

so far:

vectors ✓ Ch1

motion → 1d Ch2
2d & 3d Ch3

Ch4 & Ch5 ⇒ Newton's Law of Motion

Ch2 } motion $\frac{\Delta x}{t}$ } vector
Ch3 } $\frac{v}{a}$

how to gain/lose acceleration?

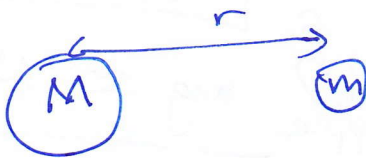
→ by force →



Force $\equiv F$

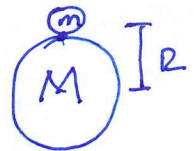
- gravitational force
 - Electromagnetic forces
 - weak force
 - strong force
- nuclear

directly
indirectly



$$F = G \frac{Mm}{r^2}$$

const.

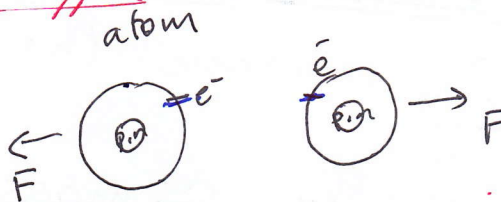


$$F = G \frac{M}{R^2} m = mg$$

$g = 9.8 \text{ m/s}^2$

$F_{\text{gravitational}} = mg$

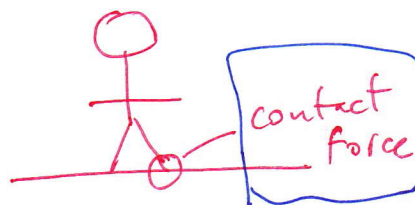
EM force →



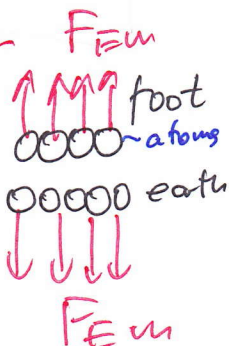
→ Contact forces
→ frictional forces.

$$F_{\text{Electrostatic}} = k \frac{q_1 q_2}{r^2}$$

(phys II)



$$F_{\text{Em}} = F_{\text{contact}}$$



$$g = \frac{MG}{R^2} = 9.8 \text{ m/s}^2 \text{ on earth.}$$

how Force is related to motion

acceleration
velocity
displacement

Force is a vector. $\rightarrow$ direction
 $\hookrightarrow$ magnitude

$$\vec{F} = [\text{Newton}]$$

gravitational forces.



my weight is $mg = \text{force}$

weight = apple $\equiv$ vector force.

scale shows $[80 \text{ kg} \equiv \text{mass}]$

weight $\neq$ mass

$$\text{weight} = 80 \times 9.8 = mg$$

$$[mg = 784 \text{ N}]$$

apple $mg \approx 1 \text{ N}$



sun earth

$$F \approx 3 \times 10^{22} \text{ N}$$

m_1
proton

m_2
 e^-

$$F_G \approx 10^{-47} \text{ N}$$

between e^- & proton

what's force? experience this "force" pulling pushing

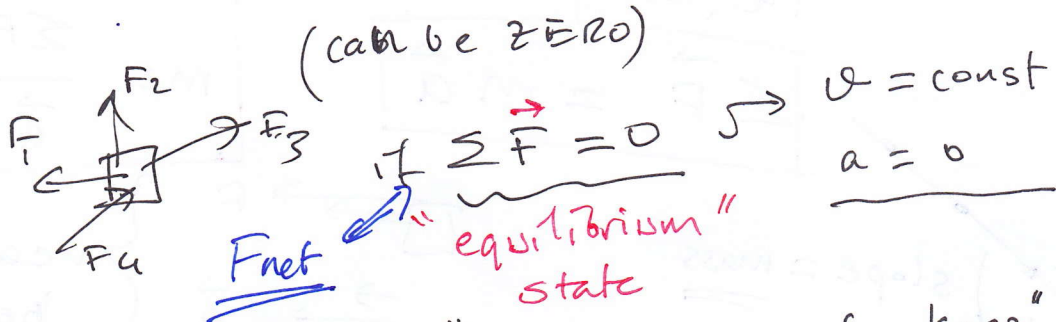
inertia $\equiv$ elements like $\approx$ mass = kille

Newton's Laws of Motion

(2)

1st Law (inertia)

A body acted on by no net force moves with constant velocity v and zero acceleration.



inertia = eylemsizlik = "the tendency of keep"
 = "durmus koruma eğilimi"

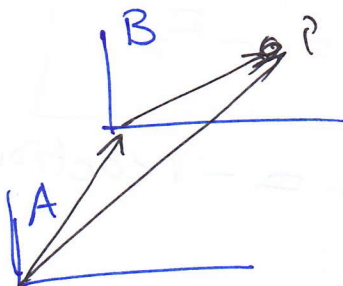
1st law $\Sigma \vec{F} = 0$ "you can keep your state"
 if $\Sigma \vec{F} \neq 0$ you can NOT keep your state
 your state will change"
 net force applied is the price you pay to win over inertia."

inertia is mass

$\Sigma \vec{F} > 0$ both masses move
 but it's HARDER to move big M.
 than the small m. (to win over inertia)

Relative motion (previous chapter)

were later



$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} \quad \downarrow \frac{d}{dt}$$

$$\vec{a}_{P/A} = \vec{a}_{P/B} + \vec{a}_{B/A}$$

$\Sigma \vec{F} = 0 \Leftrightarrow \left\{ \begin{array}{l} \vec{a}_{B/A} = 0 \text{ when} \\ \text{inertial frame} \\ \text{or reference.} \end{array} \right.$

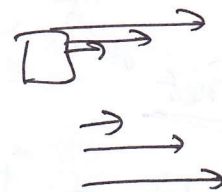
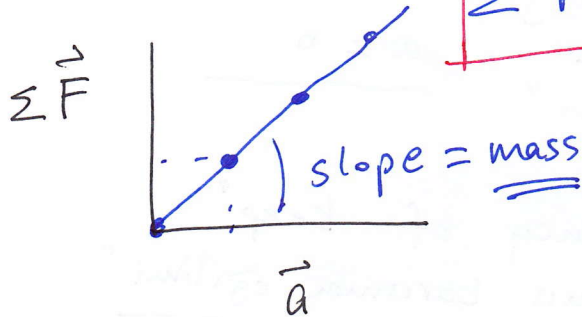
mass ~ inertia

2nd Law: if a net force acts on a body;
total

the body accelerates.

$$\sum \vec{F} = m \vec{a}$$

$$m = \frac{\sum \vec{F}}{\vec{a}}$$



measure both
calculate mass

force = (mass)(acceleration)

$$\left[\text{Newton} = \text{kg} \frac{\text{m}}{\text{s}^2} \right]$$

$$\sum \vec{F} \neq \text{mass}$$

weight $\neq$ mass
apirlık $\neq$ kütle

3rd Law: Two bodies interacting

If A exerts a force on B

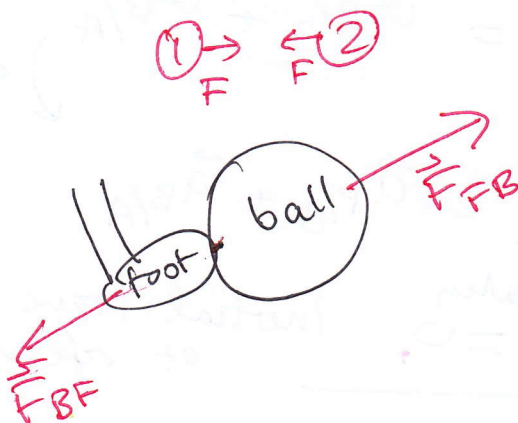
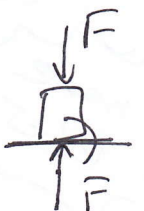
B will exert the same force on A
in the opposite direction



$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

action = - reaction



1st Law

$$\sum \vec{F} = 0 \quad v = \text{const} \\ a = 0$$

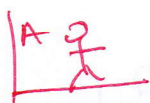
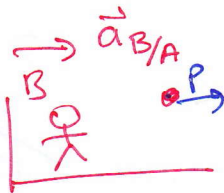
inertial

inertial reference frame

you. $\vec{a} = 0$; when you measure it.

2nd Law

$$\sum \vec{F} = m\vec{a}$$



3rd Law

action $\Rightarrow$ reaction

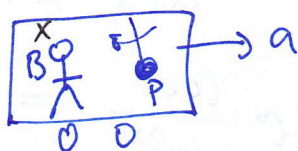
when $\vec{a}_{B/A} \neq 0$

what B measures

$\neq$

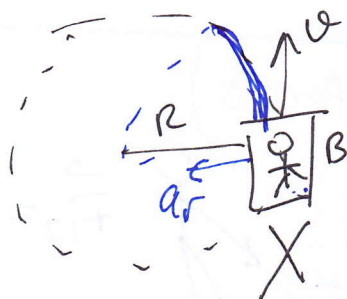
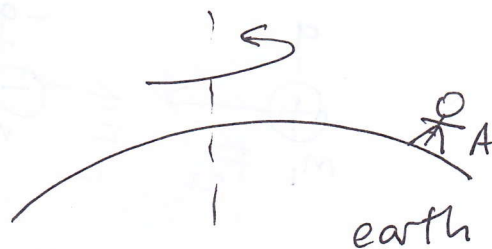
what A measures.

$\Rightarrow$ never measure something when you have acceleration.



$\checkmark$ $\underline{A} \equiv$ inertial ref. frame.

$\Rightarrow$



$\checkmark$ ground $\equiv$ earth $\equiv$ inertial reference frame.



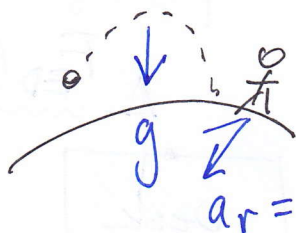
$$a_r = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2} \quad \left(\begin{array}{l} R = 6400 \text{ km} \\ T = \text{period} \end{array} \right)$$

$$a_r \approx 0.03 \text{ m/s}^2$$

$$T = 24 \times 60 \times 60 \text{ s}$$

$$g \gg a_r = 0.03 \text{ m/s}^2$$

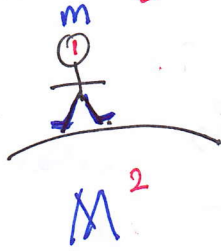
it's safe to assume that earth is an inertial frame of reference.



$$a_r = 0.03 \text{ m/s}^2$$

3rd Law :

action - reaction ~~pair~~ force pairs.



$$F \downarrow \downarrow mg \downarrow a_1$$

$$F_{21} = F = m a_1$$

$$mg = m a_1$$

$$F \uparrow \uparrow mg \text{ earth } \uparrow a_2$$

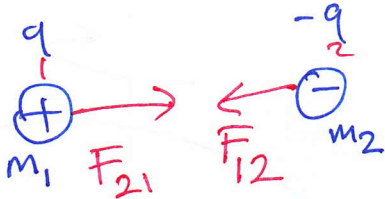
$$g = a_1$$

(2)

$$F_{12} = M a_2$$

$$mg = M a_2$$

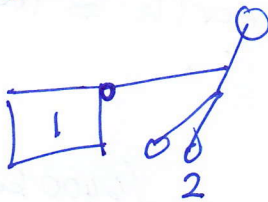
$$0 \approx 10^{-21} \frac{m}{s^2} \frac{1000}{10^{24}} = a_2$$



$$a_1 m_1 = F = a_2 m_2$$

Concat

contact forces (EM forces)

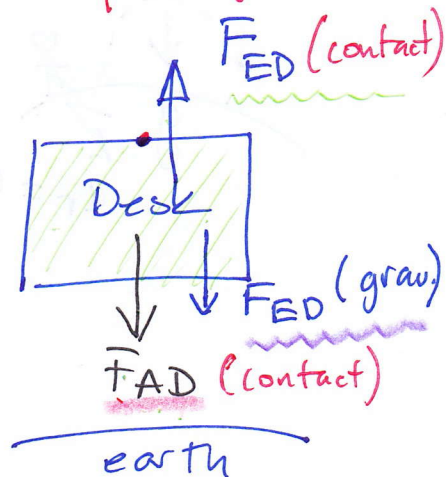
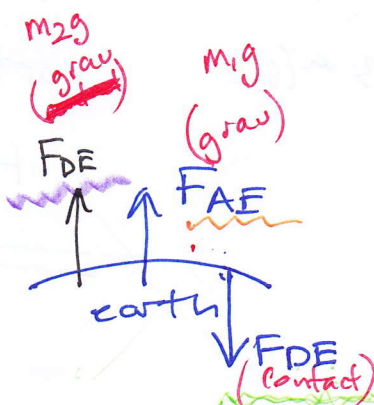
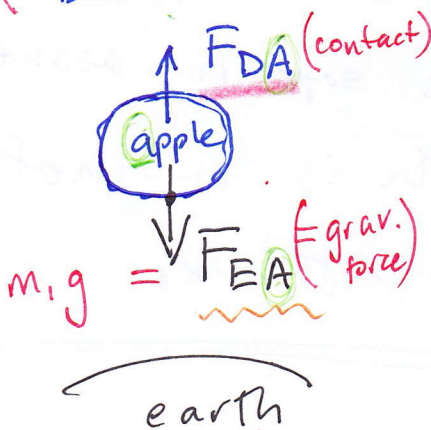


2 types of forces

↳ gravitation

↳ contact.

{ grav. force between apple & desk is neglected }



12.11.20

①

Newton's Law of Motion

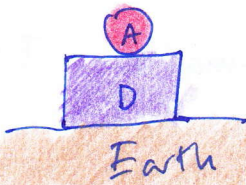
1st inertia (eg lenisiglib)

2nd $\Sigma \vec{F} = m\vec{a}$

$$m = \frac{\Sigma \vec{F}}{\vec{a}}$$

3rd action-reaction

Diagram showing two objects, 1 and 2, with forces $\vec{F}_{21}$ and $\vec{F}_{12}$ acting between them. The equation $\vec{F}_{12} = -\vec{F}_{21}$ is written below.



show forces on each object

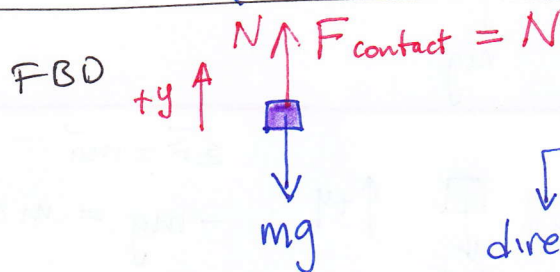
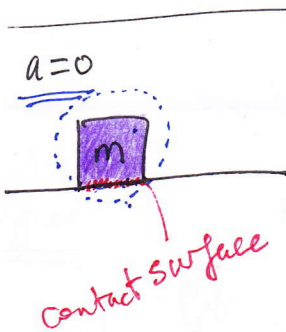
Free Body Diagram (FBD)
(serbest cisim diyagrami)

4 types of forces

- 1 - gravitational $\Rightarrow mg$
- 2 - EM forces (contact forces, frictional forces)
- 3-4 weak-strong nuclear force.

* we always neglect grav. force between small masses. ($m_1, m_2 \dots$) $\ll M_{\text{earth}}$

* contact forces $\rightarrow$ tension in the string (iptelci gicilve)
 $\rightarrow$ Normal force due to contact (perpendicular $\perp$ to the surface)



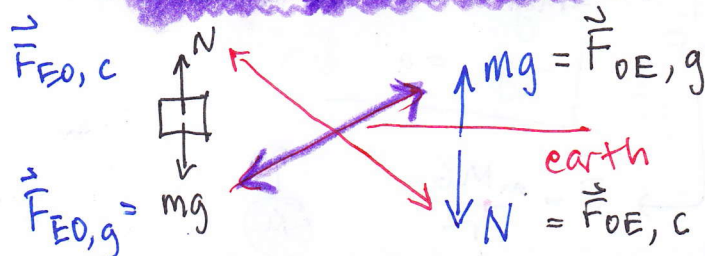
$$\Sigma \vec{F} = m\vec{a}$$

directions; decide on +y down +x

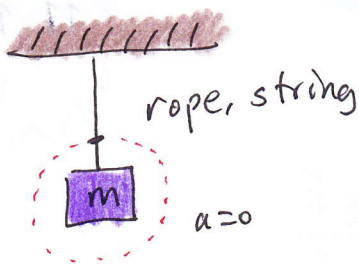
$$N - mg = m\vec{a}^0 = 0$$

$$N = mg$$

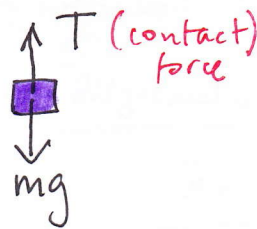
* N, mg are NOT action-reaction pairs!



action-reaction pairs will be on different objects.



FBD

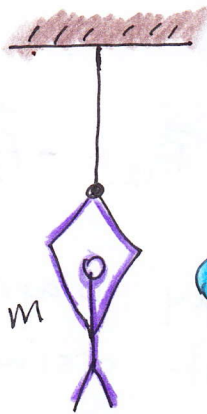


$$\Sigma \vec{F} = m\vec{a}$$

$$+\uparrow \quad T - mg = m \cdot 0$$

$$| T = mg$$

T, mg are NOT action-reaction pairs.



if you are stationary
 $a=0$



$$T - mg = 0 \quad T = mg$$

②

If you pull yourself up
 $a > 0$



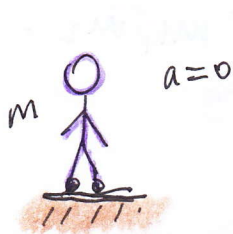
$$\Sigma \vec{F} = m\vec{a}$$

$$T' - mg = ma$$

$$T' = mg + ma$$

when you pull up!
 $T' > T$

when you stay still!

