Phys1 -04

Thursday, December 1, 2022 10:38 AM

Momentum Collisions Impulse

$\Sigma\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$ $\qquad$ $\Sigma\vec{F} = \frac{d\vec{p}}{dt}$ momentum general case

$\Sigma\vec{F}\,dt = m\,d\vec{v}$

$\underbrace{\Sigma\vec{F}\,\Delta t}_{\text{impulse }(\vec{I}\text{ve})} = \underbrace{m\,\Delta\vec{v}}_{\text{change in momentum}} = \Delta\vec{p}$; $\boxed{\vec{p} \equiv m\vec{v} \text{ momentum.}}$

$\frac{d}{dt}\left(\vec{p} = m\vec{v}\right)$

$\underline{\text{Force}} = \vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \underbrace{\frac{dm}{dt}}\vec{v}$ (rocket eqn...?!)

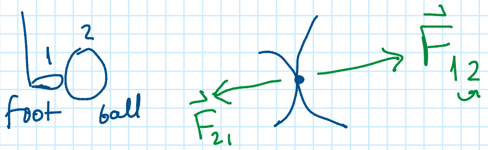
$\left\{\Sigma\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} \Rightarrow m = \text{const}\right\}$

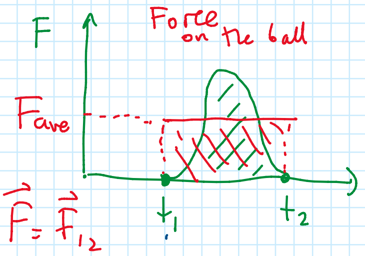
$\Sigma\vec{F}\,dt \equiv \vec{J} = \text{impulse} = [Ns] = [kg\frac{m}{s}] \equiv P = mV = [kg\frac{m}{s}]$

when two objects interact through contact forces.

action-reaction pairs

$\vec{F}_{12} = -\vec{F}_{21}$



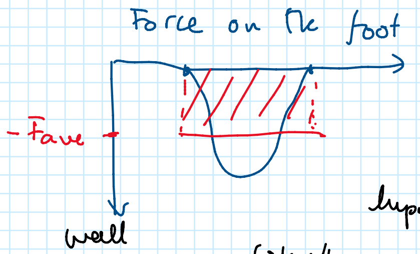


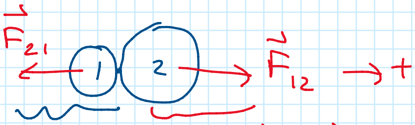
$\vec{F} = \vec{F}_{12}$

$\int\vec{F}\,dt \equiv \vec{J} \equiv \text{impulse} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$

area under curve

$\int\vec{F}\,dt = \vec{F}_{ave}\,\Delta t \equiv \vec{J} = \vec{P}_f - \vec{P}_i$



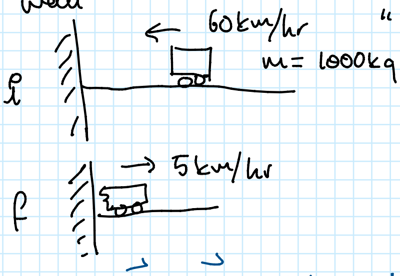


$\leftarrow \vec{J} = \Delta\vec{P}$ $\qquad$ $\rightarrow \vec{J} = \Delta\vec{P}$

Impulse on 2^{nd} obj = change in momentum of 2^{nd} obj

" " 1^{st} " = " " 1^{st} .

(ex)



the collision took 0.1s.

$\vec{F}_{ave} = ?$

$\frac{60\,km}{hr} = \frac{60\,000\,m}{3600\,s} = \frac{60}{3.6}$

$\vec{v}_i = -16.7\,m/s\,\hat{i}$

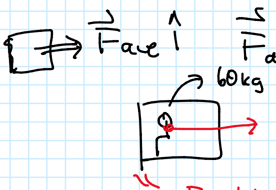
$\vec{v}_f = +1.4\,m/s\,\hat{i}$

$\vec{J} = \vec{F}_{ave}\,\Delta t = \Delta\vec{P} \quad \rightarrow \hat{i}$

$?\,(0.1) = m\vec{v}_f - m\vec{v}_i$

$\vec{F}_{ave}\,(0.1) = 1000\left(1.4 - (-16.7)\right)\hat{i}$

$\vec{F}_{ave} = +18.1\times10^4\,N\,\hat{i} = 181\,000\,N$



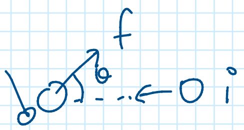
F on the person $= \frac{181\,000\,N}{(1000/60)} = 181\times$ (100kg)

" Buckle up ! "

(ex) $\qquad$ $F_{ave} = ?$ $\quad$ $\vec{J} = ?$

Buckle up!

ex



$\vec{F}_{ave} = ?$ $\vec{J} = ?$

$v_i = 20\ m/s$

$v_f = 30\ m/s$

$\theta = 45°$

$m_{ball} = 0.4\ kg$

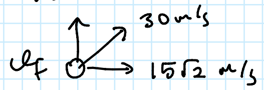
$\Delta t = 0.01\ s$

$\vec{J} = \vec{F}_{ave}\,\Delta t = \Delta\vec{P}$

$\vec{v}_i = -20\hat{i}$

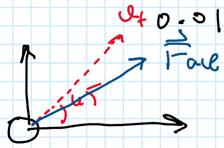
$\vec{v}_f = 15\sqrt{2}\,\hat{i} + 15\sqrt{2}\,\hat{j}$

$15\sqrt{2}\ m/s$ $\vec{F}_{ave} = ?$



$\vec{F}_{ave}(0.01) = 0.4\left(15\sqrt{2}\hat{i} + 15\sqrt{2}\hat{j} - (-20\hat{i})\right)$

$$\vec{F}_{ave} = \frac{16.5\hat{i} + 8.5\hat{j}}{0.01} = \underbrace{1650\hat{i} + 850\hat{j}}\ N$$



