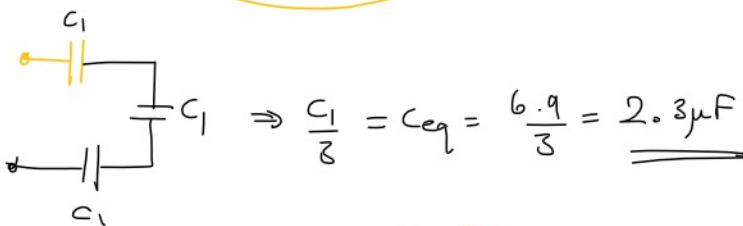
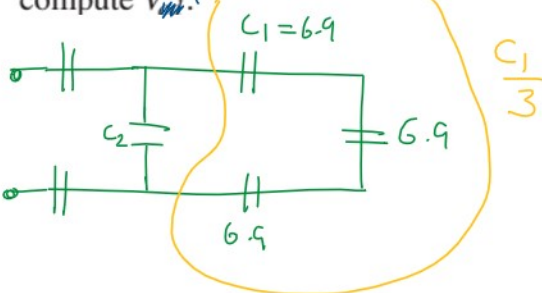
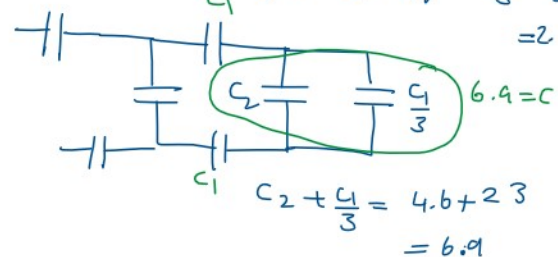
**24.63** •• In Fig. P24.63, each capacitance  $C_1$  is  $6.9 \mu\text{F}$ , and each capacitance  $C_2$  is  $4.6 \mu\text{F}$ . (a) Compute the equivalent capacitance of the network between points  $a$  and  $b$ . (b) Compute the charge on each of the three capacitors nearest  $a$  and  $b$  when  $V_{ab} = 420 \text{ V}$ . (c) With  $420 \text{ V}$  across  $a$  and  $b$ , compute  $V_{de}$ .

Figure **P24.63**



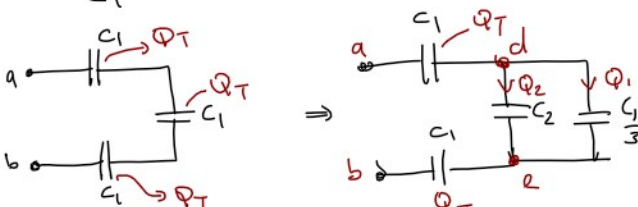
$$\left( \frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_1} \right)^{-1} = \frac{C_1}{3} = \frac{6.9}{3}$$

$$= 2.3$$



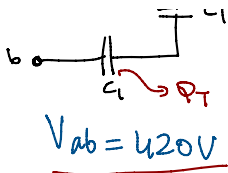
$$Q = (2.3 \mu\text{F}) (420 \text{ V})$$

$$Q_T = 966 \mu\text{C}$$



$$Q_2 + Q_1 = 966 \mu\text{C}$$

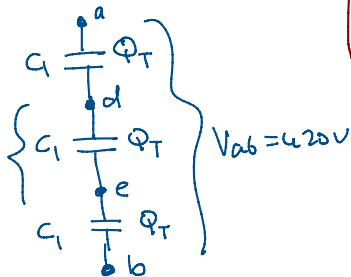
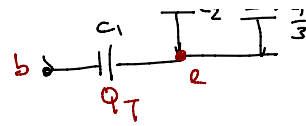
$$V_{de} = \frac{Q_2}{C_2} = \frac{Q_1}{C_1} \Rightarrow \frac{Q_2}{4.6} = \frac{Q_1}{6.9}$$



2nd way:

$$140V = V_{de} = \frac{V_{ab}}{3}$$

$$V_{de} = \frac{Q_T}{C_1} = \frac{966\mu C}{6.9\mu F} = 140V$$



$$V_{de} = \frac{Q_2}{C_2} = \frac{Q_1}{C_1/3} \Rightarrow \frac{Q_2}{4.6} = \frac{Q_1}{2.3}$$

$$Q_2 = 2Q_1$$

$$3Q_1 = 966\mu C$$

$$Q_1 = 322\mu C$$

$$Q_2 = 644\mu C$$

$$V_{de} = \frac{Q_2}{C_2} = \frac{Q_2}{4.6\mu F} = 140 \Rightarrow Q_2 = 644\mu C$$

$$V_{de} = \frac{Q_1}{C_1/3} \Rightarrow (140V)(2.3\mu F) \Rightarrow Q_1 = 322\mu C$$