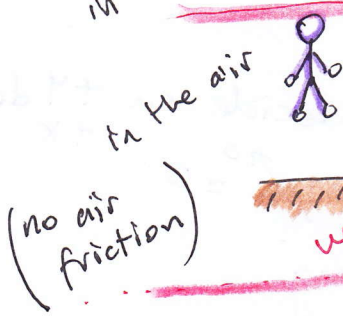


$$N = mg$$

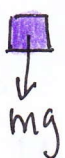
$$N' > N$$



$$N' - mg = ma \quad | \quad N' = mg + ma$$



$g \downarrow$



$+\uparrow$

$$\Sigma \vec{F} = m\vec{a}$$

$$-mg = ma$$

$$| -g = a$$

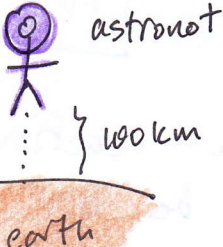
when $N=0$
weightless

similar

$$g = \frac{MG}{R^2}$$

$$R = 6400 \text{ km}$$

$$g' = \frac{MG}{(6500)^2}$$



$+\uparrow$

$$mg' = F$$

$$\Sigma \vec{F} = m\vec{a}$$

$$-mg' = ma$$

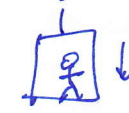
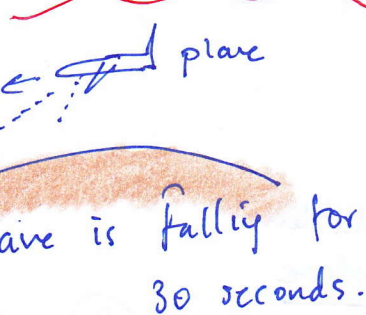
$$| -g' = a$$

weightless

$$F = \frac{mMG}{r^2}$$



weightless situations on earth



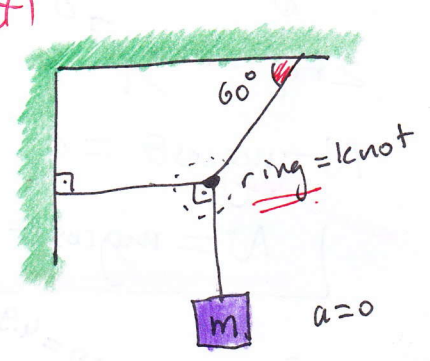
elevator falling = plane falling } weightless situations

$$h = \frac{1}{2} g t^2 \approx \frac{10}{2} \cdot 900 = 4500m$$



both person & elevator fall with g !
no contact between them!

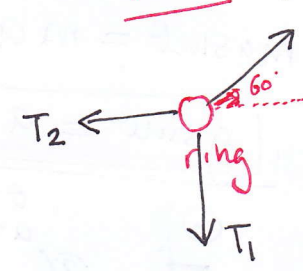
ex)



mass of the ring ≈ 0

system is in equilibrium $\Rightarrow \Sigma \vec{F} = 0$
Tensions on the ropes? $\underline{a=0}$

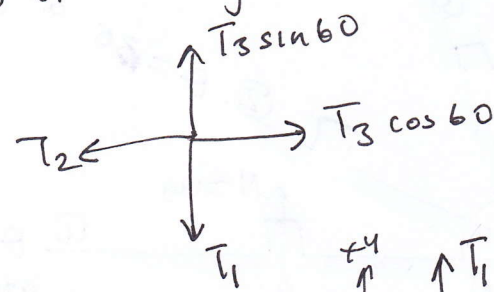
FBD of ring



$$\Sigma \vec{F} = m\vec{a}$$

$$\Sigma F_x = m a_x$$

$$\Sigma F_y = m a_y$$



FBD of the object

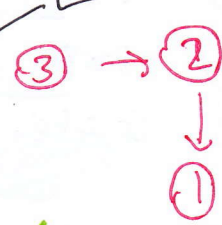
$$\Sigma \vec{F} = m\vec{a}$$

$$T_1 - mg = 0$$

$$T_3 \cos 60 - T_2 = 0 \quad (1)$$

$$T_3 \sin 60 - T_1 = 0 \quad (2)$$

$$T_1 = mg \quad (3)$$

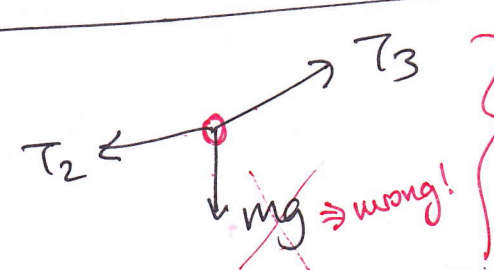


$$T_3 \cos 60 = T_2$$

$$T_3 \sin 60 = T_1 = mg$$

$$T_3 = \frac{mg}{\sin 60} = \frac{2mg}{\sqrt{3}}$$

$$\frac{mg}{\sin 60} \cos 60 = T_2 = \frac{mg}{\sqrt{3}}$$

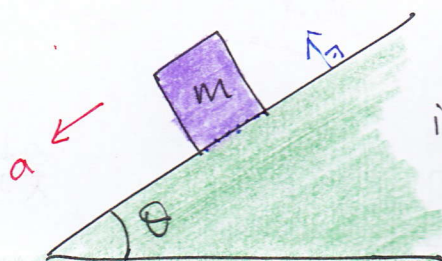


not FBD of the ring
X FALSE!



CORRECT; TRUE ✓

ex)

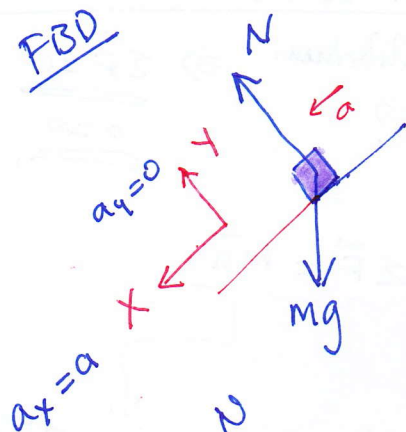


egik düzlem
inclined plane
no friction

① FBD of the object.

② acceleration of the object?

FBD



decide on
X, Y

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x$$

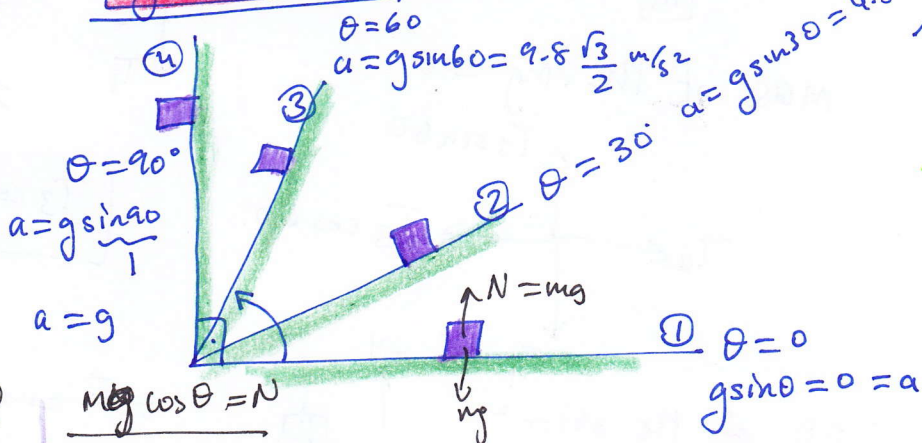
$$mg \sin \theta = ma$$

$$g \sin \theta = a$$

$$\sum F_y = ma_y = 0$$

$$N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$



	θ	$a = g \sin \theta$	$N = mg \cos \theta$
①	$\theta = 0$	$a = 0$	mg
②	$\theta = 30^\circ$	$a = \frac{g}{2}$	$mg \frac{\sqrt{3}}{2}$
③	$\theta = 60^\circ$	$a = \frac{g}{2} \sqrt{3}$	$\frac{mg}{2}$
④	$\theta = 90^\circ$	$a = g$	

falling down
without
touching
the surface.

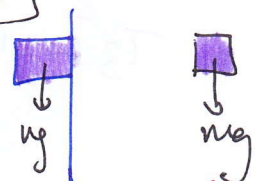
$$N = mg \cos 90 = 0$$

$$a = g$$

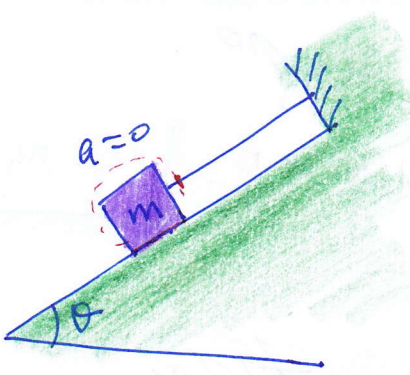
$$N = 0$$

no contact with
the surface!

FBD

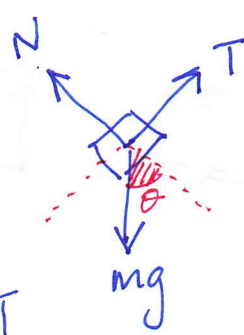


ex)



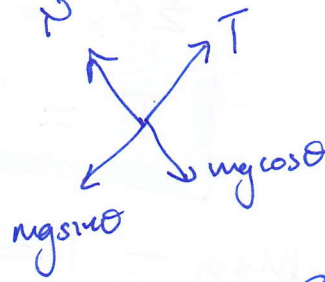
$T = ?$
on the rope

FBD?



(3)

surfaces	push
ropes	pull
strings	



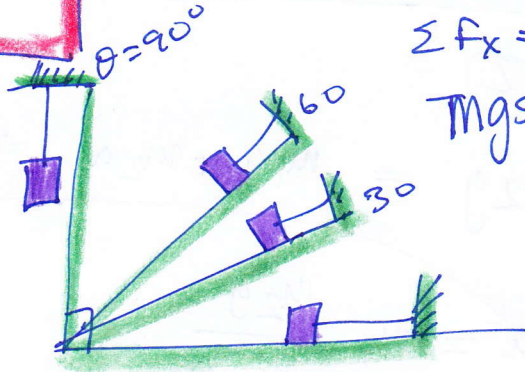
$$\sum F_y = m a_y$$

$$N = m g \cos \theta$$

$$\sum F_x = m a_x$$

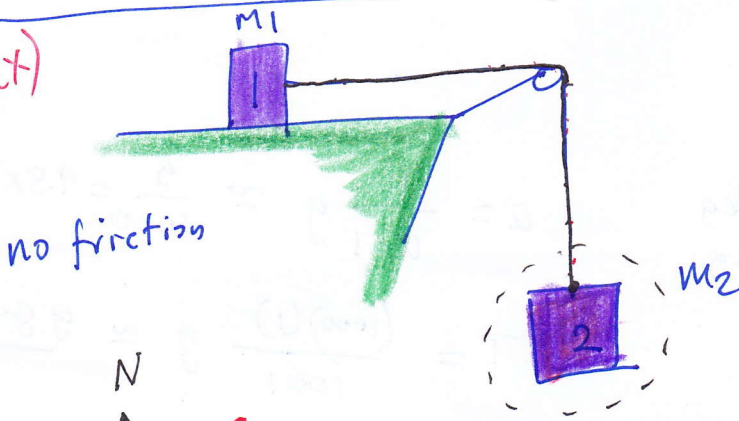
$$m g \sin \theta - T = 0$$

$$T = m g \sin \theta$$

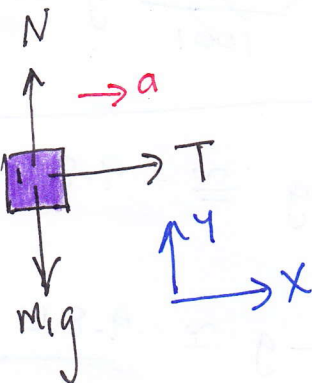


	T	N
$\theta = 0$	0	$m g$
$\theta = 30^\circ$	$\frac{m g}{2}$	$\frac{m g \sqrt{3}}{2}$
$\theta = 60^\circ$	$\frac{m g \sqrt{3}}{2}$	$\frac{m g}{2}$
$\theta = 90^\circ$	$m g$	0

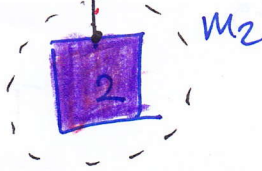
ex)



no friction



direction

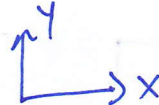


- FBD for both objects

- solve $\sum \vec{F} = m \vec{a}$ for both

find $a = ?$

find Tension on the ropes?

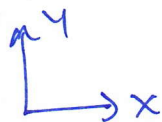
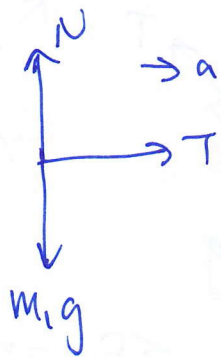


$$\sum F_y = m_2 a_y$$

$$-m_2 g + T = m_2 (-a)$$

$$T - m_2 g = -m_2 a$$

$$m_2 g - T = m_2 a$$



$$\sum F_y = m_1 a_y \rightarrow 0$$

$$N - m_1 g = 0 \quad \boxed{N = m_1 g}$$

$$\sum F_x = m_1 a_x$$

$$\boxed{T = m_1 a} \quad (1)$$

$$\boxed{m_2 g - T = m_2 a} \quad (2)$$

$$(1) + (2)$$

$$\rightarrow m_2 g = m_1 a + m_2 a \Rightarrow \boxed{a = \frac{m_2 g}{m_1 + m_2}}$$

into (1)

$$\boxed{T = m_1 a = m_1 \frac{m_2 g}{m_1 + m_2}}$$

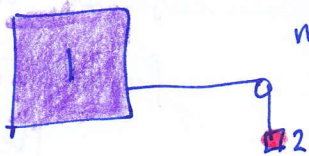
$$\boxed{T = \frac{m_1 m_2}{m_1 + m_2} g}$$

$$\boxed{a = \frac{m_2}{m_1 + m_2} g}$$

what if $m_1 \gg m_2$

$$m_1 = 1000 \text{ kg}$$

$$m_2 = 1 \text{ kg}$$



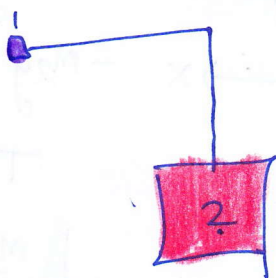
$$\underline{a = \frac{1}{1001} g \approx \frac{g}{1000} = 9.8 \times 10^{-3} \text{ m/s}^2}$$

$$\underline{T = \frac{(1000)(1)}{1001} g \approx \underline{9.8 \text{ N}}}$$

$m_2 \gg m_1$

$$m_1 = 1 \text{ kg}$$

$$m_2 = 1000 \text{ kg}$$



$$\underline{a = \frac{1000}{1001} g \approx \underline{9.8 \text{ m/s}^2}}$$

$$\underline{T = \frac{1000}{1001} g \approx \underline{9.8 \text{ N}}}$$

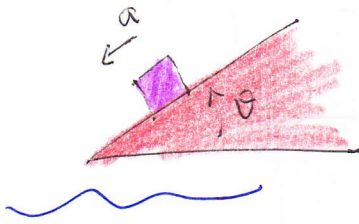
FBD

→ mg

→ contact forces

→ tension on the string
→ normal force

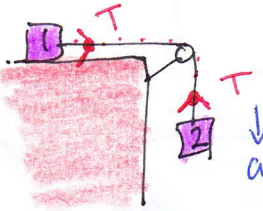
(pulls)
(pushes)



$$a = g \sin \theta$$

$$N = mg \cos \theta$$

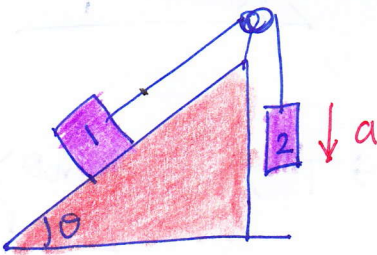
$$\theta = 0^\circ \rightarrow 90^\circ$$