$|\vec{F}_{ave}| = 1850\ N$

$\vec{J} = \Delta\vec{P} = (16.5\hat{i} + 8.5\hat{j})\ kg\,\frac{m}{s}$

Momentum Conservation in Collisions

$\vec{F}_{21} \leftarrow (1)(2) \rightarrow \vec{F}_{12}$

$\vec{F}_{12} = -\vec{F}_{21}$

$\vec{F}_{12} + \vec{F}_{21} = 0$

$(F_{12} + F_{21})\,\Delta t = 0$

$\vec{F}_{12}\Delta t + \vec{F}_{21}\Delta t = 0$

impulse on 2nd — impulse on 1st

$\Downarrow$ $\Downarrow$

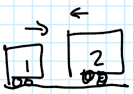
$\Delta\vec{P}_2 + \Delta\vec{P}_1 = 0$

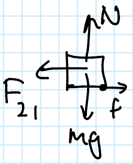
$\Delta\vec{P}_2 + \Delta\vec{P}_1 = 0$

$\vec{P}_{2f} - \vec{P}_{2i} + \vec{P}_{1f} - \vec{P}_{1i} = 0$

$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$ *

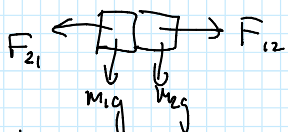
$\left\{ \Sigma\vec{P}_i = \Sigma\vec{P}_F \right\}$





$\Sigma\vec{F} = \vec{F}_{21} \Rightarrow$ mom is conserved !!

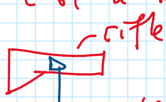
$F_{21} \gg f$ f is neglected.



$\Sigma\vec{F} \neq F_2 \equiv$ mom. is NOT conserved.

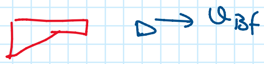
ex Recoil of a Rifle

collision ≡ explosion



$v_{Ri} = 0$

$v_{Bi} = 0$



$v_{Rf} \neq 0$

$m_{Bullet} = 5\ gr$

$m_{rifle} = 3\ kg$

$v_{BF} = 300\ m/s$

$v_{Rf} = ?$

$$\Sigma\vec{P}_i = \Sigma\vec{P}_f$$

$$m_R\vec{v}_{Ri} + m_B\vec{v}_{Bi} = m_B\vec{v}_{BF} + m_R\vec{v}_{RF} \qquad \rightarrow +\hat{i}$$

$$0 + 0 \qquad 0 = (0.005)\,300\hat{i} + 3\,\vec{v}_{RF}$$

b) what's the final Kinetic Energy of Bullet & Rifle

$$\vec{v}_{RF} = -\frac{1.5\,\hat{i}}{3} = -0.5\,\hat{i}\ m/s$$

$$\frac{1}{2} m_R v_{RF}^2 = \frac{1}{2}\,3\,(0.5)^2 = 0.375\ J \qquad \frac{1}{2} m_B v_{BF}^2 = \frac{1}{2}(0.005)(300)^2 = 225\ J$$

$$K_B \gg K_R$$

ex) $\boxed{A} \rightarrow 2\ m/s \qquad \xleftarrow{2\ m/s} \boxed{B}$

$\boxed{A} \quad \boxed{B} \rightarrow 2\ m/s$

$m_A = 0.5\ kg$
$m_B = 0.3\ kg$
$\vec{v}_{AF} = ?$

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f \qquad \Rightarrow +\hat{i}$$

$$m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{AF} + m_B \vec{v}_{BF}$$

$$0.4\hat{i} = 0.5(2\hat{i}) + 0.3(-2\hat{i}) = 0.5\,\vec{v}_{AF} + 0.3(2\hat{i})$$

$$0.4\hat{i} = 0.5\,\vec{v}_{AF} + 0.6\hat{i}$$

$$\vec{v}_{AF} = \frac{-0.2\hat{i}}{0.5} = -0.4\ m/s$$

$\xleftarrow{0.4\ m/s} \square$

Is energy conserved in this collision?

Energy $= K + U$ (U → const)

$$\frac{1}{2} m v^2 = \frac{1}{2} m v^2 \frac{m}{m} = \frac{1}{2}\frac{P^2}{m} = K$$

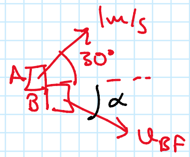
$$K_i = \frac{1}{2}(0.5)\,2^2 + \frac{1}{2}(0.3)\,2^2$$

$$K_f = \frac{1}{2}(0.5)(0.4)^2 + \frac{1}{2}(0.3)\,2^2$$

$$K_f < K_i$$

$$\left(\frac{K_f - K_i}{K_i} \times 100 = \%\ \text{of energy LOST}\right)$$

ex) $\boxed{A} \rightarrow 2\ m/s$, 20 kg $\qquad$ 12 kg $\boxed{B}$, $v_{Bi} = 0$



$\vec{v}_{Bf} = ?$
$\alpha = ?$

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$20(2\hat{i}) + 0 = 20(1)\cos 30\,\hat{i} + 20(1)\sin 30\,\hat{j} + 12\,\vec{v}_{BF}$$

$$40\hat{i} = 17.3\hat{i} + 10\hat{j} + 12\,\vec{v}_{BF}$$

$$\frac{22.7\hat{i} - 10\hat{j}}{12} = \vec{v}_{BF} = (1.89\hat{i} - 0.83\hat{j})\ m/s$$

$$|\vec{v}_{BF}| = (1.89^2 + 0.83^2)^{1/2} = 2.06\ m/s$$

$$\alpha = \tan^{-1}\left(\frac{-0.83}{1.89}\right) = -24°$$

ELASTIC & INELASTIC COLLISIONS

$\vec{P}$ is conserved when interaction forces are the main ones that contribute

$F \leftarrow \bigcirc\bigcirc \rightarrow F$ (mg, mg) $\qquad F \gg mg \Rightarrow \vec{P}$ is conserved.

