Phys1 -01

CH1
CH2
CH3

the BOOK : Sears & Zemanski University Physics

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Chapter 1: units and physical quantities
(birim)

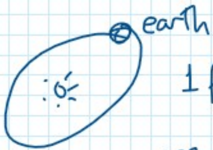
measure:
- length ~ volume
- time
- mass

} phys I

- temperature → thermodynamics
- quantity ~ avagadro number (mol) ~ chemistry
- current (ampere) ⟹ phys II
- brightness (candela) ⤳ astrophysics / astronomy / bulb

m, s, kg, K, N_A, A, Cd — SI Units

time : s



1 full turn = 365 days × 24 × 60 × 60 s

SI unit 1967   s ≡ C_s^{133} atom 9 192 631 770 ⟹ vibrates ≡ 1 s

atomic clock

length m   prototype "1m stick" ~1870's ~ 1,00001 m, 0,9998 m

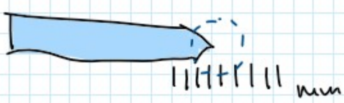
now = speed of light = c = 299 792 458 m/s
↳ 1960

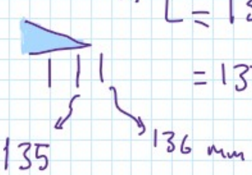
$1m = c(\Delta t)$

mass kg   prototype → 1kg of Alloy ~1870's

updated in 2019   now → h = Planck const.

Measurement : ruler ; pen ; length of the pen





?? $L = 135 \pm 1mm$

$= 135.5 \pm 0.5mm$

uncertainty

significant numbers, figures (sf)
(anlamlı sayı)

} numbers you are sure of .

significant numbers, figures (sf) (anlamlı sayı) } numbers you are sure of. Uncertainty

$135.5 \pm 0.5 \equiv 135$

$81.3 = 3\,sf \Rightarrow 813 \equiv 3\,sf = 8130 \Rightarrow 3\,sf$

$135 = 3\,sf$

zero's don't count.

$81.3 = 8.13 \times 10^{-1}$

$8130 = 8.13 \times 10^{3}$

$8013 = 4\,sf$

length of a building ; length of a pen

$L_1 = 13.5\,m = 13.5000$??? $L_2 = 11.3\,cm = 0.113\,m$

$L_1 + L_2 = 13.5 + 0.113 = 13.613\,m = 13.6\,m$

significant figure: sf

$0.0903 = 9.03 \times 10^{-2}$

0's don't count

1295 → 1300 ← 1315

$1300 = 2\,sf$

$1295 = 4\,sf$

For zeros:

$1300. = 4\,sf$

$1301 = 4\,sf$

$1300.15 = 6\,sf$

$1300.00 = 6\,sf$

$13.5? + 0.24 = 13.72 = 13.7$

$13.5 + 0.26 = 13.76 \simeq 13.8$

$13.7 + 0.26 = 13.96 \simeq 14.0$

multiplication or division

result: the least no of sf, rounded to

$\underset{2sf}{2.3} \times \underset{3sf}{5.32} = 12.236 = \underset{2sf}{12}$

$= 12.5 \simeq 13$

$x \equiv 2\,sf \qquad f(x) = y = 2\,sf$

$x = 10° \qquad \sin(10) = 0.173\ldots = 0.17 = 17 \times 10^{-2}$

ex: $\theta = 15.0°$

$\sin(\underset{3sf}{15.0}) = 0.25882$ (calc.) $= 259 \times 10^{-3} = 0.259$

$\sin(15) = 26 \times 10^{-2} = 0.26$

$a = g\sin\theta$ $g = 9.81\ m/s^2$ = 3sf

1sf

$(9.81)\sin(15) = 2.5$

$9.81 \sin(15.0) = 2.54$

$9.81 \sin(20) = 3$

$9.81 \sin(20.0) = 3.36$

$9.81 \sin(20.) = 3.4$

$$\frac{5.32 \times 2.01}{2.3} = 4.6$$

1.098 = 4sf

0.098 = 2sf 98×10^{-3}

0.0980 = 3sf $980. \times 10^{-4}$

$9.81 \cos(0) =$

$h = \frac{1}{2} g t^2$

don't count.