Tension in the string is always the same value

$$a = \frac{m_2 g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2}{(m_1 + m_2)} g$$



m_1
 m_2
 θ
 g are given
 $a = ?$
 $T = ?$

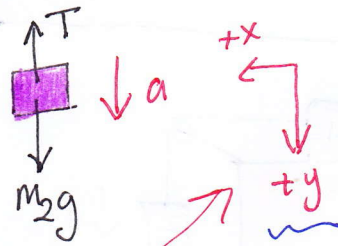
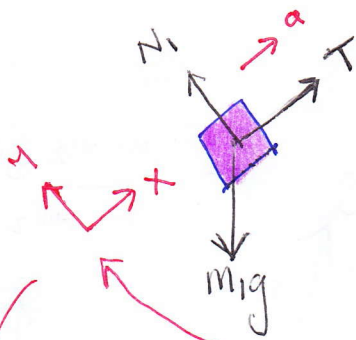
FBD

$$\Sigma \vec{F} = m \vec{a}$$

$$\Sigma \vec{F} = m_2 \vec{a}$$

$$\Sigma F_y = m_2 a_y$$

$$m_2 g - T = m_2 a \quad (1)$$



decide on coordinates separately for m_1 & m_2

$$\Sigma F_x = m_1 a_x$$

$$T - m_1 g \sin \theta = m_1 a \quad (2)$$

$$(1) + (2)$$

$$m_2 g - m_1 g \sin \theta = (m_1 + m_2) a$$

$$a = \left(\frac{m_2 - m_1 \sin \theta}{m_1 + m_2} \right) g$$

$$\Sigma F_y = m_1 a_y$$

$$N_1 - m_1 g \cos \theta = 0$$

$$N_1 = m_1 g \cos \theta \quad (3)$$

$$(2) \quad T = m_2 (g - a)$$

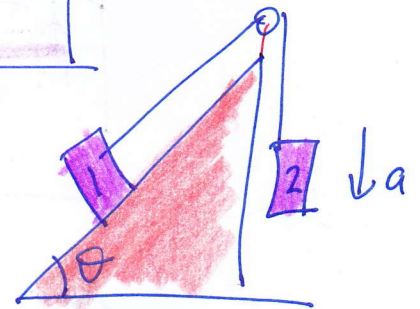
$$T = m_2 g \left(1 - \frac{m_2 - m_1 \sin \theta}{m_1 + m_2} \right)$$

$$T = m_2 g \left(1 - \frac{m_2 - m_1 \sin \theta}{m_1 + m_2} \right)$$

$$= m_2 g \left(\frac{m_1 + \cancel{m_2} - \cancel{m_2} + m_1 \sin \theta}{m_1 + m_2} \right)$$

$$T = \frac{m_1 m_2 g (1 + \sin \theta)}{m_1 + m_2}$$

$$a = \frac{m_2 - m_1 \sin \theta}{m_1 + m_2} g$$

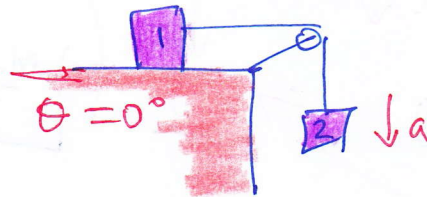


$$a > 0 ; m_2 > m_1 \sin \theta \quad \downarrow a$$

$$\text{If } a < 0 ; m_1 \sin \theta > m_2 \quad \uparrow a$$

$$T > 0 ; \sin \theta > 0 \quad [0 - 1]$$

previous lecture



$$a = \frac{m_2}{m_1 + m_2} g \quad T = \frac{m_1 m_2 g}{m_1 + m_2}$$

same

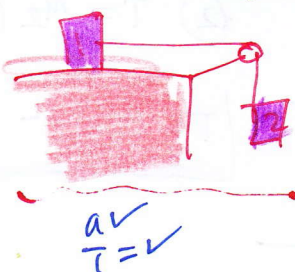
$$\rightarrow \theta = 0^\circ$$

$$a = \frac{m_2 - m_1 \sin 0}{m_1 + m_2} g = \frac{m_2 g}{m_1 + m_2} \checkmark$$

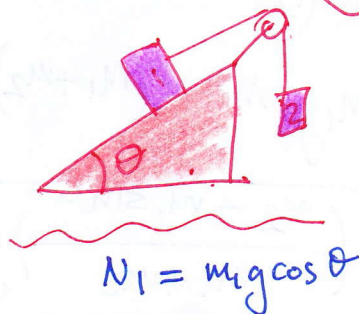
$$T = \frac{m_1 m_2 g (1 + \sin 0)}{m_1 + m_2} = \frac{m_1 m_2 g}{m_1 + m_2}$$

same

$$\theta = 0^\circ \quad N_1 = m_1 g \cos 0^\circ$$



$$\frac{a \checkmark}{T = \checkmark}$$

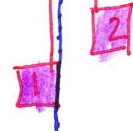


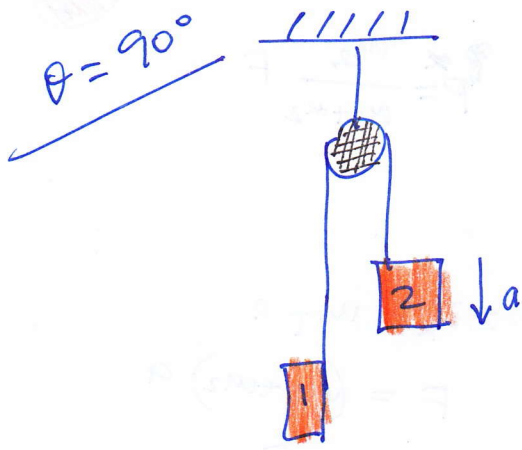
$$N_1 = m_1 g \cos \theta$$

$$\theta = 90^\circ$$

$$N_1 = m_1 g \cos 90^\circ$$

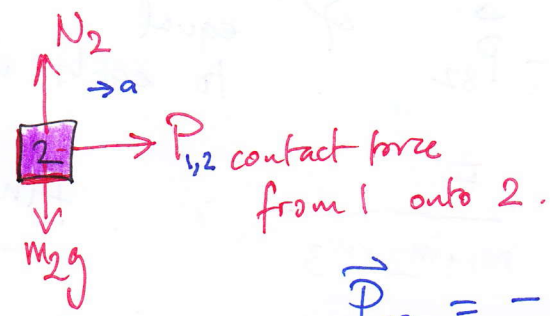
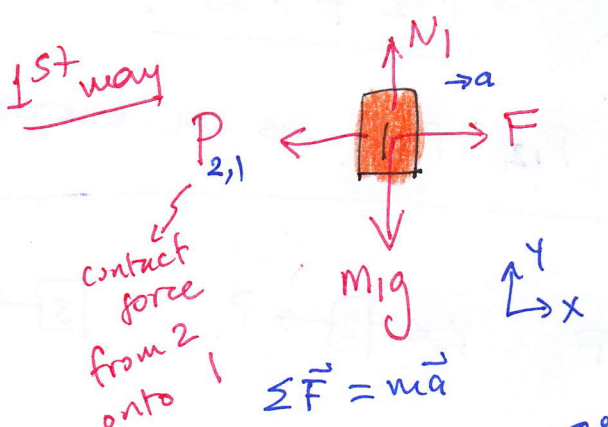
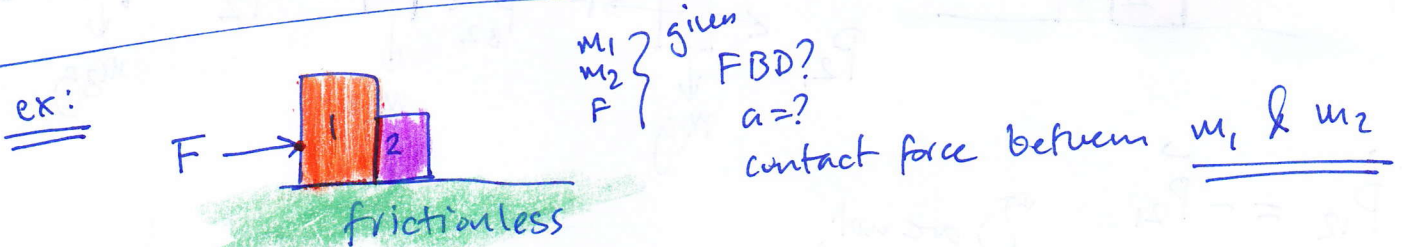
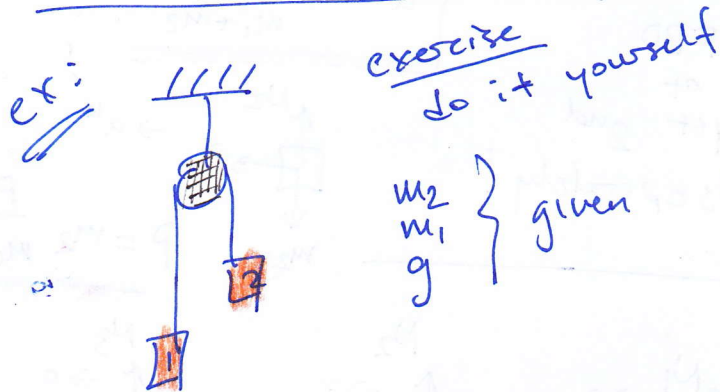
$$N_1 = 0$$





$$a = \frac{m_2 - m_1 \sin 90}{m_1 + m_2} g = \frac{m_2 - m_1}{m_1 + m_2} g$$

$$T = \frac{m_1 m_2 g (1 + \sin 90)}{m_1 + m_2} = \frac{2 m_1 m_2 g}{m_1 + m_2}$$



$\vec{P}_{12} = -\vec{P}_{21}$ (action reaction pairs)

$\sum F_x = m_1 a_x$ $\sum F_y = m_1 a_y$

① $F - P = m_1 a$ $N_1 = m_1 g$

$\sum F_x = m_2 a_x$ $\sum F_y = m_2 a_y$

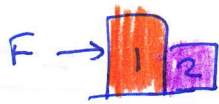
② $P = m_2 a$ $N_2 = m_2 g$

① + ② $\Rightarrow F - P + P = (m_1 + m_2) a$

$a = \frac{F}{m_1 + m_2}$

$P = m_2 \left(\frac{F}{m_1 + m_2} \right)$

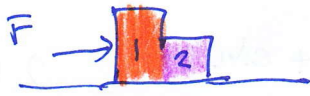
$P < F$ $P = \left(\frac{m_2}{m_1 + m_2} \right) F$



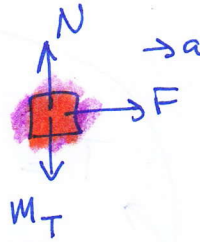
$$a = \frac{F}{(m_1 + m_2)}$$

$$P = \frac{m_2}{m_1 + m_2} F$$

2nd way



$$m_T = m_1 + m_2$$

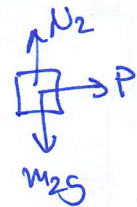


$$\Sigma F_x = m_T a$$

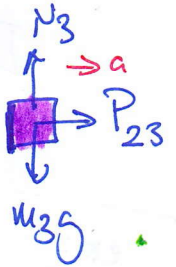
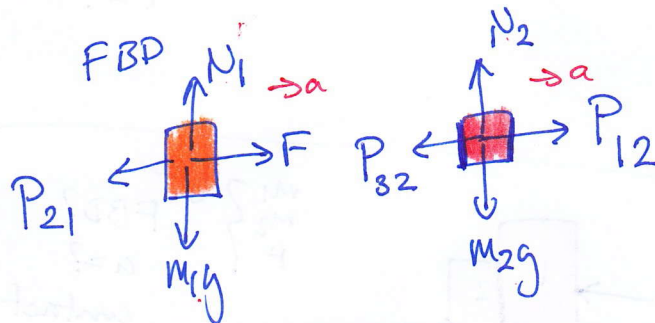
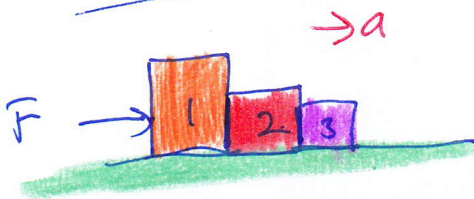
$$F = (m_1 + m_2) a$$

$$a = \frac{F}{m_1 + m_2}$$

to find the contact force } you should FBD of 1st 2nd separately!



$$P = m_2 \frac{F}{m_1 + m_2}$$



$$\vec{P}_{12} = -\vec{P}_{21}$$

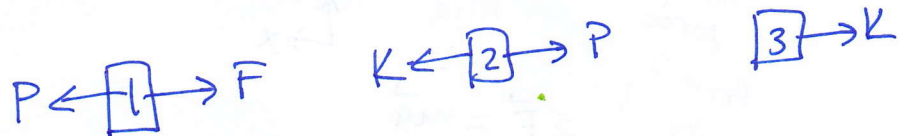
$$\vec{P}_{23} = -\vec{P}_{32}$$

are not equal to each other.

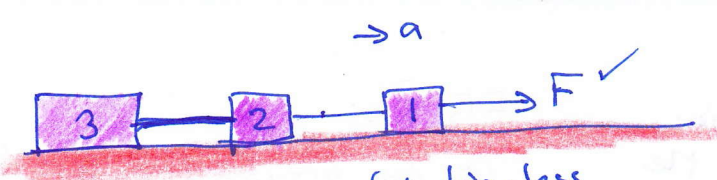
$$P_{12} \neq P_{23} !$$

$$a = \frac{F}{m_1 + m_2 + m_3}$$

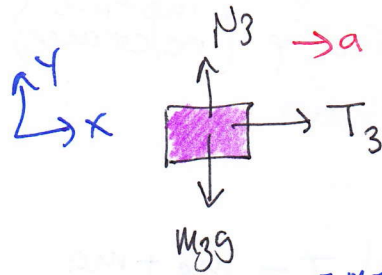
find P_{12} , P_{23} as an exercise?



ex)



same as previous example ⁽³⁾



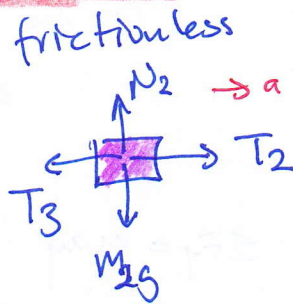
$$\sum F_y = m a_y$$

$$\sum F_x = m a_x$$

$$N_3 - m_3 g = m_3 a$$

$$N_3 = m_3 g + m_3 a$$

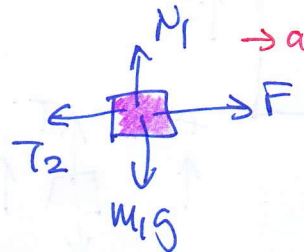
$$T_3 = m_3 a$$



$$N_2 = m_2 g$$

$$\sum F = m a$$

$$T_2 - T_3 = m_2 a$$



$$N_1 = m_1 g$$

$$F - T_2 = m_1 a$$

FBD?
a?

Tensions
on the
ropes?

a?
T₂?
T₃?

$$\textcircled{1} + \textcircled{2} + \textcircled{3} \Rightarrow$$

$$F = (m_1 + m_2 + m_3) a$$

$$a = \frac{F}{m_1 + m_2 + m_3}$$

into $\textcircled{3}$ $\textcircled{1}$

$$T_3 = m_3 \frac{F}{m_1 + m_2 + m_3}$$

$$T_2 = F - m_1 a$$

$$= F - m_1 \frac{F}{m_1 + m_2 + m_3}$$

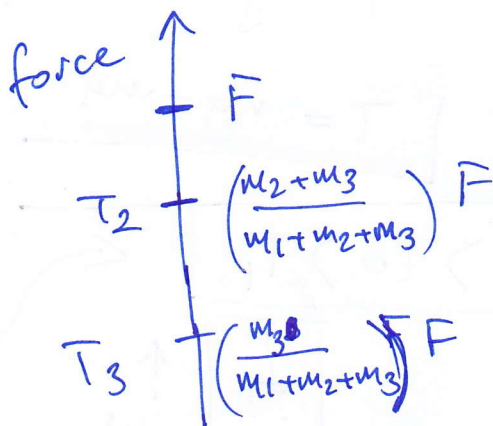
$$T_3 = P_{23}$$

$$T_2 = P_{12}$$

in the
previous
example

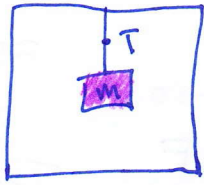
$$T_2 = F \left(\frac{m_2 + m_3}{m_1 + m_2 + m_3} \right)$$

$$T_2 = \frac{m_2 + m_3}{m_1 + m_2 + m_3} F$$



$$F > T_2 > T_3$$

ex:
elevates



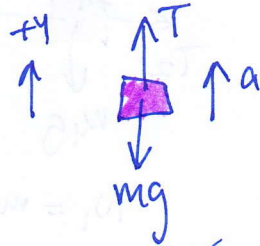
$\uparrow a$

tension of the rope?

FBD?

↳ to visualize it at a rest (inertial) frame

~~Q~~ A ✓

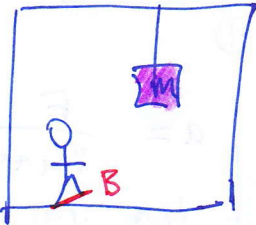


FBD ✓

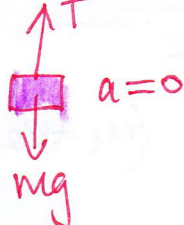
$$\sum F_y = may$$

$$T - mg = ma$$

$$T = mg + ma \quad \checkmark$$



B sees $\Rightarrow$

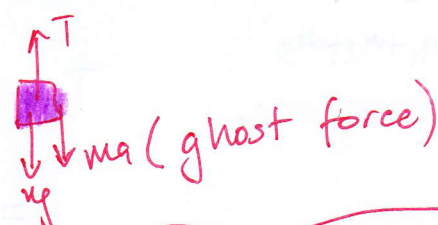


$$T - mg = m \cdot 0$$

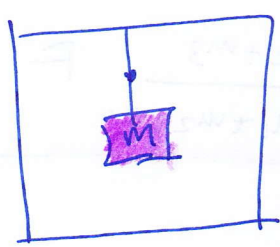
$$T = mg$$

~~wrong!!~~

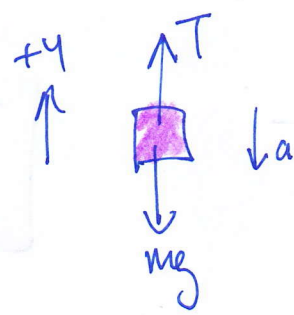
$ma \equiv$ is not a force!!



~~wrong!!~~



$\downarrow a$



$$\sum F_y = may$$

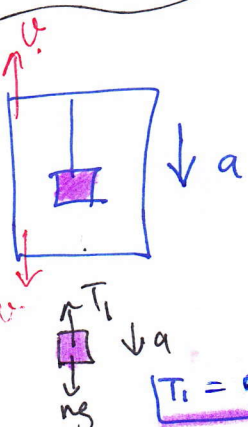
$$T - mg = m(-a)$$

$$T = mg - ma$$

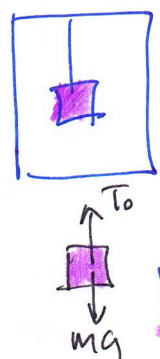
FBD? Tension?