In our cases $\vec{P}$ will be conserved always.

Energy
- conserved: Elastic — $\Sigma K_i = \Sigma K_f$
- not conserved: Inelastic — $\Sigma K_i \neq \Sigma K_f$

Elastic Inelastic

$\Sigma K_i = \Sigma K_f$ $\Sigma K_i \neq \Sigma K_f$

Inelastic collisions

i f

inelastic collision $\Rightarrow$

Rifle + Bullet

$v_i = 0$

$\Sigma K_i \neq \Sigma K_f$

$\rightarrow 2\,m/s$ $\leftarrow 2\,m/s$

0.5 kg 0.3 kg

$\leftarrow 0.4\,m/s$ $\rightarrow 2\,m/s$

$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$

0.64 J

1.6 J > 0.64 J

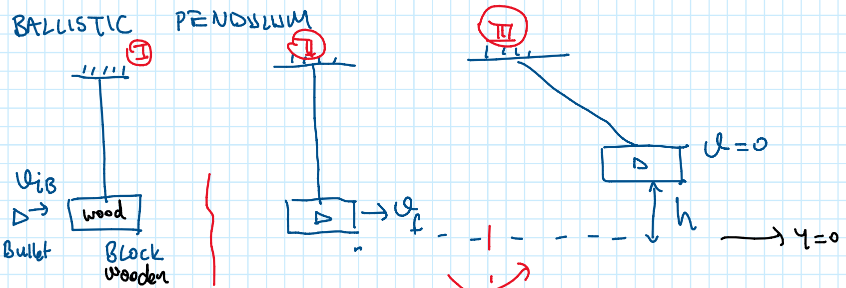
energy lost $\frac{\Delta K}{K_i} = \frac{0.96}{1.6}$

Inelastic!!

ELASTIC COLLISION: (must be stated in the question!)

spring is storing the energy during collision

BALLISTIC PENDULUM



inelastic collision

$\Sigma \vec{P}_i = \Sigma \vec{P}_f$

energy conserved

$\Sigma E_i = \Sigma E_f$ $E = K + U$

① → ② P is conserved

K is NOT conserved

② → ③ total energy is conserved

what's the Bullet's initial velocity?

$m_B v_{Bi} + \underbrace{m_w v_{wi}}_{0 \text{ at rest}} = (m_B + m_w) v_f$ } $* \; m_B \underset{?}{v_{Bi}} = (m_B + m_w) v_f$

$m_B v_{Bi} = (m_B + m_w)\sqrt{2gh}$

② → ③ $\Sigma E_i = \Sigma E_f$

$K_i + U_i = K_f + U_f$

$\frac{1}{2}(m_B + m_w) v_f^2 + 0 = 0 + (m_B + m_w) g h \Rightarrow v_f^2 = 2gh$

$$\boxed{v_{Bi} = \frac{m_B + m_w}{m_B}\sqrt{2gh}}$$

$m_B = 5\,gr$ $m_w = 2\,kg$

$h = 3\,cm$

$v_{Bi} = \left(\frac{2.005}{0.005}\right)\sqrt{2(9.8)(0.03)}$

$$v_{Bi} = \left(\frac{2.005}{0.005}\right)\sqrt{2(9.8)(0.03)}$$

$$= 307 \text{ m/s} \checkmark$$

How much energy is lost?

$$K_i = \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2}(0.005)(307)^2$$

$$= 236 \text{ J}$$

$$v_f = \sqrt{2gh} = 0.76 \text{ m/s}$$

$$K_f = \frac{1}{2}(m_B + m_w) v_f^2 = (m_B + m_w) g h = U_f = 0.59 \text{ J} \simeq 0.6 \text{ J}$$

$$\frac{\Delta K}{K_i} = \frac{236 - 0.6}{236} = 0.997 ; \quad \underline{99.7\% \text{ is lost}}$$

* where did the energy go?

f d

$$W_f = -f d = (0.6 - 236) \text{ J}$$

$d = 10$ cm

$$\underline{f = 2354 \text{ N}}$$

ELASTIC COLLISION

A v_{Ai} B v_{Bi} | A v_{AF} B v_{BF}

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$\Sigma K_i = \Sigma K_f$$

$$m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$$

$$\frac{m_A v_{Ai}^2}{2} + \frac{m_B v_{Bi}^2}{2} = \frac{m_A v_{Af}^2}{2} + \frac{m_B v_{Bf}^2}{2}$$

m_A m_B v_{Ai} v_{Bi} v_{AF} v_{BF} $\Rightarrow$ 6 parameters

1 dimensional elastic collision

A v_{Ai}, m_A B m_B, $v_{Bi} = 0$ | A v_{AF} B v_{Bf} $\rightarrow$ $\hat{i}$

m_A m_B v_{Bi} $\checkmark$

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$m_A v_{Ai} + m_B \cancel{v_{Bi}}^{\,0} = m_A v_{AF} + m_B v_{Bf}$$

$$\cancel{\tfrac{1}{2}} m_A v_{Ai}^2 + 0 = \cancel{\tfrac{1}{2}} m_A v_{Af}^2 + \cancel{\tfrac{1}{2}} m_B v_{Bf}^2$$

$$\rightarrow m_B v_{BF} = m_A (v_{Ai} - v_{AF}) \quad \text{— (1)}$$

$$\rightarrow m_B v_{BF}^2 = m_A (v_{Ai}^2 - v_{AF}^2) \quad \text{— (2)}$$

$$(v_{Ai} + v_{AF})(v_{Ai} - v_{AF})$$

$$\frac{(2)}{(1)} \Rightarrow \boxed{v_{BF} = v_{Ai} + v_{AF}} \rightarrow \text{into mom. eqn.}$$