1sf 1sf

$9.81 \cos(0.) = 9.81 \simeq 10$

$\cos(0.0) = \simeq 9.8$

2sf. 2sf

1sf

$13.5 \cos(0.) = 10$

$14.5 \cos(0.) = 10$

$15.5 \cos(0.) = 20$

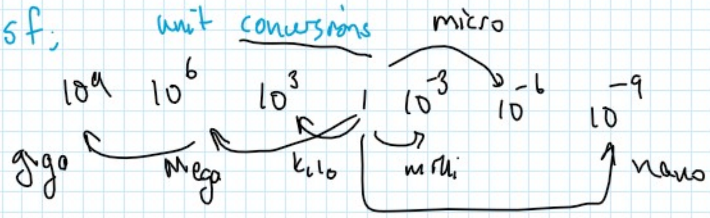
1sf

$(160.7)\cos(0.) = 200$

Gravitational constant	G	$6.67428(67) \times 10^{-11}$ N·m²/kg²
Planck's constant	h	$6.62606896(33) \times 10^{-34}$ J·s

$$\left\{ \begin{array}{l} h = 6.62606896 \\ \mp 0.00000033 \end{array} \right\}$$

SI units ; sf; unit conversions



Unit analysis

$h = a^{\alpha} t^{\beta}$

length acc. time

$s^0 m^1 = \left[\frac{m}{s^2}\right]^{\alpha} s^{\beta}$

$s^0 = s^{-2\alpha} s^{\beta}$

$-2\alpha + \beta = 0$

$\boxed{\beta = 2\alpha}$ $\leftarrow m^1 = m^{\alpha}$

$\boxed{\alpha = 1}$

$\boxed{\beta = 2}$

$h = a^1 t^2$

orders of magnitude calculations (fast; envelope calc.)

$10^{?}$ 10^3 10^{-9}

→ average no of breath a human take/give in a lifespan?

→ average no of breath a human take/give in a lifespan?

$$\#\text{ of breaths} = \frac{\text{lifetime}}{\text{time of breath}} = \frac{80 \text{ yrs}}{5_s} = \frac{80 \times 365 \times 24 \times 60 \times 60}{5}$$

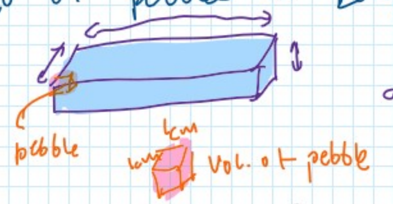
$$= \frac{80 \times 400 \times 25 \times 4000}{5}$$

$$= \underbrace{8 \times 4}_{32} \times \underbrace{5 \times 4}_{20} \times 10^6$$

$$504 \times 10^6 = 5 \times 10^8$$

$$\simeq 600 \times 10^6 = 6 \times 10^8$$

→ no of pebbles in Konyaaltı beach?



10 km, 200 m, depth = 10 m

$$\text{no of pebbles} = \frac{V_{beach}}{V_{pebble}} = \frac{\overbrace{10 \times 10^3}^{10\,km} \times 10 \times 200 \; m^3}{1\,cm^3}$$

$$\frac{2 \times 10^7 \; m^3}{1 \; cm^3} \quad m^3 \to (10^2)^3 \, cm^3 \quad ; \quad m = 100\,cm$$

$$\frac{2 \times 10^7 \times 10^6 \; cm^3}{1 \; cm^3} = \underset{\sim}{10^{13}}$$