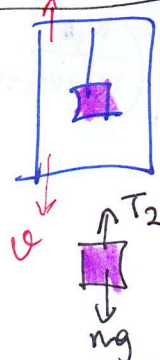
$$T_2 > T_0 > T_1$$



$$T_1 = mg - ma$$



$$T_0 = mg$$

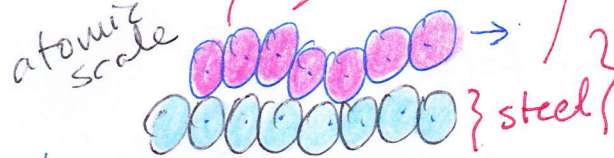
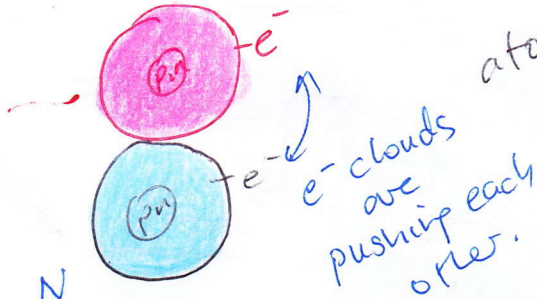
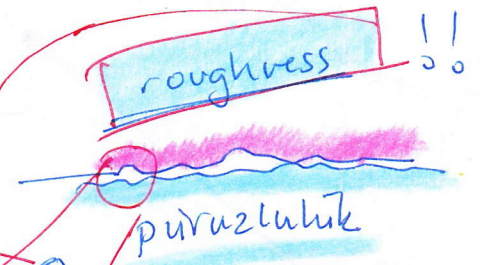
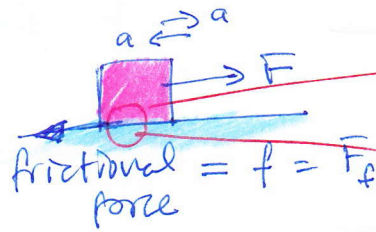


$$T_2 = mg + ma$$

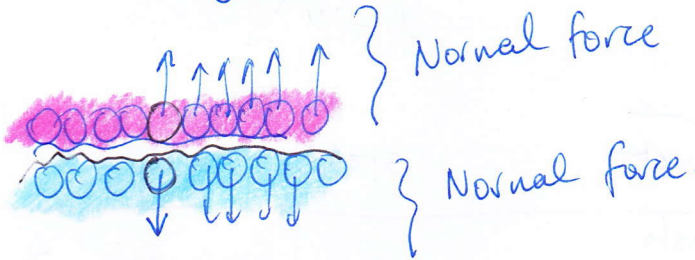
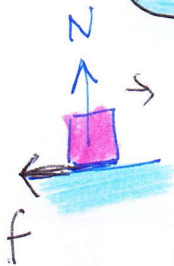
FBD

Newton's Laws & Applications

Frictional forces:



Electromagnetic interaction
① ⇒ interaction



friction $\propto$ normal force.

$$f \propto N$$

$$f = \mu N$$

μ = friction constant depends on type/roughness surface.
(μ)

materials

steel - steel

Al on steel

Cu on steel

glass on glass

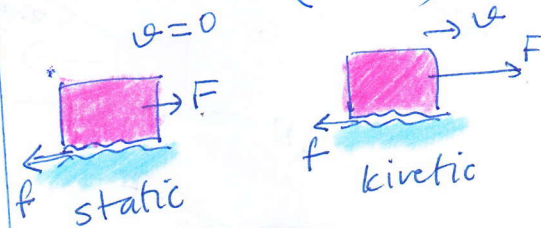
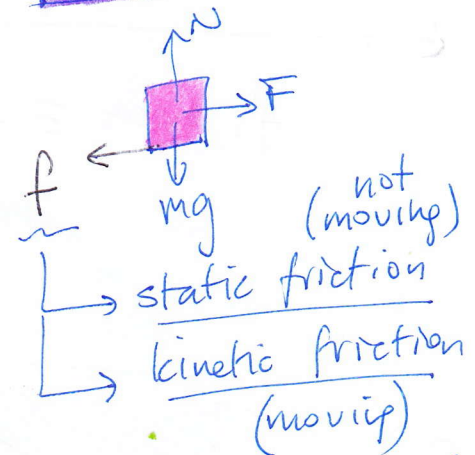
Cu on glass

Teflon on Teflon

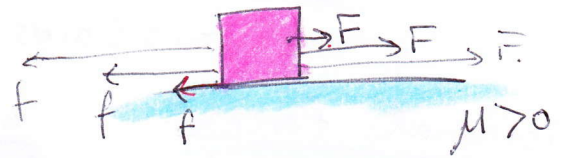
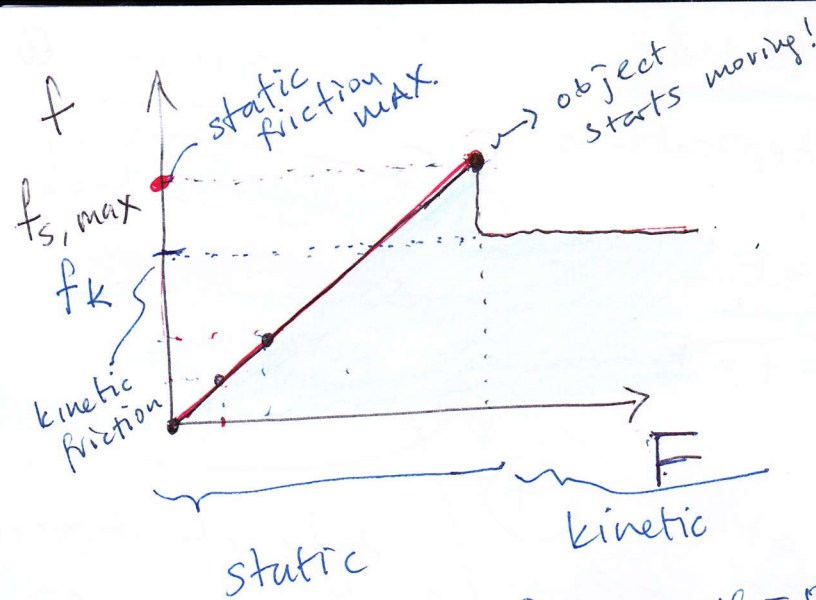
Human joints (elements)

Rubber on concrete (dry)
Rubber on concrete (wet)

μ_s	μ_k
0.74	0.57
0.61	0.47
0.53	0.36
0.94	0.40
0.68	0.53
0.04	0.04
0.004	0.004



Drive slowly & carefully in the rain !!



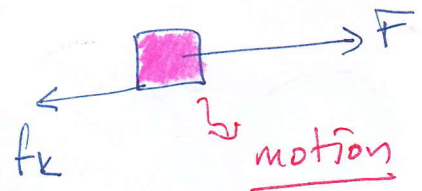
$$\sum F_x = 0$$

$$F - f = 0$$

static / not moving

$f_s = \text{static fric. force} \quad v = 0$

$f_k = \text{kinetic fric. force.} \quad v > 0$



$f_{s,max} > f_k$

static > kinetic

$f = \mu N$

static kinetic

ex: it's easier to push a desk if it's already moving.

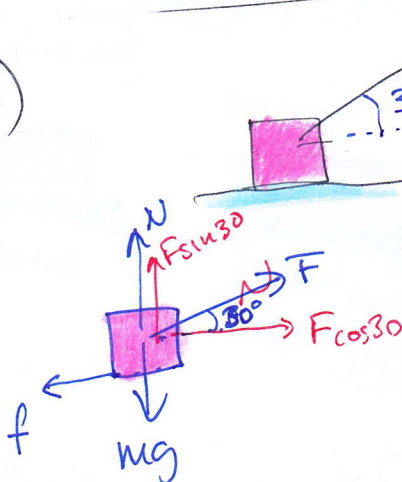
static f. force > kinetic f. force

$$f_{s,max} \equiv f_s = \mu_s N$$

$$f_k = \mu_k N$$

$$\mu_s > \mu_k$$

Ex)



$\theta = 30^\circ$ $mg = 500 \text{ N}$

$F = ?$

$\sum F_x = m a_x$

$F \cos 30 - f = 0 \quad (1)$

$F \cos 30 = f = \mu N$

$\sum F_y = m a_y$

$N + F \sin 30 - mg = 0 \quad (2)$

obj. moving

$\mu_k = 0.4$

what's Force you apply so that object is moving with const. velocity?

$(3) f_k = \mu_k N \quad \checkmark$

$$F \cos 30 = \mu N$$

$$N = \frac{F \cos 30}{\mu}$$

$$N = \frac{188 \frac{\sqrt{3}}{2}}{0.4}$$

$$N = 406 \text{ N}$$

$$f = \mu N = (0.4)(406)$$

$$f = 162.4 \text{ N}$$

$$N + F \sin 30 = mg$$

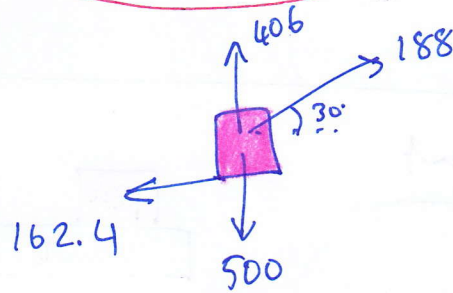
$$\frac{F \cos 30}{\mu} + F \sin 30 = mg$$

$$mg = 500 \text{ N}$$

$$\mu = 0.4$$

$$F = \left(\frac{\cos 30}{\mu} + \sin 30 \right) = \left(\frac{\frac{\sqrt{3}}{2}(0.4) + \frac{1}{2}}{1} \right)$$

$$F = 188 \text{ N}$$



$$f = \mu N$$

$$1 > \mu > 0$$

$$\mu_s \rightarrow \mu_s$$

$$\mu_k$$

m is moving down with a.

$$\mu_k = \mu$$

$$a = ?$$

FBD

$$\Sigma F_x = ma$$

$$mg \sin \theta - f = ma$$

$$f = \mu N$$

$$\Sigma F_y = ma$$

$$N - mg \cos \theta = 0$$

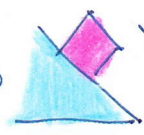
$$N = mg \cos \theta$$

$$mg \sin \theta - \mu mg \cos \theta = ma$$

$$g(\sin \theta - \mu \cos \theta) = a$$

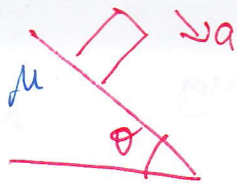
* remember

$$\mu = 0$$



$$a = g \sin \theta$$

if no friction
 $\mu = 0$



$$a = g(\sin\theta - \mu\cos\theta)$$

what's the situation when $a=0$

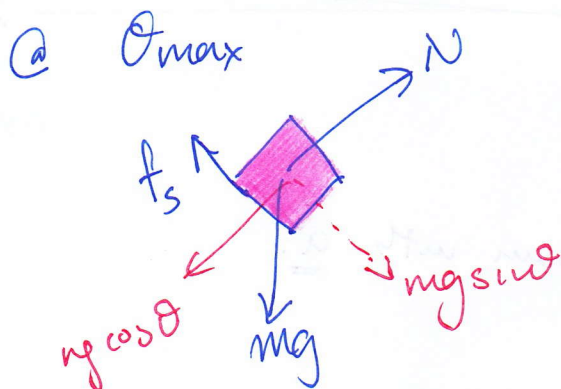
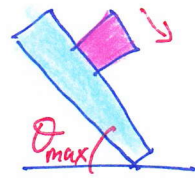
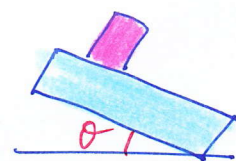
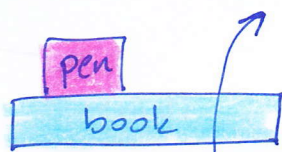
$$a=0 = g(\sin\theta - \mu\cos\theta)$$

$$\mu = \frac{\sin\theta}{\cos\theta} = \underline{\tan\theta}$$

$$\tan 45^\circ = 1 = \mu$$

$$\tan 46^\circ > 1 \neq \underline{\mu = 1 \text{ max!}}$$

home experiment



static friction.

$$\sum F_x \geq 0$$

$$mg\sin\theta - f_s = 0$$

$$\underline{mg\sin\theta_m = f_s = \mu_s N = \mu_s mg\cos\theta_m}$$

$$\underline{\underline{\mu_s = \tan\theta_m}}$$

- do this expt.

- find $\theta_{\max}$ angle

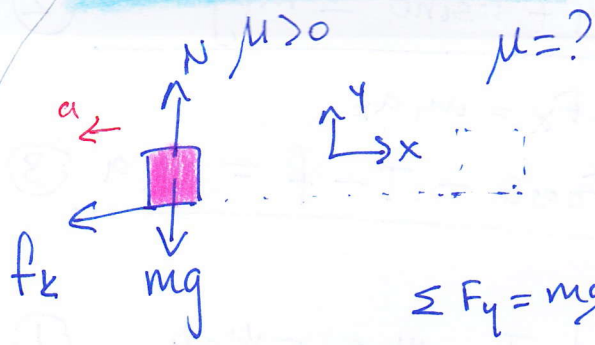
- measure $\theta_{\max}$ " when pen starts to move!

$$\underline{\underline{\tan\theta_m = \mu_s}}$$

$$\underline{\underline{\theta_m \leq 45^\circ}}$$

ex) $\rightarrow v_i$ d $v_f = 0$

object with v_i velocity stops d distance away on a ~~friction~~ surface with friction



f_k opposite to velocity

$$\sum F_y = m a_y \Rightarrow N = mg$$

$$\sum F_x = m a_x$$

$$-f_k = m a$$

$$f_k = \mu_k N = \mu_k mg$$

$$a = -\frac{f_k}{m} = -\frac{\mu_k mg}{m}$$

$$a = -\mu_k g$$

$$v_f^2 = v_i^2 + 2a \Delta x$$

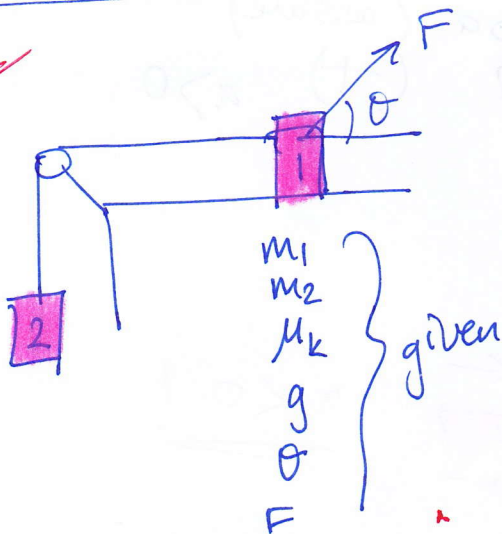
$$0 = v_i^2 + 2a d \Rightarrow v_i^2 - 2\mu_k g d = 0$$

$$\mu_k = \frac{v_i^2}{2gd}$$

if $v_i = 20 \text{ m/s}$
 $d = 115 \text{ m}$
 $g = 9.8 \text{ m/s}^2$

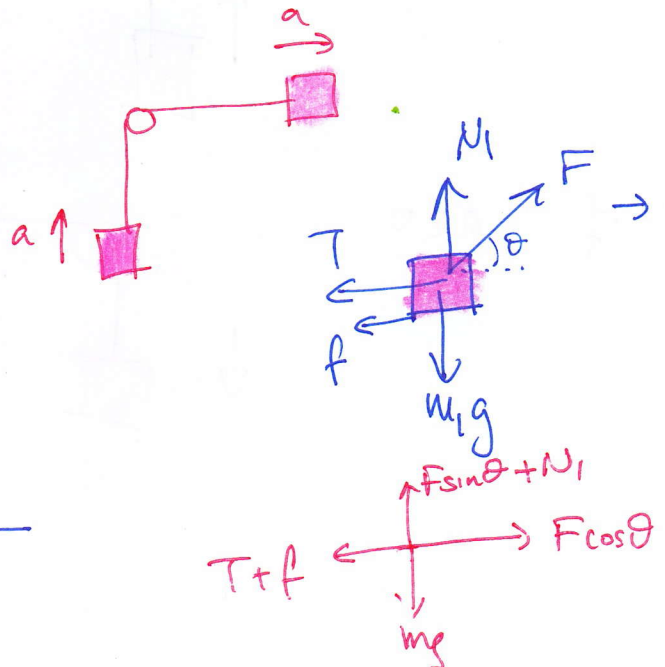
$$\mu_k = \frac{400}{2(9.8)115} = 0.17$$

Example

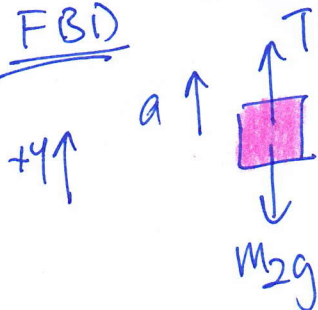


a = ?
 let's

decide on direction of acceleration.