$\left.\begin{array}{l} m_A \\ m_B \\ v_{Bi} \\ v_{Ai} \end{array}\right\}$ known

$v_{AF} = ?$

$v_{BF} = ?$

$$m_B v_{Ai} + m_B v_{AF} = m_A v_{Ai} - m_A v_{AF}$$

$$(m_B + m_A) v_{AF} = (m_A - m_B) v_{Ai}$$

$$\boxed{v_{AF} = \frac{m_A - m_B}{m_A + m_B} v_{Ai}}$$

$$v_{AF} + v_{Ai} = v_{BF}$$

$$\left(\frac{m_A - m_B}{m_A + m_B} + 1\right) v_{Ai} = v_{BF}$$

$$v_{BF} = \frac{2 m_A}{\quad} v_{Ai}$$

$$\left(\frac{m_A - m_B}{m_A + m_B} + 1\right) v_{Ai} = v_{BF}$$

$$\boxed{v_{BF} = \frac{2m_A}{m_A + m_B} v_{Ai}}$$

$\overset{v_{Ai}}{A \rightarrow}$ B, $v_{Bi} = 0$ $\Big\{$ $O \rightarrow v_{AF}$ $\quad$ $O \rightarrow v_{BF}$

if $m_A = m_B = ?$ $\quad v_{AF} = \frac{m-m}{2m} v_{Ai} = \underline{0}$ $\qquad v_{BF} = \frac{2m}{m+m} v_{Ai} = v_{Ai}$

$\boxed{\text{1st case}}$

$\overset{m}{O} \xrightarrow{v}$ $\overset{m}{O}$ $\Big|$ $\overset{m}{O}$ $\overset{m}{O} \rightarrow v$

2nd case $\quad m_A >> m_B$

$1000 >> 1$

$A \xrightarrow{v} O$ $\Big|$ $A \rightarrow v$ $\quad O \xrightarrow{2v}$

$$v_{AF} = \frac{m_A - m_B}{m_A + m_B} v = \frac{999}{1001} v \simeq \underline{v}$$

$$v_{BF} = \frac{2(1000)}{1001} v \simeq 2v$$

3rd case $\quad m_A << m_B$

$1 << 1000$

$O \xrightarrow{v}$ O $\Big|$ $\overset{v}{\leftarrow O}$ $\quad \overset{v \simeq 0}{O}$

$$v_{AF} = \frac{1-1000}{1001} v = -\frac{999}{1001} v \simeq -v$$

$$v_{BF} = \frac{2(1)}{1001} v = 0.002v \simeq \underline{0}$$

$A \rightarrow v$ $\quad O$, $v_{Bi} = 0$ $\Big\{$ $O \rightarrow v_{AF}$ $\quad O \rightarrow v_{BF}$

$$v_{AF} = \frac{m_A - m_B}{m_A + m_B} v \, ; \qquad v_{BF} = \frac{2m_A}{m_A + m_B} v$$

$A \rightarrow v_{Ai}$ $\quad O \rightarrow v_{Bi}$

$\Updownarrow$

$A \rightarrow v'_{Ai}$ $\quad O$, $v_{Bi} = 0$

$$v'_{Ai} \neq v_{Ai}$$

$$v'_{Ai} = v_{Ai} - v_{Bi}$$

$$v'_{AF} = \frac{m_A - m_B}{m_A + m_B} \underline{v'_{Ai}} \, ; \qquad v'_{BF} = \frac{2m_A}{m_A + m_B} \underline{v'_{Ai}}$$

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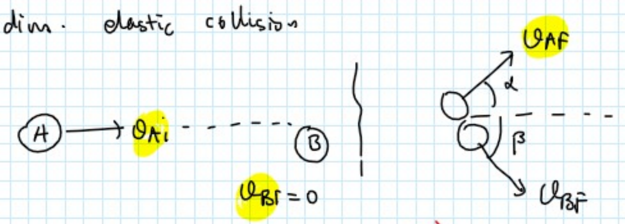
2 dim. elastic collision



m_A, m_B, v_{Bi}, v_{Ai}, v_{AF} given $\qquad v_{BF}$, α, β

$\underbrace{\qquad\qquad}_{\text{8 parameters}}$

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$m_A v_{Ai} \hat{i} + 0 \hat{i} = m_A v_{AF} \cos\alpha \, \hat{i} + m_B v_{BF} \cos\beta \, \hat{i} \quad \Big\} \text{ 2 eqn.}$$

$\Sigma \bar{P}_i = \Sigma P_f$ γ parallel:

$m_A v_{Ai} \hat{i} + 0 \hat{j} = m_A v_{AF} \cos\alpha \hat{i} + m_B v_{BF} \cos\beta \hat{i}$
$m_A v_{AF} \sin\alpha \hat{j} - m_B v_{BF} \sin\beta \hat{j}$ } 2 eqn.