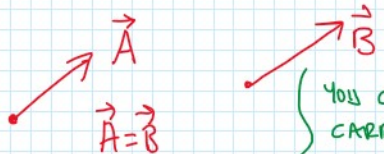
⇒ VECTORS :

why?
- motion
- force
- torque
- ⋮

} need vectors

→ magnitude (büyüklük) = number

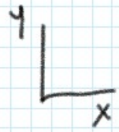
→ direction ≡ number ≡ angle

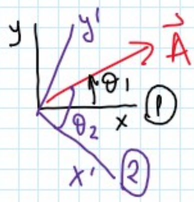


$\vec{A} = \vec{B}$

YOU CAN CARRY the VECTOR ANYWHERE w/o CHANGING the DIRECTION IT WILL BE THE SAME VECTOR

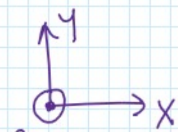
direction = angle?



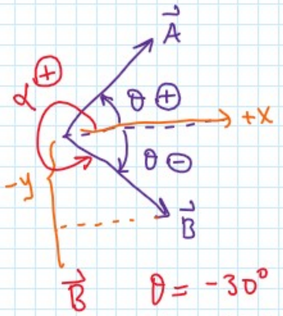


depending on coord. sys. direction angle can change.

$\theta_1 \neq \theta_2$ different coord. sys.

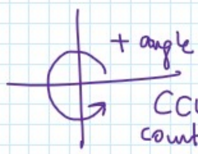


z out of page

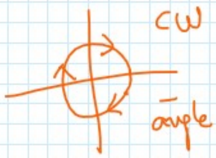


$\theta = -30°$

$\alpha = 330°$



CCW counter clock wise



ADDITION of TWO VECTORS

$$\boxed{A^2 = A_x^2 + A_y^2}$$

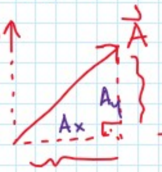
$\vec{A} + \vec{B}$

$\alpha = 330°$

ADDITION OF TWO VECTORS

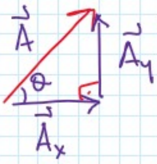
Magnitude

$|\vec{A}| = A$



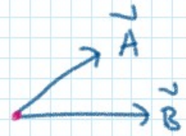
$\boxed{A^2 = A_x^2 + A_y^2}$

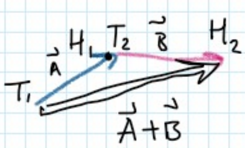
$\Downarrow$



$\boxed{\vec{A} = \vec{A}_x + \vec{A}_y}$

$\vec{A} + \vec{B}$





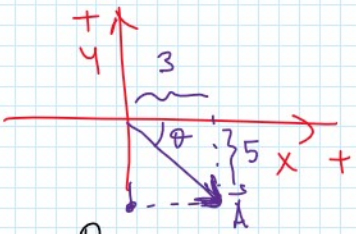
T H T H .... H

Direction $\tan\theta = \dfrac{A_y}{A_x}$; $\sin\theta = \dfrac{A_y}{A}$; $\cos\theta = \dfrac{A_x}{A}$; $A = \sqrt{A_x^2 + A_y^2}$

$\theta = ?$

$\boxed{\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)}$ $\theta = \sin^{-1}\dfrac{A_y}{A}$; $\theta = \cos^{-1}\dfrac{A_x}{A}$

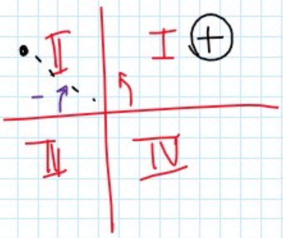
A = positive value A_x, A_y → + / → −

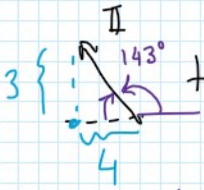


$\theta = \tan^{-1}\left(\dfrac{-5}{3}\right) = -59°$

$|\vec{A}| = (3^2 + 5^2)^{1/2} = 5.8$

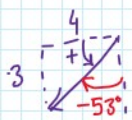
θ





$\tan^{-1}\left(\dfrac{\text{accross} = A_y}{\text{neighbor} = A_x}\right) = \theta$