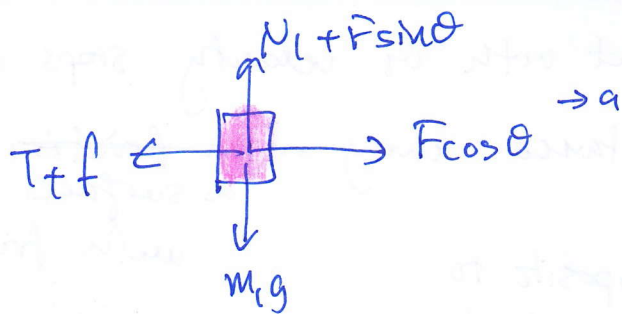


FBD



$$\sum F_y = m_2 a_y$$

$$T - m_2 g = m_2 a$$



$$\sum F_y = m_1 a_y \quad \nearrow$$

$$N_1 + F \sin \theta = m_1 g \quad (2)$$

$$\sum F_x = m_1 a_x$$

$$F \cos \theta - T - f = m_1 a \quad (3)$$

$$(4) \quad f = \mu N_1 =$$

(2) into (4)

$$N_1 = (m_1 g - F \sin \theta)$$

$$f = \mu (m_1 g - F \sin \theta)$$

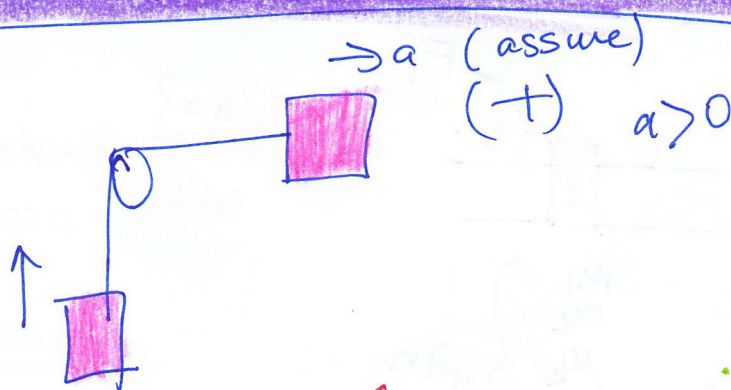
$$T - m_2 g = m_2 a \quad (1)$$

(1) + (3)

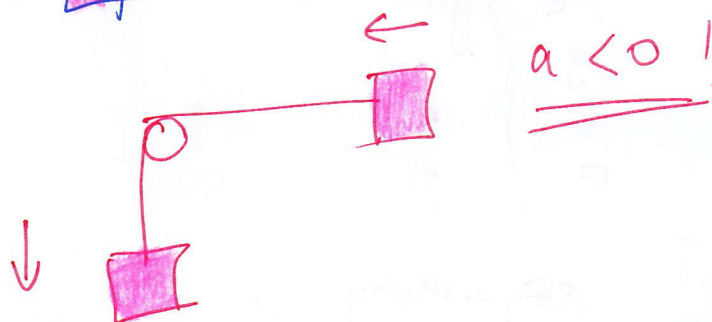
$$F \cos \theta - f - m_2 g = (m_1 + m_2) a$$

$$F \cos \theta - \mu (m_1 g - F \sin \theta) - m_2 g = (m_1 + m_2) a$$

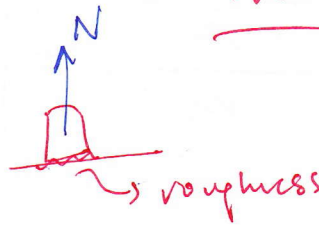
$$a = \frac{F \cos \theta - m_2 g - \mu (m_1 g - F \sin \theta)}{m_1 + m_2}$$



if a < 0



Frictional Forces



$$f = \mu N$$

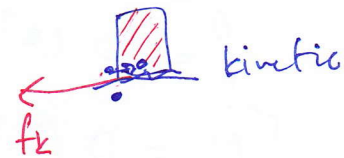
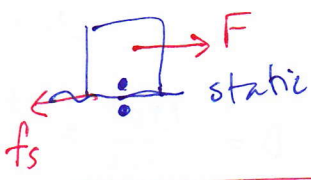
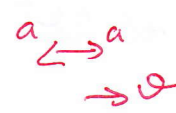
normal force

fric. forces. $[0 < \mu < 1]$; $\mu \equiv$ unitless

$$f = \mu N$$

$$[N = 1. N]$$

- ① kinetic $\longrightarrow \mu_k$
- ② static $\longrightarrow \mu_s$



Frictional forces
 $\longrightarrow$ in air/gas
 $\longrightarrow$ in liquid. } more complicated than $f = \mu N$

Liquid

$f \sim \text{Velocity}$

$f = k v$
 $\downarrow$
 constant

release $v_i = 0$
an iron ball

$v_i = 0$

$f_1 = 0$ $\Sigma \vec{F} = m\vec{a}$ $\downarrow +y$

$\Sigma F > 0$
 $mg - f_2 = ma$
 $f_2 = k v_2$

const. velocity = terminal velocity

$\Sigma F = ma$
 $mg - f_4 = 0$
 $f_4 = \text{max} = f_3$
 fric. force

$f_3 = k v_3$
 $f_3 = \text{friction force} = mg$
 $\Sigma F = ma$
 $mg - f_3 = 0 = m a_3$
 $a_3 = 0$

$v_3 = \text{max velocity}$
 $v_T = \text{terminal velocity}$

terminal velocity conditions
 $\Sigma F = 0 = mg - f_{\text{max}} = 0$
 $mg = f_{\text{max}} = k v_T$

$$v_T = \frac{mg}{k}$$

in liquid

$$f = k v$$

$$N = ? \frac{m}{s}$$

$$k = \left[\frac{Ns}{m} \right] = \frac{kg \frac{m}{s^2} s}{m} = \left[\frac{kg}{s} \right]$$

$$f = k v \text{ (in liquid; when } v \text{ is small)}$$

Air resistance

air

$$f \propto v^2 \text{ (} v \text{ values are higher than in liquid } v \text{ values)}$$

$$f = D v^2$$

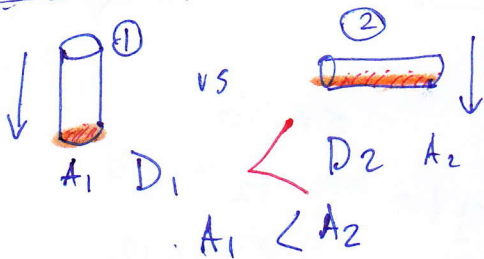
$$\left[N = D \left(\frac{m}{s} \right)^2 \right] \Rightarrow D = \frac{Ns^2}{m^2} = \frac{kg \frac{m}{s^2} s^2}{m^2}$$

$D \neq k$
in air in liquid

$$\left[D = \frac{kg}{m} \right]$$

D depends on the

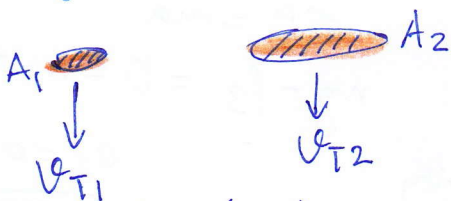
- Temperature
- shape of the object (crosssectional area)
- orientation of the object



$A_1 = \text{crosssectional area (kesit ala)}$

$$D \propto A \quad f = D v^2$$

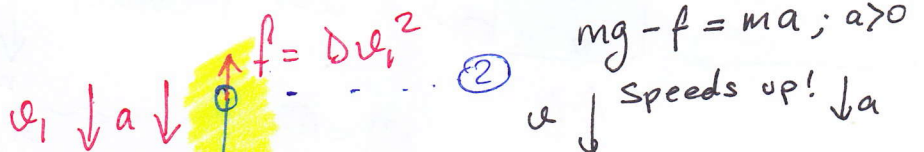
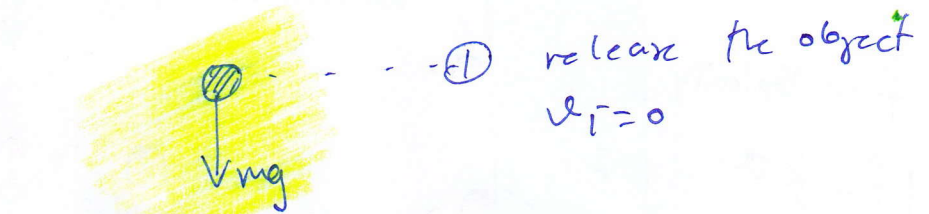
$$mg = f = D v_T^2$$



$$A_1 < A_2$$

$$D_1 < D_2$$

$$v_{T1} > v_{T2}$$



@ terminal velocity

$$f = D v_T^2 = mg$$

$$\Sigma F = ma$$

$$mg - f = 0 \quad | \quad a = 0$$

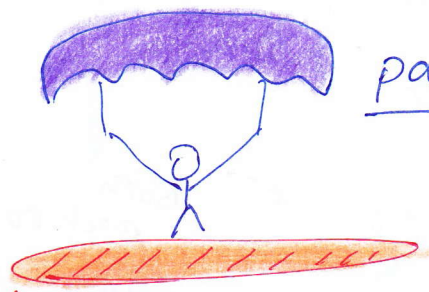
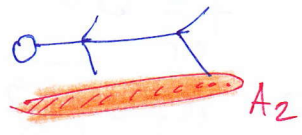
$$mg = f = D v_T^2$$

$$v_T = \sqrt{\frac{mg}{D}} \text{ air}$$

$$v_T = \frac{mg}{k} \text{ in liquid}$$



FREE FALL



parachute

$$f = D v^2$$

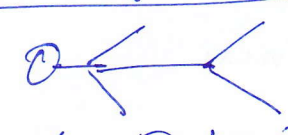
$$A_1 < A_2 < A_3$$

$$v_{T1} > v_{T2} > v_{T3}$$

survive!

$$100 \frac{m}{s} > 50 \frac{m}{s} > 5 \frac{m}{s}$$

Ex): $D = 0.25 \frac{kg}{m}$ for spread-eagle position for a human body



$v_T = ?$ of a person of mass of 50 kg?

terminal

$$mg - f = 0$$

$$mg = f = D v_T^2$$

$$v_T = \sqrt{\frac{mg}{D}}$$

$$v_T = \sqrt{\frac{50 \times 9.8}{0.25}} = 44 \frac{m}{s} = ? \frac{km}{hr}$$

how to convert $\frac{m}{s} \rightarrow \frac{km}{hr}$

$$1 hr = 60 min = 3600 s$$

$$\frac{1 m}{1 s} = \frac{10^{-3} km}{1/3600 hr} = 3.6 \frac{km}{hr}$$

$$44 \times 3.6 = 158.4 \frac{km}{hr} \approx 160 \frac{km}{hr}$$

v_T for $m = 50 kg$

$$v_T = \left(\frac{mg}{D \sim Area} \right)^{1/2}$$

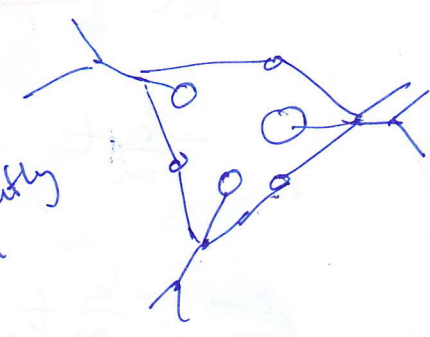
if $m = 100 kg$

$$v_T = \sqrt{2} \cdot 160 \frac{km}{hr} \approx 230 \frac{km}{hr}$$

for $m = 100 kg$

air acrobats

use this area efficiently to catch



in liquid

$$f = k v$$

gas/oil

water/su

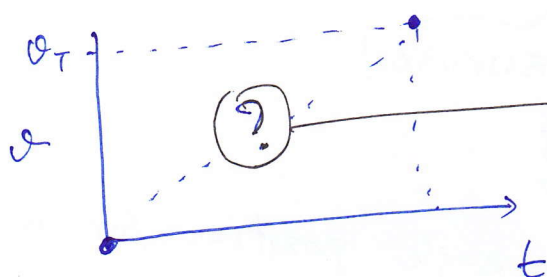
$$k_1 \neq k_2$$

Ug. $f = k v$
 $v_T = \frac{mg}{k}$

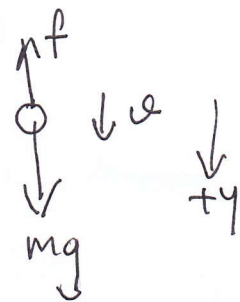
$f_{\text{air}} = D v^2$
 $v_T = \sqrt{\frac{mg}{D}}$

air

Both reach to terminal/max velocity



$v_i = 0$



$\Sigma F = ma$

$mg - f = ma$

$mg - kv = ma$ } differential eqn.

$\frac{dv}{dt} = a$

$\left\{ mg - kv = m \frac{dv}{dt} \right\}$

$dt = \frac{m dv}{mg - kv} \Rightarrow dt = \frac{dv}{\left(g - \frac{k}{m} v\right)}$; $\int \frac{du}{u} = \ln u$

$-dt = \frac{dv}{\left(\frac{k}{m} v - g\right)} \Rightarrow v = \frac{k}{m} v - g \Rightarrow -dt = \frac{\frac{m}{k} dv}{u}$

$\int -\frac{k}{m} dt = \int \frac{du}{u} \Rightarrow -\frac{k}{m} t \Big|_{t_i}^{t_f} = \ln u \Big|_{u_i}^{u_f} = \ln \left(\frac{k}{m} v - g \right) \Big|_{u_i}^{u_f}$

if $t_i = 0$; $v_i = 0$ (release) $-\frac{k}{m}(t-0) = \ln \left(\frac{k}{m} v - g \right) - \ln(-g)$
 $t_f = t$ $v_f = v$

$-\frac{k}{m} t = \ln \left(\frac{\frac{k}{m} v - g}{-g} \right)$

$\leftarrow -\frac{k}{m} t = \ln \left(-\frac{k}{mg} v + 1 \right)$

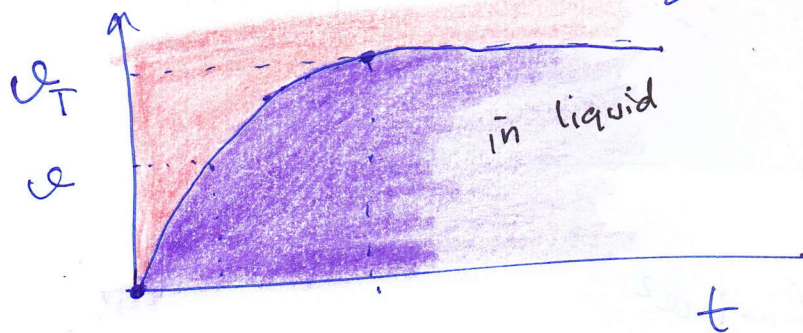
$e^{-\frac{k}{m} t} = -\frac{k}{mg} v + 1$

$$\frac{k}{mg} v = 1 - e^{-\frac{k}{m} t}$$

$$\Rightarrow v = \frac{mg}{k} \left[1 - e^{-\frac{k}{m} t} \right] \quad (3)$$

in liquid

$$v(t) = v_T \left(1 - e^{-\frac{k}{m} t} \right)$$



when $t=0$

$$v(0) = v_T \left(1 - e^{-\frac{k}{m} \cdot 0} \right)$$

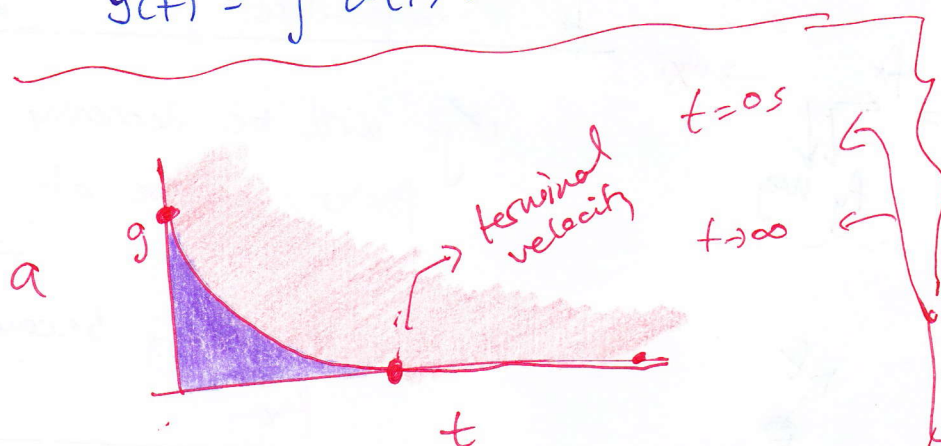
{ 1 - 1 }

0

$$v_i = \underline{v(0) = 0}$$

$v(t)$

$$y(t) = \int v(t) dt$$

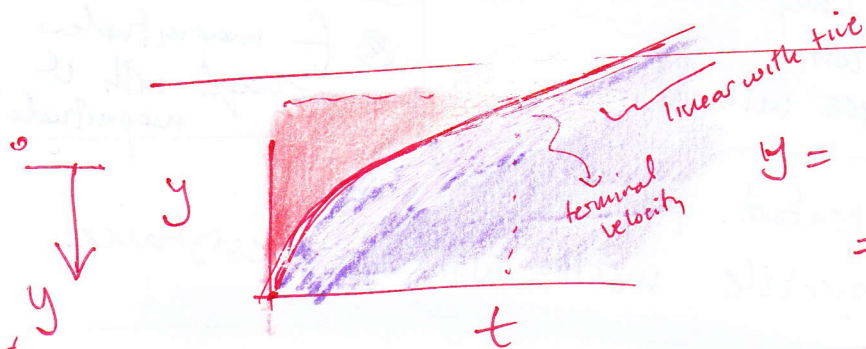


$$\frac{d}{dt} \left(v_T (1 - e^{-\frac{k}{m} t}) \right)$$

$$a = (-v_T) \left(-\frac{k}{m} \right) e^{-\frac{k}{m} t}$$

$$= \frac{mg}{k} \frac{k}{m} e^{-\frac{k}{m} t}$$

$$a(t) = g e^{-\frac{k}{m} t}$$



$$y = \int v(t) dt$$

$$= \int \left(v_T - v_T e^{-\frac{k}{m} t} \right) dt$$

$$= v_T \Delta t - v_T \left(-\frac{m}{k} \right) e^{-\frac{k}{m} t}$$

$$= v_T \left(t + \frac{m}{k} e^{-\frac{k}{m} t} \right)$$

velocity of the object at any time t .