($v_{Ai} \to 4$, $v_{AF} \to 2$)

$$\frac{m_A v_{Ai}^2}{\cancel{2}} + 0 = \frac{m_A v_{AF}^2}{\cancel{2}} + \frac{m_B v_{Bf}^2}{\cancel{2}}$$

$m_A = 0.5$ kg $\quad v_{Bi} = 0$ $\quad \alpha?$

$m_B = 0.3$ kg $\quad v_{Ai} = 4$ m/s $\quad v_{AF} = 2$ m/s $\quad \beta?$

$v_{Bf}?$

① $(0.5)\, 4\hat{i} = \left[(0.5)\, 2\cos\alpha + 0.3\, v_{Bf} \cos\beta\right] \hat{i}$

② $0\hat{j} = \left[(0.5)(2)\sin\alpha - (0.3)\, v_{Bf} \sin\beta\right] \hat{j}$

③ $(0.5)\, 4^2 = (0.5)\, 2^2 + (0.3)\, v_{Bf}^2$

① $2 = \cos\alpha + 0.3\, v_{Bf} \cos\beta$

② $\sin\alpha = 0.3\, v_{Bf} \sin\beta$

③ $\frac{6}{0.3} = v_{Bf}^2 \Rightarrow v_{BF} = \sqrt{20}$ m/s ✓

$2 = \cos\alpha + 0.3\sqrt{20} \cos\beta$

$\sin\alpha = 0.3\sqrt{20} \sin\beta$

$\sin^2\beta + \cos^2\beta = 1$

$$\frac{\sin^2\alpha}{0.3^2(20)} + \frac{(2-\cos\alpha)^2}{0.3^2(20)} = 1 \Rightarrow \frac{\sin^2\alpha + 4 + \cos^2\alpha - 4\cos\alpha}{0.3^2(20)} = 1$$

$$5 - 4\cos\alpha = 0.3^2(20)$$

$$\cos\alpha = \frac{5 - 0.3^2(20)}{4} = \frac{5 - 1.8}{4} = \frac{3.2}{4}$$

$$\alpha = \cos^{-1}\left(\frac{3.2}{4}\right) = \cos^{-1}(0.8) = 36.9°$$

$\beta = ?$ $\quad \sin\alpha = 0.3\sqrt{20} \sin\beta$

$$\frac{\sin(36.9°)}{0.6} = \quad \beta = \sin^{-1}\left(\frac{0.6}{0.3\sqrt{20}}\right) = \sin^{-1}\left(\frac{2}{\sqrt{20}}\right) = 26.6° \; \beta$$

GENERAL FORMULA for α

$m_A v_{Ai} \hat{i} + 0 \hat{j} = m_A v_{AF} \cos\alpha \hat{i} + m_B v_{BF} \cos\beta \hat{i}$
$m_A v_{AF} \sin\alpha \hat{j} - m_B v_{BF} \sin\beta \hat{j}$ } 2 eqn.

$$\frac{m_A v_{Ai}^2}{\cancel{2}} + 0 = \frac{m_A v_{AF}^2}{\cancel{2}} + \frac{m_B v_{BF}^2}{\cancel{2}}$$

$$m_A v_{Ai} = m_A v_{AF}\cos\alpha + m_B v_{BF}\cos\beta$$

v_{AF}; m_A, m_B, v_{Ai} given

$$m_A v_{AF}\sin\alpha = m_B v_{BF}\sin\beta$$

$$m_A v_{Ai}^2 = m_A v_{AF}^2 + m_B v_{BF}^2 \Rightarrow v_{BF} = \sqrt{\frac{m_A}{m_B}\left(v_{Ai}^2 - v_{AF}^2\right)}$$

solved!

$$\frac{m_A\left(v_{Ai} - v_{AF}\cos\alpha\right)}{m_B v_{BF}} = \cos\beta \qquad \sin\beta = \frac{m_A v_{AF}}{m_B v_{BF}}\sin\alpha$$

(we know v_{BF} now!)

$$\cos^2\beta + \sin^2\beta = 1$$

$$\frac{m_A^2}{m_B^2 v_{BF}^2}\left[v_{Ai}^2 + v_{AF}^2\cos^2\alpha - 2v_{Ai}v_{AF}\cos\alpha\right] + \frac{m_A^2}{m_B^2 v_{BF}^2}\left[v_{AF}^2\sin^2\alpha\right] = 1$$

$$\frac{m_A^2}{m_B^2 v_{BF}^2}\left[v_{Ai}^2 + \underbrace{v_{AF}^2\cos^2\alpha + v_{AF}^2\sin^2\alpha}_{v_{AF}^2} - 2v_{Ai}v_{AF}\cos\alpha\right] = 1$$

$$\underbrace{\frac{m_A^2}{m_B^2 v_{BF}^2}}_{c^2}\left[v_{Ai}^2 + v_{AF}^2 - 2v_{Ai}v_{AF}\cos\alpha\right] = 1$$

$$c^2\left(v_{Ai}^2 + v_{AF}^2 - 2v_{Ai}v_{AF}\cos\alpha\right) = 1$$

$$c^2\left(v_{Ai}^2 + v_{AF}^2\right) - 1 = 2c^2 v_{Ai}v_{AF}\cos\alpha$$

$$\boxed{\alpha = \cos^{-1}\left(\frac{c^2\left(v_{Ai}^2 + v_{AF}^2\right) - 1}{2c^2 v_{Ai}v_{AF}}\right) \quad c = \frac{m_A}{m_B v_{BF}} \quad v_{BF} = \sqrt{\frac{m_A}{m_B}\left(v_{Ai}^2 - v_{AF}^2\right)}}$$

$$m_A v_{Ai} = m_A v_{AF}\cos\alpha + m_B v_{BF}\cos\beta$$

$$\cos\beta = \frac{m_A v_{Ai} - m_A v_{AF}\cos\alpha}{m_B v_{BF}} = \frac{m_A v_{Ai} - m_A v_{AF}\left(\frac{c^2\left(v_{Ai}^2 + v_{AF}^2\right) - 1}{2c^2 v_{Ai}v_{AF}}\right)}{m_B v_{BF}} = \frac{m_A v_{Ai} - m_A v_{AF}\frac{\left(m_A^2\left(v_{Ai}^2 + v_{AF}^2\right) - m_B^2 v_{BF}^2\right)}{2m_A^2 v_{Ai}v_{AF}}}{m_B v_{BF}}$$