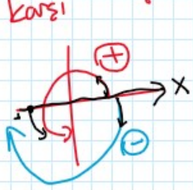
$\tan^{-1}\left(\dfrac{+3}{-4}\right) = -37°$



$\theta = \tan^{-1}\left(\dfrac{-3}{-4}\right) = +37°$

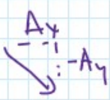
across $\equiv A_y$
karşı

III



$\left\{ \alpha = \tan^{-1}\left(\dfrac{-A_x}{-A_y}\right) = +53 ?? \right\}$

açı y'den başlıyor.

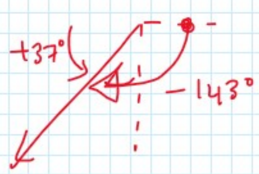


REGION	arctan(....)	θ
I	$\left(\dfrac{+A_y}{+A_x}\right)$	+
II	$\left(\dfrac{+A_y}{-A_x}\right)$	−
III	$\left(\dfrac{-A_y}{-A_x}\right)$	+
IV	$\left(\dfrac{-A_y}{+A_x}\right)$	−

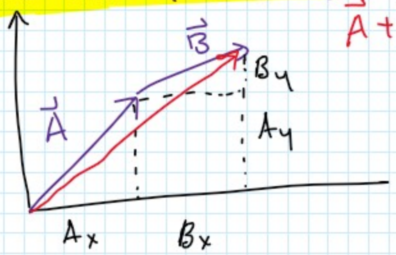
{ angle always start FROM X axis }

$\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) = \sin^{-1}\left(\dfrac{A_y}{A}\right)$

$\tan = \dfrac{\sin}{\cos}\left(\dfrac{A_y/\ldots}{A_x/\ldots}\right)$



ADDITION of VECTORS



$\vec{A}+\vec{B}$

$$|\vec{A}+\vec{B}| = \sqrt{(A_x+B_x)^2 + (A_y+B_y)^2}$$

$$|\vec{A}+\vec{B}| \neq \underbrace{|\vec{A}| + |\vec{B}|}$$

$\leq$ larger than

SUBTRACTION of VECTORS

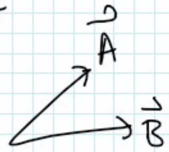
} definition

$-\vec{A}$ 180° flipped of $\vec{A}$

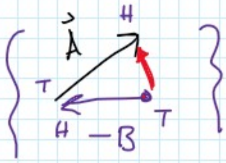
$\vec{A}-\vec{B} = \vec{A} + (-\vec{B})$

- I flip $\vec{B}$
- II perform + TH TH alignment

start



$\vec{A}-\vec{B} \Rightarrow$

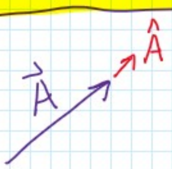


$\vec{A}-\vec{B} \neq \vec{B}-\vec{A}$

$\vec{A}-\vec{B} = -(\vec{B}-\vec{A})$

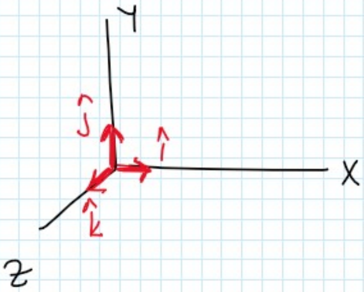
UNIT VECTORS

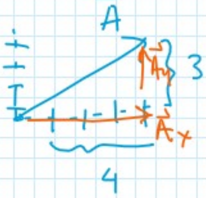
$\Rightarrow$ { * magnitude = length $\equiv$ 1 = unit
* unitless $\Rightarrow$ birim yok }



$$\hat{A} \equiv \frac{\vec{A}}{A}$$

$\hat{A}$; $\vec{A}$ has the same direction





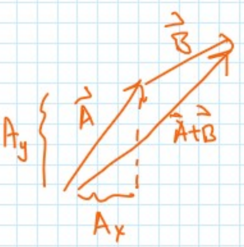
$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$