$$v_i = 0 \text{ m/s} ; \quad mg = f = kv_T$$

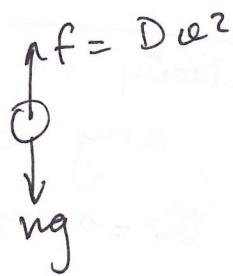
$$v_T = \frac{mg}{k}$$

$t \rightarrow \text{large} \rightarrow \infty$

$$\frac{e^{-\frac{k}{m} \infty}}{0} = \frac{1}{e^{\infty}} = 0$$

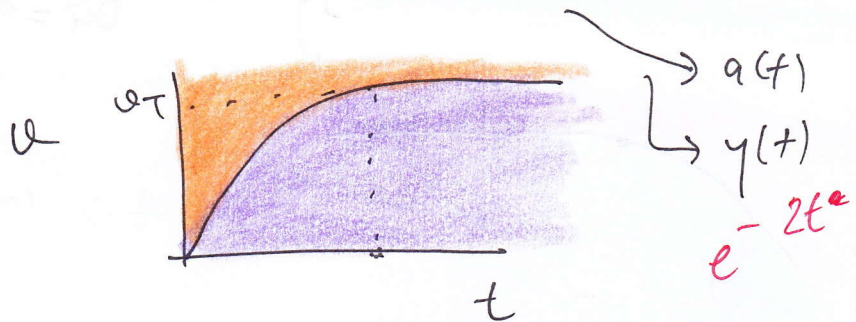
$$v_T (1 - 0) = \underline{v_T = v(t)}$$

air resistance



$$mg - Dv^2 = ma = m \frac{dv}{dt}$$

$v(t) \sim$ exponential function



A ball
motion
in air;

Football
baseball

$$f \sim Dv^2$$

car on highway
truck

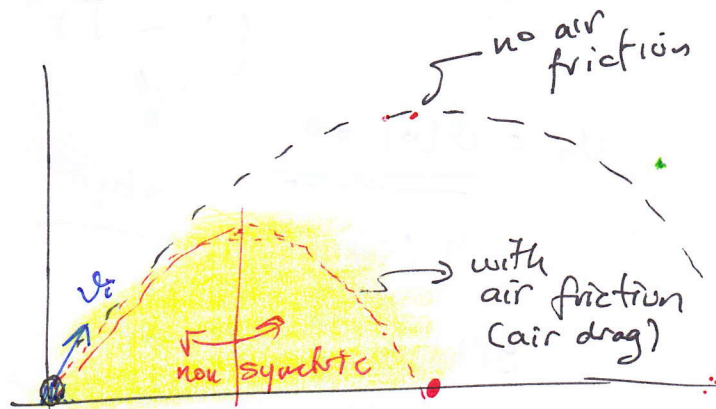
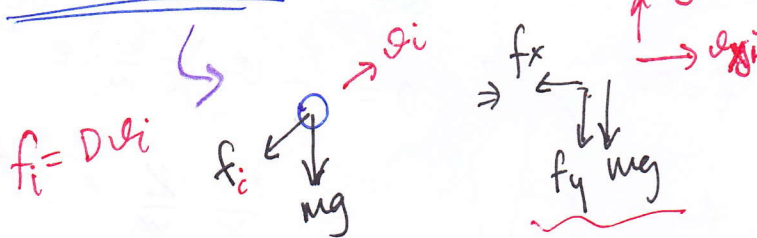
$$f \sim Dv^2$$

projectile motion



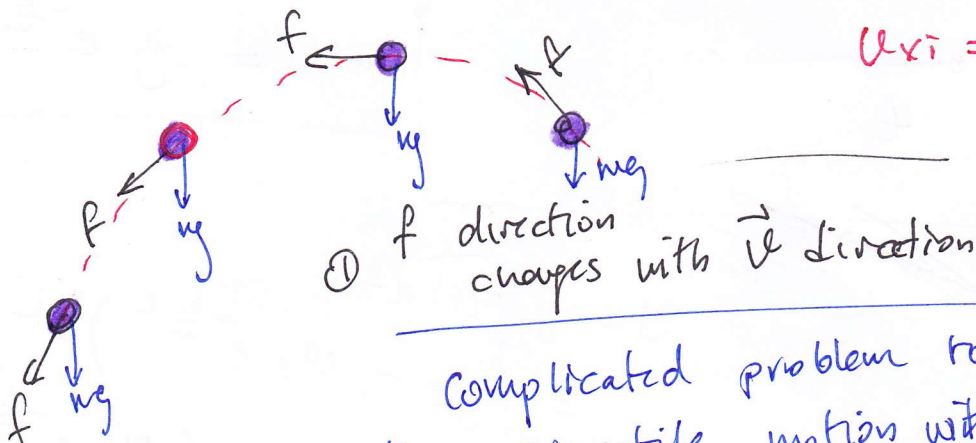
at start

when air friction



v_{yi} will be decreasing
faster because of f_y

$v_{xi} \Rightarrow$ decreasing because of f_x



① f direction changes with $\vec{v}$ direction

② f magnitude changes with v magnitude

Complicated problem to solve
the projectile motion with air resistance!

Dynamics of Circular Motion

25.11.20

(1)

Application of Newton's law in circular motion

Circular motion

uniform
c.m.

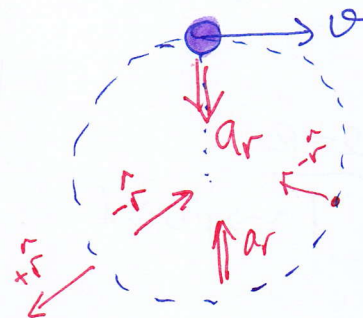
non uniform
c.m.

$$v = \text{const}$$

$$R = \text{const}$$

$$\vec{a}_{\text{rad}} = \frac{v^2}{R} (-\hat{r})$$

towards to center



$$\sum \vec{F} = m \vec{a}$$

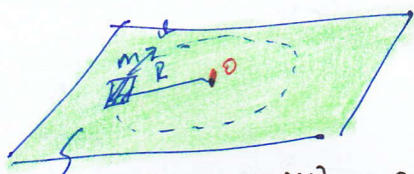
$$\vec{a} = \vec{a}_{\text{rad}}$$

$$\sum \vec{F} = m \frac{v^2}{R} (-\hat{r})$$

radial acc = a_{rad}

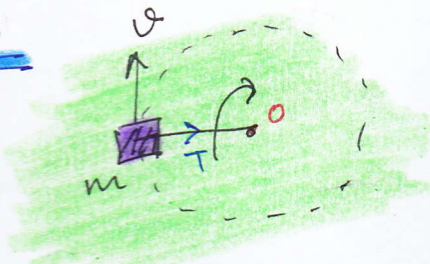
(merkezci ivme = merkez depre)

ex:) uniform circular motion on a frictionless table.
m mass has been tied with a rope; with a fixed speed v
radius R and experiencing UCM with const. ~~velocity~~ v



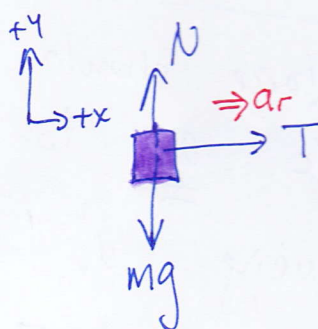
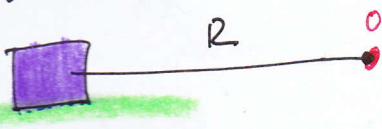
$\mu=0$
no friction!

TOP VIEW



- m, v, R given
- draw FBD.
- what's net force; $F=?$

side view
 $\vec{v}$ into the page
 $\otimes$



$$\left\{ \begin{array}{l} \sum F_y = mg \\ N - mg = 0 \\ N = mg \end{array} \right\} \quad \left\{ \begin{array}{l} \sum F_x = m a_r \\ T = m a_r \\ T = m \frac{v^2}{R} \end{array} \right.$$

$$\sum \vec{F} = \vec{T}; \text{ tension on the rope.}$$

Tension on the rope is the net force towards to the center

T is the reason for circular motion.

If $m = 25 \text{ kg}$; mass is rotating 5 revolutions (rotations) per minute.

$$R = 5 \text{ m}$$

value for $T = ?$

$$\omega = \frac{2\pi R}{T \rightarrow \text{period}}$$

$$F = T = m \frac{\omega^2}{R}$$

↑
tension

$$\rightarrow \underline{5 \text{ rpm}} = \frac{5 \times (2\pi R)}{60 \text{ s}} = \omega$$

$$= \frac{5 \times 2 \times 3.14 \times 5}{60} = \underline{2.62 \text{ rad/s}}$$

tension on the rope

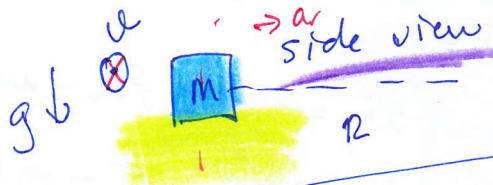
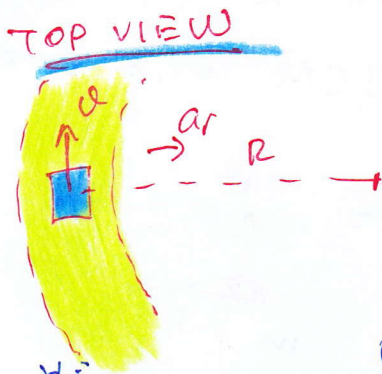
$$T = m \frac{\omega^2}{R} = \frac{25 (2.62)^2}{5} = \underline{34 \text{ N}}$$

$$a_r = \frac{\omega^2}{R} = 1.37 \text{ m/s}^2$$

ex:) Rounding a flat curve on the road.
car is on the curve. (egimisy viraj)

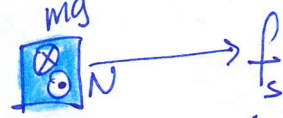
there must be friction on the road to keep the car on the curved path.

DCM



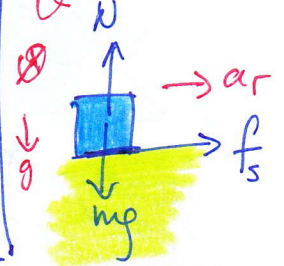
⊗ into the page
⊙ out of page

FBD top view



static friction!

side view FBD



f : frictional force should be forward to center b/c a_r is forward to center.

f_s is producing a_r !

$$\sum F_y = mg$$

$$N - mg = 0 \quad N = mg$$

$$\sum F_x = m a_r$$

$$f_s = m a_r = m \frac{\omega^2}{R}$$

$$f = m \frac{v^2}{R} ; f_s = \mu_s N ; N = mg$$

$$\mu mg = m \frac{v^2}{R} \Rightarrow v^2 = \mu_s g R$$

$$v = \sqrt{\mu_s g R}$$

if R const. ; road is dry ; $\mu = \text{const.}$

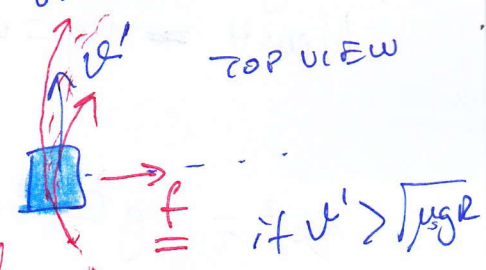
$\mu_s g R = \text{fixed number} = \text{const. relation between } v \text{ and } \mu_s R$

$$v = \sqrt{\mu_s g R}$$

max value of speed the car before

sliding / skidding

the car deviates from curved path.



Dimensional analysis on the formula.

$$v = \sqrt{\mu_s g R}$$

$$\left[\frac{m}{s} = \left(1 \frac{m}{s^2} m \right)^{1/2} \right]$$

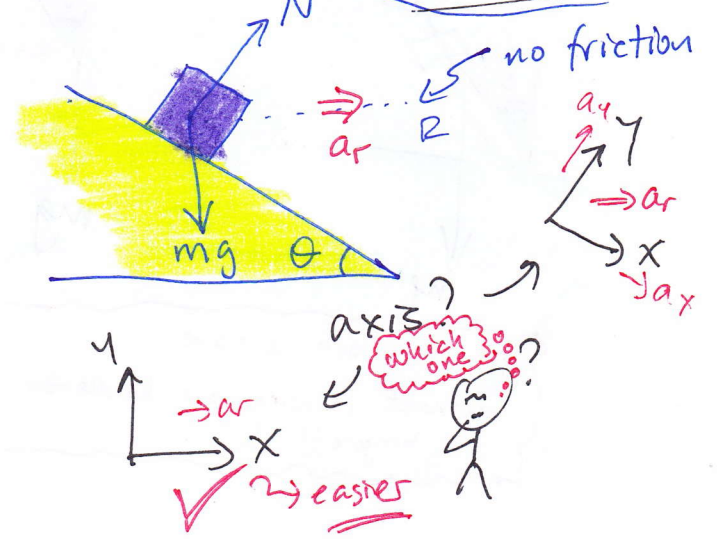
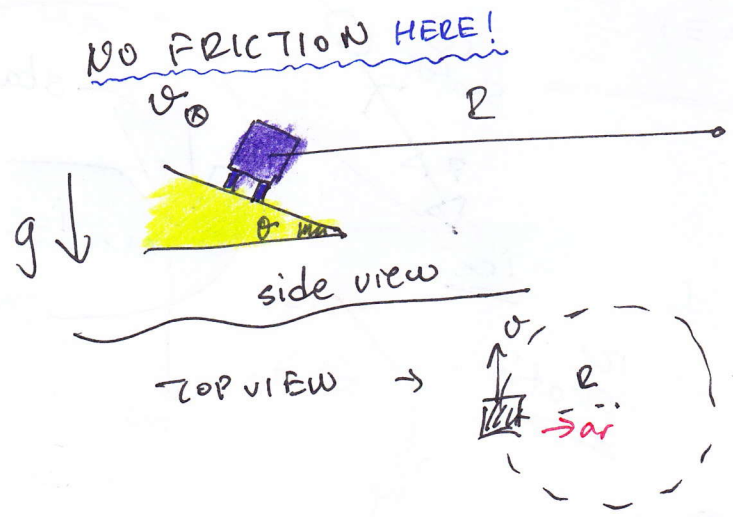
if $\mu_s = 0.96 ; g = 9.8 m/s^2 ; R = 230m$

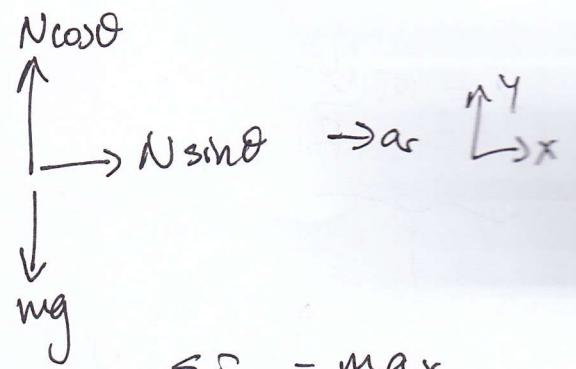
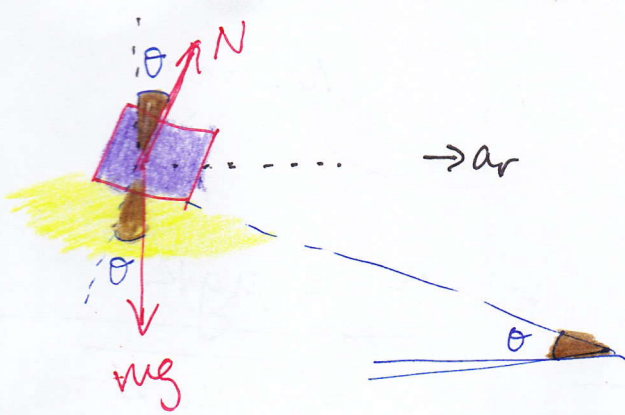
$$v_{max} = \sqrt{(0.96)(9.8)(230)} = 47 m/s = 47 \times 3.6 \frac{km}{hr}$$

$$v_{max} \approx 170 km/hr$$

Ex) Rotation on a surface without friction.
"Roundup on a banked curve."

NASCAR race
there is friction X





$$\Sigma F_x = m a_x$$

$$\Sigma F_y = m a_y = 0$$

$$N \cos \theta = mg$$

$$N = \frac{mg}{\cos \theta}$$

$$N \sin \theta = m a_r = m \frac{v^2}{R}$$

$$\frac{mg}{\cos \theta} \sin \theta = m \frac{v^2}{R}$$

$$v^2 = g R \tan \theta$$

$$v = \sqrt{g R \tan \theta}$$

rounding on a banked curve.