$$\cos\beta = \frac{2m_A^3 v_{Ai}^2 v_{AF} - m_A^3 v_{AF}^3 - m_A^3 v_{Ai}^2 v_{AF} + m_A m_B^2 v_{AF}v_{BF}^2}{2m_A^2 v_{Ai}v_{AF}\, m_B v_{BF}} = \frac{m_A^{\cancel{3}\,2} v_{Ai}^2 v_{AF} + \cancel{m_A} m_B^2 v_{AF}v_{BF}^2 - m_A^{\cancel{3}\,2} v_{AF}^3}{2m_A^{\cancel{2}} v_{Ai}v_{AF}\, m_B v_{BF}}$$

$$\boxed{\beta = \cos^{-1}\left(\frac{m_A^2 v_{Ai}^2 v_{AF} + m_B^2 v_{AF}v_{BF}^2 - m_A^2 v_{AF}^3}{2m_A v_{Ai}v_{AF}\, m_B v_{BF}}\right) \; ; \; v_{BF} = \sqrt{\frac{m_A}{m_B}\left(v_{Ai}^2 - v_{AF}^2\right)}}$$

$$\beta = \cos^{-1}\left(\frac{0.5^2(4^2)\,2 + 0.3^2\,2(20) - 0.5^2\,2^3}{2\,(0.5)\,4\,(2)\,(0.3)\sqrt{20}}\right) = \cos^{-1}\left(\frac{48/5}{2.4\sqrt{20}}\right) = \cos^{-1}\left(\frac{20}{5\sqrt{20}}\right) = \cos^{-1}\left(\frac{4}{\sqrt{20}}\right)$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{5}}\right) = 26.56^\circ \simeq 27^\circ$$

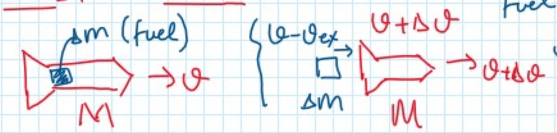
$$v_{BF} = \sqrt{\frac{0.5}{0.3}\left(4^2 - 2^2\right)} = \sqrt{\frac{6}{0.3}} = \sqrt{20} \text{ m/s}$$

$$C = \frac{0.5}{0.3\sqrt{20}} \Rightarrow C^2 = \frac{25}{9(20)} = \frac{5}{36} \qquad \alpha = \cos^{-1}\left(\frac{\frac{5}{36}\left(4^2+2^2\right) - 1}{(2)\frac{5}{36}(4)(2)}\right) = \cos^{-1}\left(\frac{64/36}{80/36}\right)$$

$$= \cos^{-1}\left(\frac{8}{10}\right) = 36.9^\circ$$

ROCKET PROPULSION:

fuel is exhausted / ejected with v_{ex} velocity w.r.t. the rocket.



$i \qquad\qquad f$

$$\sum \vec{P}_i = \sum \vec{P}_f \qquad \begin{matrix}\rightarrow v + \Delta v \\ \rightarrow v - v_{ex}\end{matrix}$$

$$(M + \Delta m)\, v = M(v + \Delta v) + m(v - v_{ex})$$

$$Mv + \Delta m v = Mv + M\Delta v + \Delta m v - \Delta m v_{ex}$$

$$M\Delta v = +\Delta m\, v_{ex}$$

(M = mass of the rocket body
Δm = " " " fuel
Δv = gain in velocity
v_{ex} = exhaust velocity

M, m are related.

$$M_T = M_{body} + M_{fuel}$$
$$1000\text{ kg} = 200\text{ kg} + 800\text{ kg}$$

M_T: 1000, 999) −1 | Δm fuel: +1 kg

$\Delta M = -1$ kg

$$\Delta m = -\Delta M$$
$$1\text{ kg} = -1\text{ kg}$$
$$M\Delta v = \Delta m\, v_{ex} = -\Delta M\, v_{ex}$$

$$\boxed{M\Delta v = -\Delta M\, v_{ex}} \;*$$
$$\boxed{M\, dv = -v_{ex}\, dM}$$

$$\int_i^f -\frac{dv}{v_{ex}} = \int_i^f \frac{dM}{M} =$$

$$\frac{1}{v_{ex}}\left(-\Delta v\Big|_{v_i}^{v_f}\right) = \Big[\ln M\Big]_{M_i}^{M_f}$$

$$\frac{1}{v_{ex}}(v_i - v_f) = \ln M_f - \ln M_i = \ln\frac{M_f}{M_i}$$

$$v_f - v_i = v_{ex}\ln\frac{M_i}{M_f}$$

velocity of the rocket
$$\boxed{v_f = v_i + v_{ex}\ln\frac{M_i}{M_f}}$$

$$\hookrightarrow v(t) = v_0 + v_{ex}\ln\frac{M_0}{M(t)}$$

$$\frac{dv}{dt} = a = \qquad \frac{dM}{dt} \equiv \text{know this!!}$$

$$\frac{1}{\Delta t}(M\Delta v) = -v_{ex}\frac{\Delta M}{\Delta t}$$

$$\vec{F} = \frac{\Delta\vec{P}}{\Delta t} = -\vec{v}_{ex}\frac{\Delta M}{\Delta t}$$

$$\vec{J} = \vec{F}\Delta t = \Delta\vec{P}$$
$$\vec{F} = \frac{\Delta\vec{P}}{\Delta t}$$

$$Ma = \vec{F} = -\vec{v}_{ex}\frac{\Delta M}{\Delta t} = Ma$$

Force on the rocket

$$P = mv$$

$$\frac{dP}{dt} = F = \frac{dm}{dt}v + m\frac{dv}{dt}$$

ma

ex) A rocket ejects fuel ...