$\vec{A}_x = A_x\hat{i}$ $\quad = 4\hat{i} + 3\hat{j} = \vec{A}$

$\vec{A}_y = A_y\hat{j}$

$\vec{A}+\vec{B} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j})$; $\hat{i}$ $\hat{j}$ $\perp$ to each other

$$= (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} = \vec{A}+\vec{B}$$



$$\vec{A}-\vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

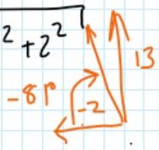
$$|\vec{A}-\vec{B}| = \left[(A_x-B_x)^2 + (A_y-B_y)^2\right]^{1/2}$$

ex) $\vec{A} = 6\hat{i} + 3\hat{j}$
$\vec{B} = 4\hat{i} - 5\hat{j}$ } $|\vec{A}-2\vec{B}|$; direction of $\vec{A}-2\vec{B}$

ex) $\vec{A} = 6\hat{i} + 3\hat{j}$
$\vec{B} = 4\hat{i} - 5\hat{j}$ } $|\vec{A} - 2\vec{B}|$; directio of $\vec{A} - 2\vec{B}$

$\vec{A} - 2\vec{B} = [6 - 2(4)]\hat{i} + [3 - 2(-5)]\hat{j}$

$|\vec{A} - 2\vec{B}| = \sqrt{13^2 + 2^2}$



$\vec{A} - 2\vec{B} = -2\hat{i} + 13\hat{j} = \vec{C}$ $|\vec{C}|$

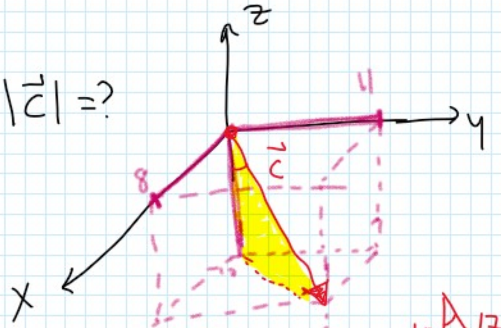
$\theta = \tan^{-1}\left(\frac{13}{-2}\right) = -81°$

ex) $\vec{D} = 6\hat{i} + 3\hat{j} - 1\hat{k}$
$\vec{E} = 4\hat{i} - 5\hat{j} + 8\hat{k}$

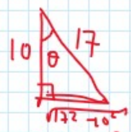
$2\vec{D} - \vec{E} = ?\ \vec{C}$
$|2\vec{D} - \vec{E}| = ?$ direction ??

$$\begin{array}{rl} 12\hat{i} + 6\hat{j} - 2\hat{k} & = 2D \\ + \quad -4\hat{i} + 5\hat{j} - 8\hat{k} & = -E \\ \hline 8\hat{i} + 11\hat{j} - 10\hat{k} & \equiv \vec{C} \end{array}$$

$|\vec{C}| = ?$

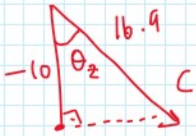


$|\vec{C}| = [8^2 + 11^2 + (-10)^2]^{1/2} = 16.9$



$\sqrt{17^2 - 10^2}$

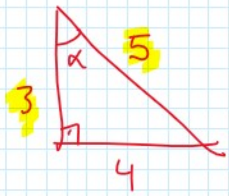
$\cos\theta = \frac{10}{17}$



$\theta_z = \cos^{-1}\left(\frac{-10}{16.9}\right) =$

$\theta_x, \theta_y, \theta_z$ are all differnt values

$\theta = \tan^{-1}\left(\frac{\sqrt{17^2 - 10^2}}{10}\right)$



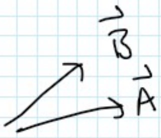
$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = \cos^{-1}\left(\frac{3}{5}\right) = \sin\left(\frac{4}{5}\right)$

vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ $\left\{ \begin{array}{l} \hat{i} = \hat{x} \\ \hat{j} = \hat{y} \\ \hat{k} = \hat{z} \end{array} \right\}$