$v > v_{\max}$ car slides

unit analysis

$$v = \frac{m}{s} = \sqrt{\frac{m}{s^2} m} = \sqrt{g R}$$

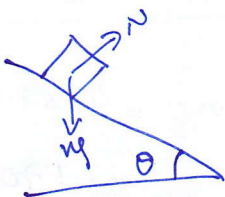
previously

on a flat surface

$$v = \sqrt{\mu_s g R}$$

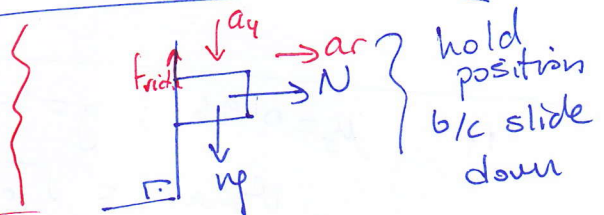
further analysis of

$$v = \sqrt{g R \tan \theta} \quad \mu = 0$$



$$\tan 45^\circ = 1$$

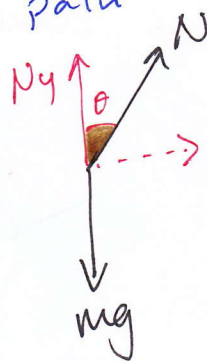
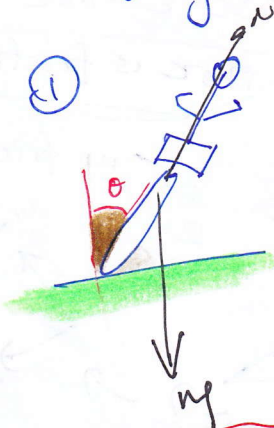
$$\tan 90^\circ \rightarrow \infty$$



if $\theta = 90^\circ$ introduce friction $\mu > 0$
must be

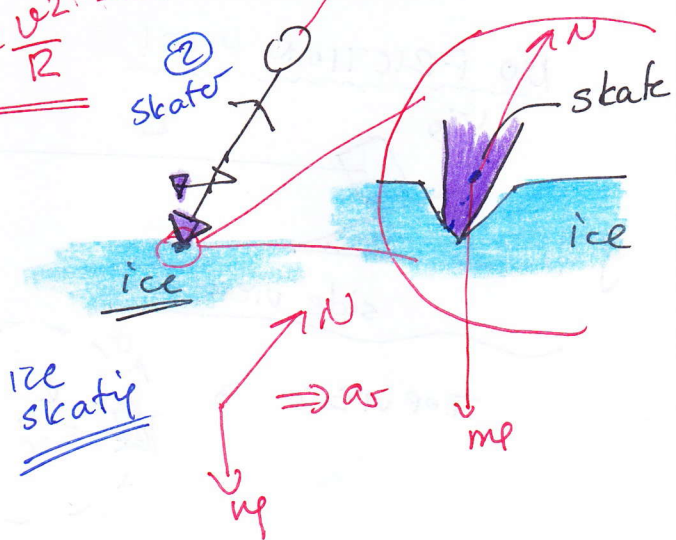
bicycle; skater; motorcycle curved path

①



$$N_x = m \frac{v^2}{R}$$

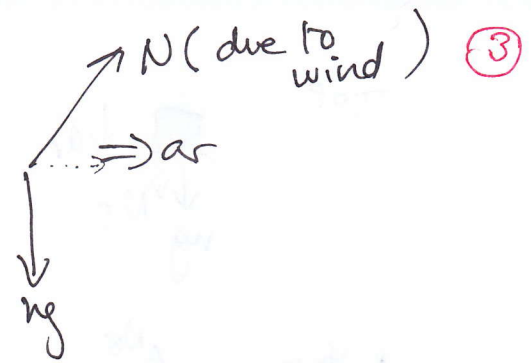
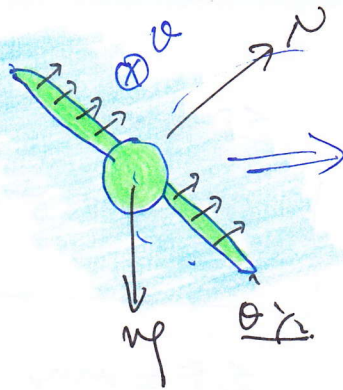
② skater



you can turn a curve without rotating your handle-bar on bicycle!!

ice skating

plane rotate.



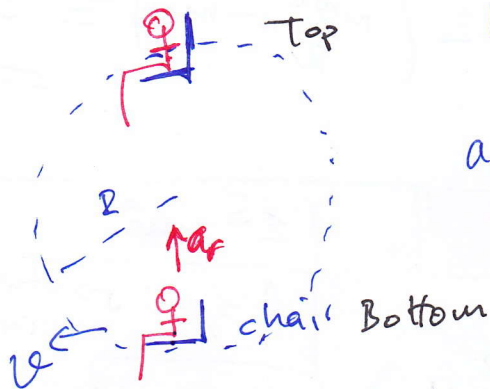
$$v = \sqrt{gR \tan \theta}$$

Ex: ferris wheel

(linear loop)

LCM

(normal) analyze contact forces TOP; BOTTOM



$$a_r = \frac{v^2}{R}$$



Bottom



Top

Bottom $\sum \vec{F} = m\vec{a}$ Top $\uparrow +y$

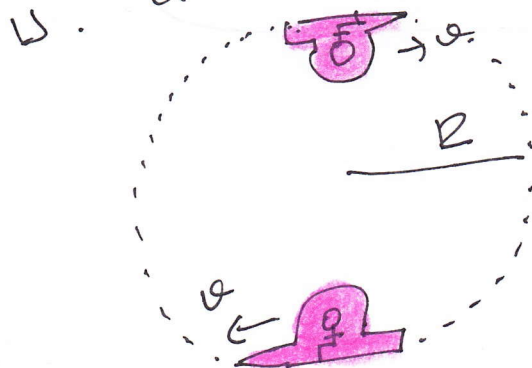
$$N_B - mg = mar \quad \left\{ \begin{array}{l} N_T - mg = m(-a) \\ N_T = mg - mar \end{array} \right.$$

$$N_B = mg + mar$$

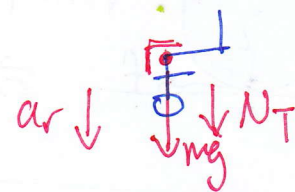
$$N_B - N_T = 2mar = 2m \frac{v^2}{R}$$

d. force between contact forces

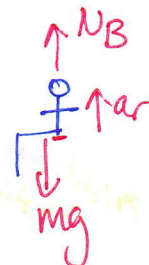
Ex: Jet-pilot circular motion $\neq$ ferris wheel



study the normal force applied to the pilot

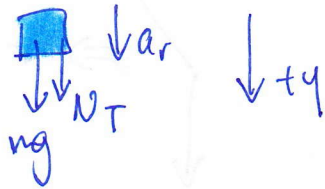


Top



Bottom

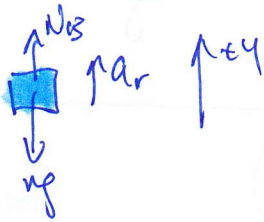
TOP



$$\Sigma \vec{F} = m\vec{a}_r$$

$$mg + N_T = m \frac{v^2}{R}$$

Bottom



$$\Sigma F = ma_r$$

$$N_B - mg = m \frac{v^2}{R}$$

$$N_B - N_T = mg + m \frac{v^2}{R} - \left(m \frac{v^2}{R} - mg \right) = \underline{\underline{2mg}}$$



End of Ch5

$$\Sigma \vec{F} = m\vec{a}$$

$f = \mu N \rightarrow$ static
kinetic

UCM; cm

flat curve on UCM ; $\mu > 0$



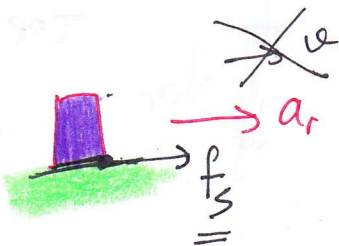
kinetic? or
no!

static?
yes ✓

since R const;
it's NOT moving toward center

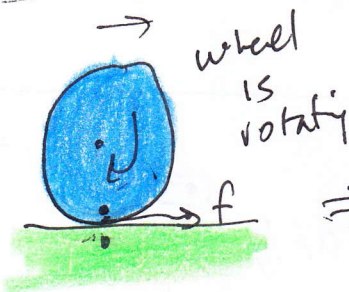
no velocity

center of the motion



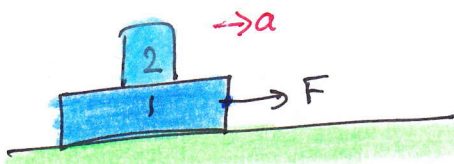
$$f_s = \mu_s N = \mu_s mg = m \frac{v^2}{R}$$

$$v = \sqrt{\mu_s g R}$$



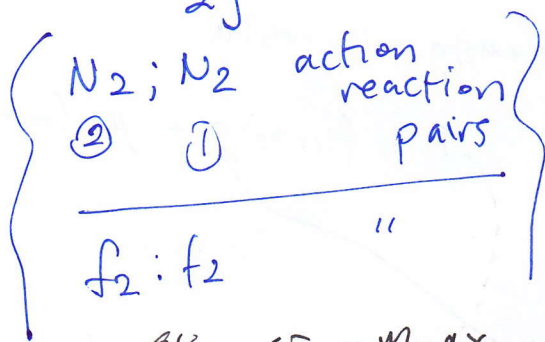
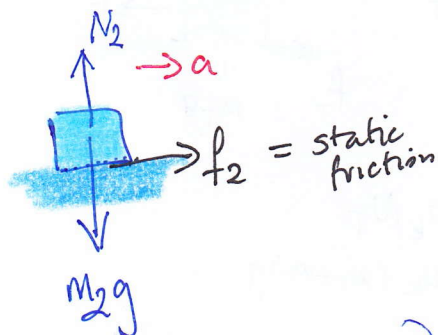
$\Rightarrow$ friction static $\Rightarrow$ Ch9 - Ch10

e)



what's acceleration? $F = ?$

draw FBD for each object.



$$\sum F_y = m_2 a_y \quad \sum F_x = m_2 a_x$$

$$N_2 = m_2 g \rightarrow f_2 = m_2 a$$

$$f_2 = \mu_s N_2$$

$$\mu_s m_2 g = m_2 a$$

$$a = \mu_s g$$

equal to each other

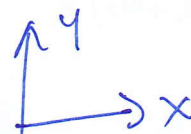
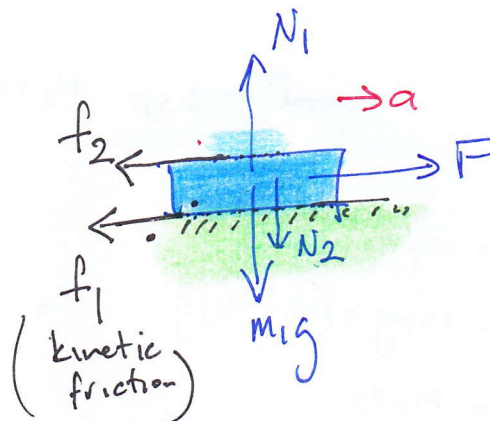
$$m_1 \mu_s g = F - \mu_k m_1 g - m_2 g (\mu_s + \mu_k)$$

$$F = m_1 g (\mu_k + \mu_s) + m_2 g (\mu_s + \mu_k)$$

$$F = (m_1 + m_2) (\mu_s + \mu_k) g$$

max value of force to apply ① & ② move together.

1 & 2 are moving together with same acceleration.
 F is applied as a max. force to move them together.
 μ_s, μ_k are given
 m_1, m_2



$$\sum F_y = m_1 a_y = 0$$

$$N_1 - m_1 g - N_2 = 0$$

$$N_1 = m_1 g + N_2 = (m_1 + m_2) g$$

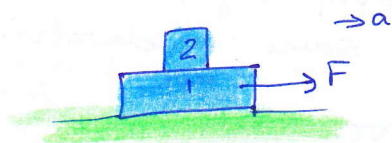
$$\sum F_x = m_1 a_x$$

$$F - f_1 - f_2 = m_1 a$$

$$F - \mu_k N_1 - \mu_s N_2 = m_1 a$$

$$F - \mu_k (m_1 + m_2) g - \mu_s m_2 g = m_1 a$$

$$a = \frac{F - \mu_k m_1 g - m_2 g (\mu_s + \mu_k)}{m_1}$$

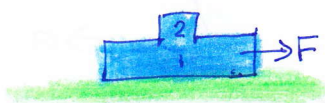


$$a = \mu_s g$$

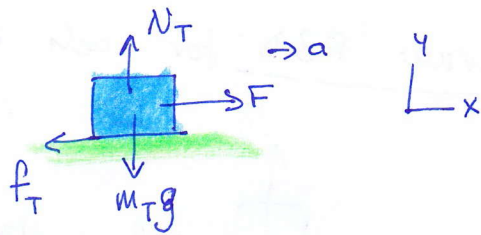
$$F = (m_1 + m_2)(\mu_s + \mu_k)g$$

check!

since 2 & 1 moving together



$$m_2 + m_1 = m_T$$



$$\sum F_y = m_T a_y^{\rightarrow 0}$$

$$N_T = m_T g = (m_1 + m_2)g$$

$$\rightarrow f_T = \mu_k N_T$$

$$f_T = \mu_k (m_1 + m_2)g$$

$$\sum F_x = m_T a_x$$

$$F - f_T = (m_1 + m_2)a \Rightarrow F - \mu_k (m_1 + m_2)g = (m_1 + m_2)a$$

$$F = (m_1 + m_2)a + \mu_k (m_1 + m_2)g$$

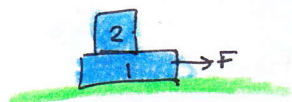
since we know

m_2 moves with

frictional forces

we found

$$\mu_s m_2 g = m_2 a \Rightarrow a = \mu_s g$$



$$F = (m_1 + m_2)\mu_s g + (m_1 + m_2)\mu_k g$$

Let's give some values to variables!

if

$$\mu_s = 0.8$$

$$\mu_k = 0.6$$

$$a = (9.8)(0.8)$$

$$a = 7.84 \text{ m/s}^2$$

this is the max. acc.

they move together.

$$F = (m_1 + m_2)(\mu_s + \mu_k)g$$

same as before!

$$a = \mu_s g$$

if

$$m_1 = 10 \text{ kg}$$

$$m_2 = 5 \text{ kg}$$

$$F = (10 + 5)(0.8 + 0.6)(9.8) = 205.8 \text{ N}$$

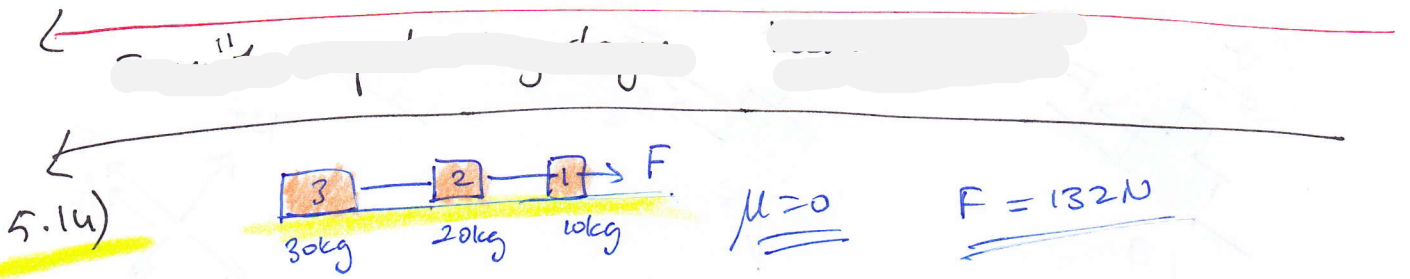
any force greater than 205.8 N is applied;

$\rightarrow$ then m_2 slides ^{back} m_2 will have a_2 acceleration

$\rightarrow m_1$ will have a_1 acceleration

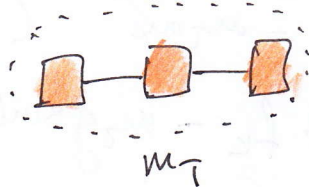
$$a_1 > a_2 !!$$

this is the max force one can apply to m_1 so that m_1 & m_2 move together; (m_2 is not sliding!!!)

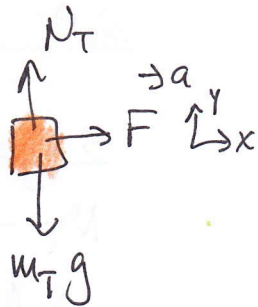
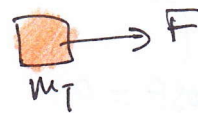


- a) $a = ?$
 b) Tension between 1st & 2nd object?