$\frac{d\,}{dt} = a = \qquad \frac{dm}{dt} \equiv$ know this!!

ex) A rocket ejects fuel in the 1st second; it ejects its $\frac{1}{120}$ of its initial mass at a speed of $2400\,m/s$.
what's the acceleration of the rocket?

$$F = \frac{dM}{dt} v_{ex} = Ma \Rightarrow$$

$$= \frac{(M_0/120)}{1s} 2400 = M_0\, a \qquad a = \frac{2400}{120} = 20\ m/s^2$$

b) if $v_i = 0$ and $\frac{3}{4}$ of the mass of the rocket is fuel.
all the fuel is consumed at a constant rate in 90 seconds. $\left(\frac{dm}{dt} = \text{const}\right)$

$v_f = ?$

$$v_f = v_i + v_{ex} \ln \frac{M_0}{(M_0/4)} \quad ; M_f = M_0 - \frac{3}{4} M_0$$

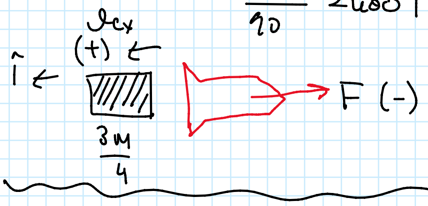
$$v_f = 0 + 2400 \ln 4 = 3327\ m/s \quad @\ 90s$$

c) if $M = 1000\,kg$; what's force on the rocket for 90s.

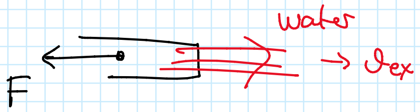
$$\vec{F} = \frac{dM}{dt}\vec{v}_{ex} = \frac{\Delta M}{\Delta t}\vec{v}_{ex} = \frac{M_f - M_i}{t_f - t_i}\vec{v}_{ex} = \frac{\frac{1000}{4} - 1000}{90 - 0} 2400$$

$$= \frac{-750}{90} 2400\,\hat{i} = -20\,000\,N = -20\,kN\,\hat{i}$$

opposite direction with v_{ex}



Fire hose



$$F = \frac{dM}{dt} v_{ex}$$

$$F = \left[\frac{kg}{s}\ \frac{m}{s}\right] = \left[kg\ \frac{m}{s^2}\right] = N$$

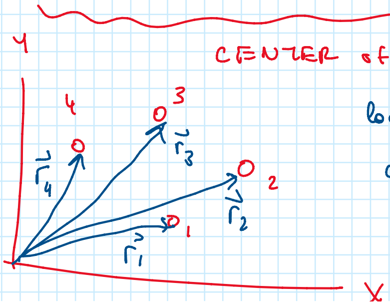
ex) if water is exhausted at $3600\,\frac{L}{min}$ from a fire hose
force applied on the hose 600 N. $v_{ex} = ?$

$$F = \frac{dM}{dt} v_{ex} \qquad \frac{3600\,L}{60\,s} \quad (L \to 1kg) \quad = \frac{3600\,kg}{60\,s} = 60\,\frac{kg}{s}$$

$$600 = (60)\, v_{ex}$$

$$\left\{ v_{ex} = 10\ m/s \right\}$$

CENTER of MASS

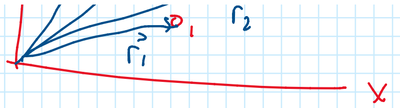


$$M_T = \sum m_i$$

location of center of mass

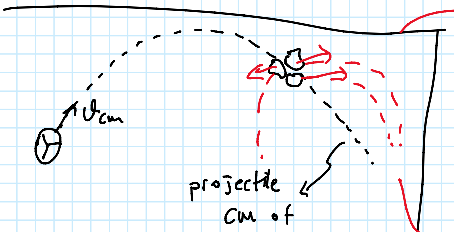
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{\underbrace{m_1 + m_2 + \cdots}_{\text{total mass} = M_T}}$$

$$M_T\,\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots$$



mass $m_1 + m_2 + \dots$ {total mass $= M_T$

$$M_T \vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \dots$$



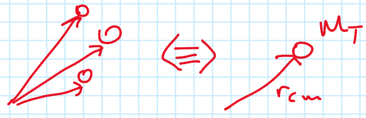
$$* \left[M_T \vec{r}_{cm} = \sum m_i \vec{r}_i \right]$$

$\frac{d}{dt}$

$$M_T \frac{d\vec{r}_{cm}}{dt} = \sum m_i \frac{d\vec{r}_i}{dt}$$

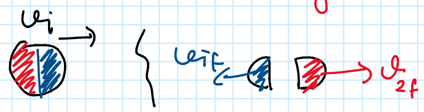
$$\left[M_T \vec{v}_{cm} = \sum m_i \vec{v}_i \right]$$

total momentum = addition of mom. of the little pieces.



If $\sum \vec{F}_{external}$ on the system is ZERO $\Rightarrow$ $\sum \vec{P} = \text{constant}$

$$M_T \vec{v}_{cm} = \sum m_i \vec{v}_i$$

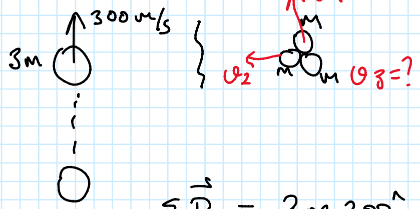


$t=0$ $\quad$ $t>0$

i $\quad$ f

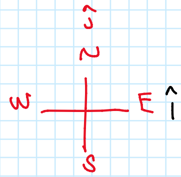
$$\sum \vec{P}_i = \sum \vec{P}_f$$

ex



1st part $\quad$ 450 m/s $\hat{j}$

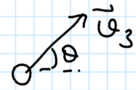
2nd " $\quad$ 240 m/s to West



$v_3 = ?$

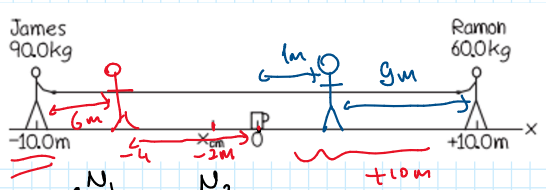
$$\sum \vec{P}_i = 3m\, 300\hat{j} = m\, 450\hat{j} + m(-240\hat{i}) + m\vec{v}_3$$