FBD



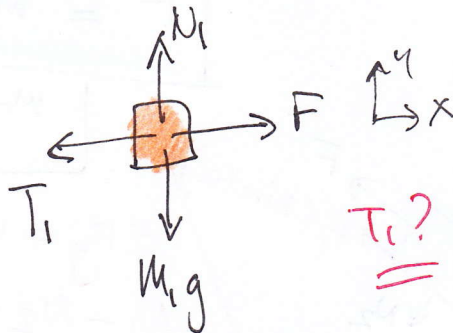
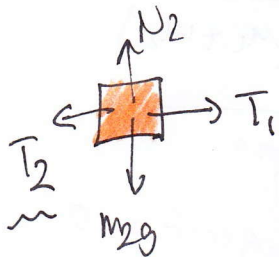
$\Rightarrow$



$$\sum F_y = m_T a_y \Rightarrow N_T = m_T g$$

$$\sum F_x = m_T a_x \quad 132 = (60) a$$

$$a = 2.2 \text{ m/s}^2$$



$T_1 = ?$



$$\sum F_y = m_1 a_y \Rightarrow N_1 = m_1 g$$

$$\sum F_x = m_1 a_x \quad F - T_1 = m_1 a$$

$$132 - T_1 = 10(2.2)$$

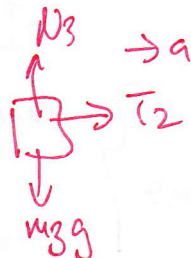
$$\underline{T_1 = 110\text{N}}$$

$T_2 = ?$

look @ 3rd object

$$\sum F_x = m_3 a$$

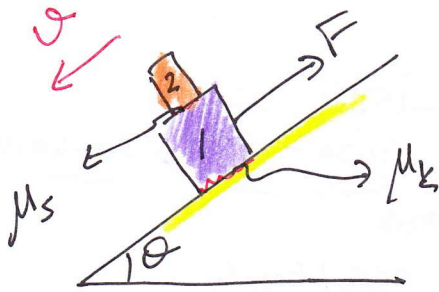
$$T_2 = (30)(2.2)$$



$$\underline{T_2 = 66\text{N}}$$

5.33)

given
 m_2
 m_1
 μ_k
 θ
 g

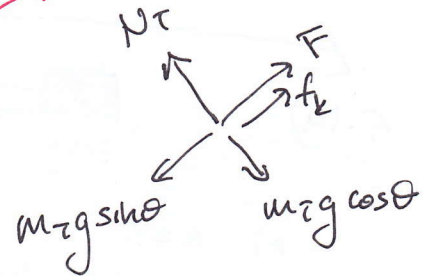
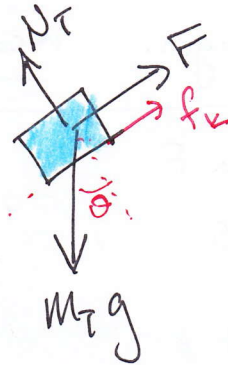


what is $F = ?$

objects are moving down together
 with const. v
 $\mu_k > 0$ $\mu_s > 0$

Draw FBD

if objects are moving together; you can study them as a one object



$$\sum F_y = m_T a_T \rightarrow 0$$

$$N_T - m_T g \cos \theta = 0$$

$$N_T = m_T g \cos \theta$$

$$f_k = \mu_k N_T = \mu_k m_T g \cos \theta$$

$$\sum F_x = m_T a_x \rightarrow 0 \quad (v = \text{const})$$

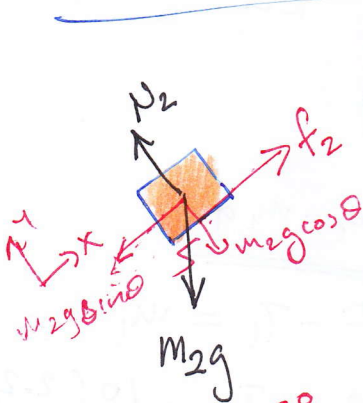
$$F + f_k - m_T g \sin \theta = 0$$

$$F = m_T g \sin \theta + \mu_k m_T g \cos \theta$$

$$F = m_T g (\sin \theta + \mu_k \cos \theta)$$

$$m_T = (m_1 + m_2)$$

Draw FBD of both

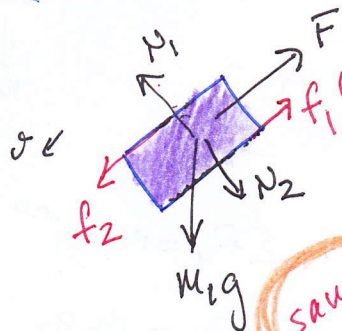


$$\sum F_x = m_2 a_x \rightarrow 0$$

$$f_2 - m_2 g \sin \theta = 0$$

$$f_2 = m_2 g \sin \theta$$

$$f_2 = \mu_s N_2 = \mu_s m_2 g \cos \theta$$



$$\sum F_x = m_1 a_x \rightarrow 0$$

$$F + f_1 - f_2 - m_1 g \sin \theta = 0$$

$$F = m_1 g \sin \theta + m_2 g \sin \theta - \mu_k (m_1 + m_2) g \cos \theta$$

$$m_2 g \sin \theta = \mu_s m_2 g \cos \theta$$

$$\mu_s = \tan \theta$$

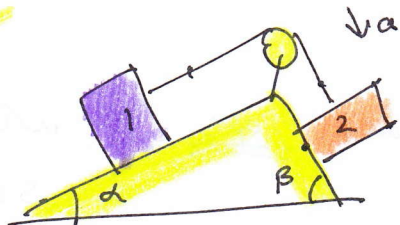
$$\sum F_y = m_1 a_y \rightarrow 0$$

$$N_1 - N_2 - m_1 g \cos \theta = 0$$

$$N_1 = m_1 g \cos \theta + m_2 g \cos \theta$$

$$f_1 = \mu_k N_1 = \mu_k (\dots)$$

7.90)



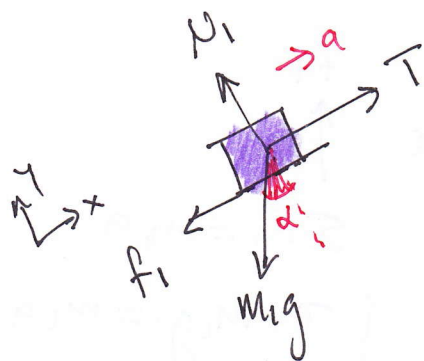
$\mu > 0$

$\frac{m_1}{m_2}$
 μ

FBD?

$a_z?$
 $=$

$T=?$
on the rope.

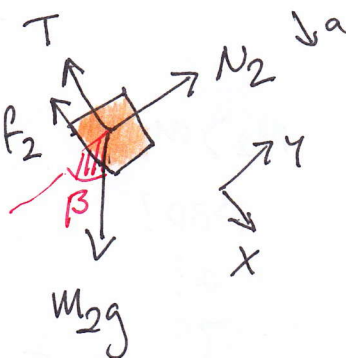


$$\sum F_y = m_1 a_y^{\text{ro}}$$

$$N_1 = m_1 g \cos \alpha$$

$$\sum F_x = m_1 a_x$$

$$\textcircled{1} \quad T - f_1 - m_1 g \sin \alpha = m_1 a$$



$$\sum F_y = m_2 a_y^{\text{ro}}$$

$$N_2 = m_2 g \cos \beta$$

$$\sum F_x = m_2 a_x$$

$$\textcircled{2} \quad m_2 g \sin \beta - T - f_2 = m_2 a$$

$\textcircled{1} + \textcircled{2}$

$$m_2 g \sin \beta - m_1 g \sin \alpha - f_1 - f_2 = (m_1 + m_2) a$$

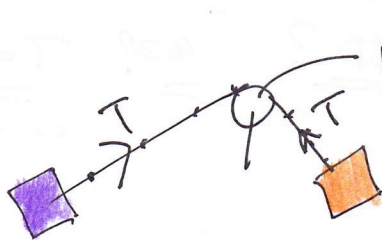
$$\rightarrow m_2 g \sin \beta - m_1 g \sin \alpha - \mu m_1 g \cos \alpha - \mu m_2 g \cos \beta = (m_1 + m_2) a$$

$$a = g \frac{m_2 \sin \beta - m_1 \sin \alpha - \mu (m_1 \cos \alpha + m_2 \cos \beta)}{(m_1 + m_2)}$$

$T=?$ plug in a into $\textcircled{1}$

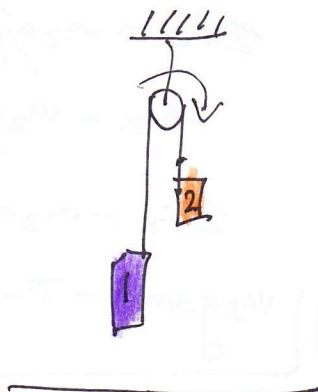
$$T = m_1 a + f_1 + m_1 g \sin \alpha$$

$$= m_1 a + \mu m_1 g \cos \alpha + m_1 g \sin \alpha$$



pulley makara

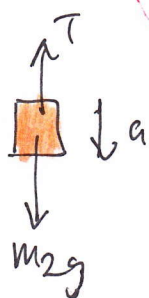
if it's the same rope it will have same Tension value.



$m_2 > m_1$
FBD?
 $a?$
 $T?$



$$\begin{aligned} \sum F_y &= m_1 a \\ T - m_1 g &= m_1 a \quad (1) \end{aligned}$$



$$\begin{aligned} \sum F_y &= m_2 a_y \\ m_2 g - T &= m_2 a \quad (2) \end{aligned}$$

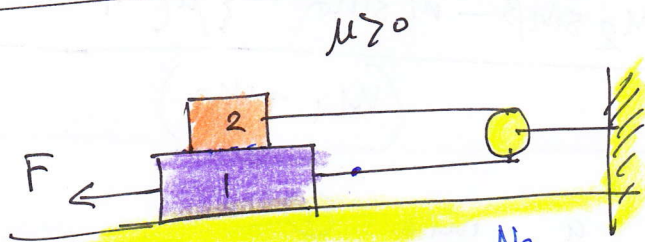
① + ②

$$m_2 g - m_1 g = (m_1 + m_2) a$$

$$a = \frac{m_2 - m_1}{m_1 + m_2} g$$

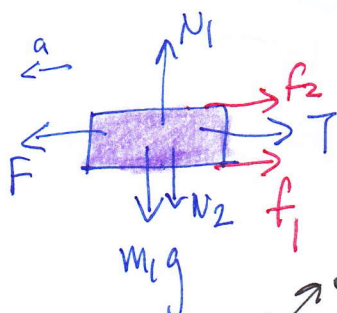
$$T = m_1 a + m_1 g \quad \checkmark$$

5.87)



F
 m_1
 m_2
 μ
g

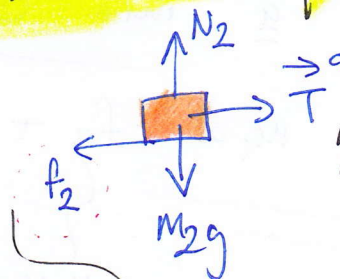
$a=?$
 $T=?$



$$\sum F_y = m_1 a_y$$

$$N_1 - N_2 - m_1 g = 0$$

$$N_1 = m_1 g + m_2 g$$



$$N_2 = m_2 g$$

$$\sum F_x = m_2 a$$

$$(1) \quad T - f_2 = m_2 a$$

$$T - \mu m_2 g = m_2 a$$

$$\sum F_x = m_1 a_x$$

$$F - T - f_1 - f_2 = m_1 a \quad (2)$$

① + ② $\Rightarrow F - f_1 - 2f_2 = (m_1 + m_2) a$

$$f_1 = \mu N_1$$

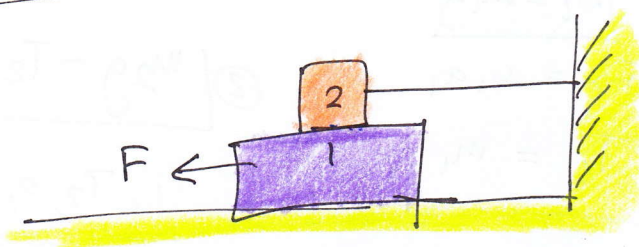
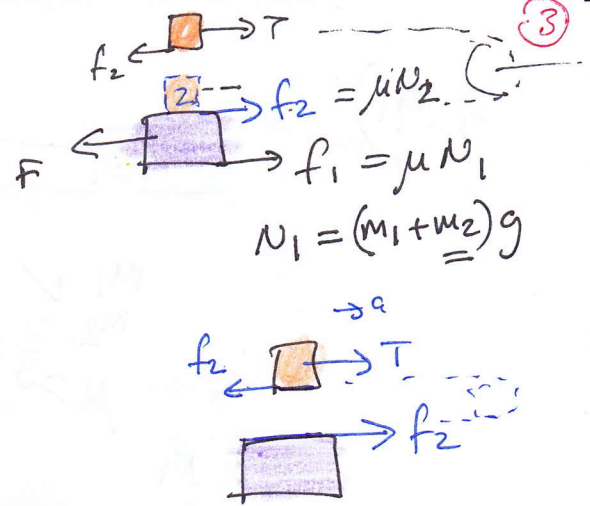
$$f_2 = \mu N_2$$

$$\boxed{\frac{F - \mu(m_1 + m_2)g - 2\mu(m_2 g)}{m_1 + m_2} = a}$$

$$a = \frac{F - \mu m_1 g - 3\mu m_2 g}{m_1 + m_2}$$

$$T = m_2 a + \mu m_2 g$$

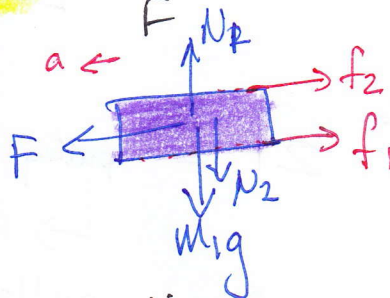
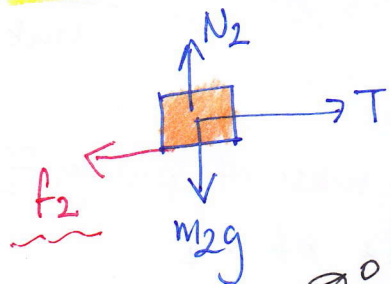
$$T = m_2 \left(\frac{F - \mu m_1 g - 3\mu m_2 g}{m_1 + m_2} + \mu g \right)$$



m_2
 m_1
 $\mu \Rightarrow$ kinetic
 g
 F

a of m_1 ?

Tension on the rope.



$$N_2 = m_2 g$$

$$N_1 = (m_1 + m_2)g$$

$$f_1 = \mu N_1$$

$$f_2 = \mu N_2$$

$$\sum F_x = m_2 a_x$$

$$T - f_2 = 0 \Rightarrow T = f_2 = \mu N_2 = \underline{\underline{\mu m_2 g}}$$

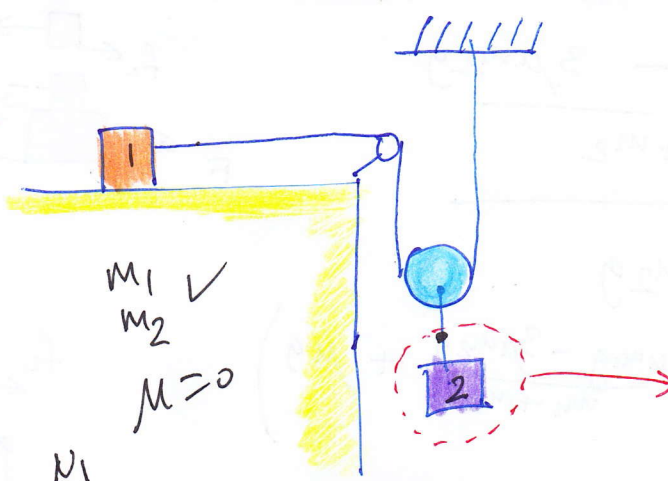
1st obj

$$\sum F_x = m_1 a$$

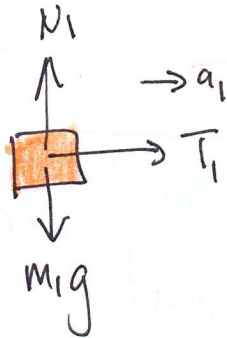
$$F - f_2 - f_1 = m_1 a ; a = \frac{F - f_2 - f_1}{m_1}$$

$$a = \frac{F - \mu m_2 g - \mu (m_1 + m_2)g}{m_1} = \underline{\underline{\frac{F - \mu m_1 g - 2\mu m_2 g}{m_1}}}$$

5.91)



$m_1 \checkmark$
 $m_2 \checkmark$
 $\mu = 0$



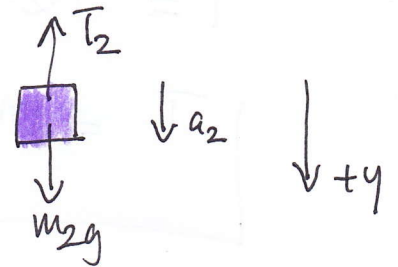
$$\sum F_y = m_1 a_1^{\rightarrow 0}$$

$$| N_1 = m_1 g$$

$$\sum F_x = m_1 a_1$$

$$\textcircled{1} \quad T_1 = m_1 a_1$$

FBD ✓
 $a = ?$ of each body?
 T on each rope?

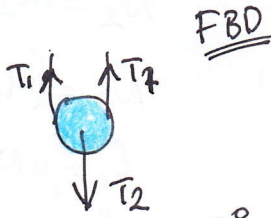


$$\sum F_y = m_2 a_y$$

$$\textcircled{2} \quad m_2 g - T_2 = m_2 a_2$$

T_1, T_2, a_1, a_2
Unknown

pulley moving



$$\sum F_y = m_{\text{pulley}} a$$

$$2T_1 - T_2 = 0$$

$$\sum F = m_{\text{pulley}} (a \neq 0)$$

↳ assume mass of pulley ≈ 0

a of pulley = a of m_2

(ignore the mass of pulleys!)

$$\textcircled{3} \quad 2T_1 = T_2$$

$$T_1 = m_1 a_1$$

$$m_2 g - 2T_1 = m_2 a_2$$

need one more eqn!!
 $a_1 \leftrightarrow a_2$

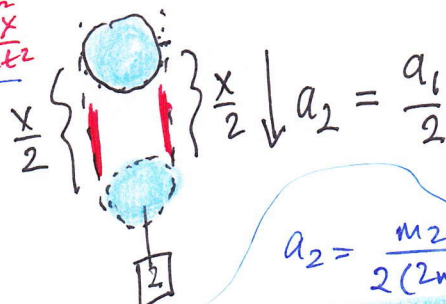
$$a_1 = a_2 = a$$

$$a_1 = \frac{d^2 x}{dt^2} = a$$

$$a_2 = \frac{1}{2} \frac{d^2 x}{dt^2} = \frac{a}{2}$$

$$\frac{a_1}{a_2} = 2$$

$x \rightarrow \frac{dx}{dt} = v \rightarrow \frac{dv}{dt} = a$
 $x_1 \Rightarrow x$
 $x_2 \Rightarrow \frac{x}{2}$
 $a_1 = \frac{d^2 x}{dt^2}$
 $a_2 = \frac{1}{2} \frac{d^2 x}{dt^2}$



$$\textcircled{1} \quad T_1 = m_1 a_1$$

$$\textcircled{2} \quad m_2 g - 2T_1 = m_2 \frac{a_1}{2}$$

$$2 \times \textcircled{1} + \textcircled{2}$$

$$m_2 g = 2m_1 a_1 + m_2 a_1$$

$$\frac{m_2}{2m_1 + m_2} g = a_1$$