$$\vec{v}_3 = 240\hat{i} + 450\hat{j}$$

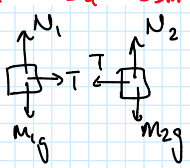


$$v_3 = \sqrt{450^2 + 240^2}$$

$$\theta = \tan^{-1}\left(\frac{450}{240}\right) \dots$$



James (mass 90.0 kg) and Ramon (mass 60.0 kg) are 20.0 m apart on a frozen pond. Midway between them is a mug of their favorite beverage. They pull on the ends of a light rope stretched between them. When James has moved 6.0 m toward the mug, how far and in what direction has Ramon moved?



$$\sum \vec{F}_{ext} = 0 \qquad \sum \vec{P}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\Downarrow$$

$$v_{cm} = 0$$

not move

$$= X_{cm} = \text{const} = \frac{60(10) + 90(-10)}{60+90} = \frac{-300}{150}$$

$$= -2m$$

so 60 kg person gets the cup first !!

$$X_{cm} = -2 = \frac{90(-4) + 60X}{150} \Rightarrow X = \frac{60}{60} = 1m$$

* when James moves 6m towards cup, Ramon moves 9m toward cup

* Jaes can never go beyon $X = -2m$

8.106 •• A 45.0-kg woman stands up in a 60.0-kg canoe 5.00 m long. She walks from a point 1.00 m from one end to a point 1.00 m from the other end (Fig. P8.106). If you ignore resistance to motion of the canoe in the water, how far does the canoe move during this process?

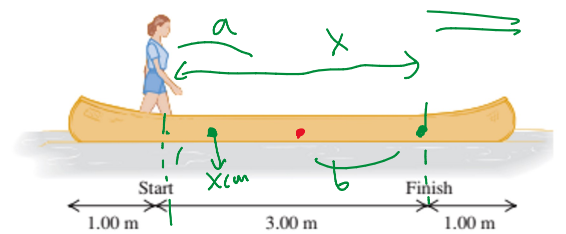
45 kg 60 kg

1 2

1m

2.5m

Figure **P8.106**



$a+b=3$

$$X_{cm} = \frac{45(1) + 2.5(60)}{105}$$

$$= 1.86\ m = \frac{45(1+x) + 60y}{105}$$

2.25m cance slips

$45(3) = 60X \quad x \Rightarrow 2.25m?($

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$0 = 0 = m_1 v_1 + m_2 v_2$$

$$0 = m_1 \frac{x_1}{t} + m_2 \frac{x_2}{t}$$

$$\frac{a}{b} = \frac{60}{45} = \frac{4}{3} \quad a+b=3 \quad b = \frac{9}{7}$$

$$\frac{4b}{3} + b = \frac{7b}{3} = 3$$

X_1

X_2

1m

$$m_1 X_1 = -m_2 X_2 \qquad \longrightarrow +x$$

$$45 X_1 = -60 X_2 \qquad \frac{X_1}{X_2} = -\frac{4}{3}$$

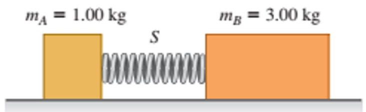
$$X_1 + X_2 = 3$$

$$\frac{4}{3} X_2 + X_2 = 3$$

$$X_1 = \frac{4}{3} X_2 = \frac{4}{3}\frac{9}{7} = \frac{12}{7} \qquad X_2 = \frac{9}{7} \approx 1.26m$$

S between them; then the system is released from rest on a level, frictionless surface. The spring, which has negligible mass, is not fastened to either block and drops to the surface after it has expanded. Block *B* acquires a speed of 1.20 m/s. (a) What is the final speed of block *A*? (b) How much potential energy was stored in the compressed spring?

Figure **E8.24**



$v = 0$

A B i

v_{AF} A B 1.2m/s f

$$\sum \vec{P}_i = \sum \vec{P}_f \qquad \rightarrow \hat{i}$$

$$0 = m_A \vec{v}_{AF} + m_B \vec{v}_{BF}$$

$$= 1\ \vec{v}_{AF} + 3(1.2)\hat{i}$$

$$\vec{v}_{AF} = -3.6\ m/s\ \hat{i} \qquad \leftarrow v_{AF}$$

Elastic collision

$$E_i = E_f$$

$$\frac{1}{2} m v_{Ai}^2 + \frac{1}{2} m v_{Bi}^2 + \frac{1}{2} k X^2 = \frac{1}{2} m v_{Af}^2 + \frac{1}{2} m v_{Bf}^2 + 0$$

$$0 + 0 + U_{el} = \frac{1}{2} 1 (3.6)^2 + \frac{1}{2} 3 (1.2)^2 = 6(1.2)^2 \sim 9\ J$$

1

2

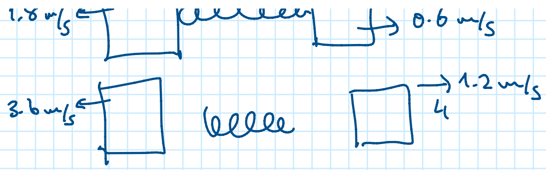
$$E_1 = E_2 = E_3 = E_4$$

$$0 = P_1 = P_2 = P_3 = P_4$$

1.8 m/s 3 0.6 m/s

$$3(0.6)\hat{i} + 1(\vec{v}_{AF}) = 0 \quad v_{AF} = -1.8 m/s$$

1.2 m/s



$3(0.6)\hat{i} + 1(v_{AF}) = 0 \quad v_{AF} = -1.8\ m/s$