$\vec{A} = 3\hat{i} - 4\hat{j} + 1\hat{k}$

linear algebra $\begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} = \vec{A}$

addition & subtraction ✓

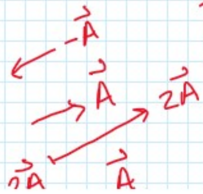


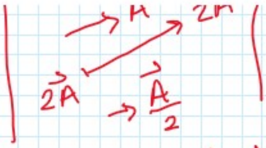
MULTIPLICATION

DOT PRODUCT
SCALAR "
$\vec{A} \cdot \vec{B}$

CROSS PRODUCT
VECTOR "
$\vec{A} \times \vec{B}$

LATER!!





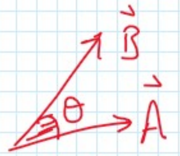
$\vec{A}\bullet\vec{B}$

SCALAR = number

$\vec{A}\times\vec{B}$

$\vec{A}\cdot\vec{B} = C \equiv$ number

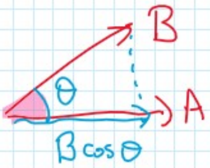
$\left\{ * \quad \vec{A}\cdot\vec{B} = AB\cos\theta \right\} \Rightarrow$



$\hookrightarrow$ work $\Rightarrow \vec{F}\cdot\vec{s} = W$

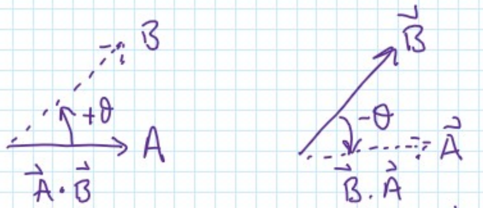
$\hookrightarrow$ what it means is....

how 2 vectors are projected onto eachother. (izdüşüm)



$AB\cos\theta \longleftrightarrow BA\cos\theta$

$$\left\{ \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} \right\}$$



$\vec{A}\cdot\vec{B}$ $\quad$ $\vec{B}\cdot\vec{A}$

$AB\cos\theta = BA\cos(-\theta)$

$\vec{A}\cdot\vec{B}$

$$(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\bullet(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A}\cdot\vec{B} = A_xB_x\,\hat{i}\cdot\hat{i} + A_xB_y\,\hat{i}\cdot\hat{j} + A_xB_z\,\hat{i}\cdot\hat{k}$$
$$+ A_yB_x\,\hat{j}\cdot\hat{i} + A_yB_y\,\hat{j}\cdot\hat{j} + A_yB_z\,\hat{j}\cdot\hat{k}$$
$$+ A_zB_x\,\hat{k}\cdot\hat{i} + A_zB_y\,\hat{k}\cdot\hat{j} + A_zB_z\,\hat{k}\cdot\hat{k}$$

$\hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos 90$
$\quad 1 \quad 1 \quad 0$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = \hat{k}\cdot\hat{i} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = 0$$

$$\hat{i}\cdot\hat{i} = |\hat{i}|\,|\hat{i}|\cos 0 = 1\,1\,1 = 1 = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k}$$

$$\left\{ \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta = \sqrt{A_x^2+A_y^2+A_z^2}\,\sqrt{B_x^2+B_y^2+B_z^2}\,\cos\theta \right\}$$

ex $\left.\begin{array}{l}\vec{A} = 2\hat{i} + 3\hat{j} + 1\hat{k} \\ \vec{B} = -4\hat{i} + 2\hat{j} - 1\hat{k}\end{array}\right\}$ $\vec{A}\cdot\vec{B} = ?$ $\hookrightarrow$ I $\;AB\cos\theta \;\; (\theta = ?)$

$\hookrightarrow$ II $\;A_xB_x + A_yB_y + A_zB_z$

$$\vec{A}\cdot\vec{B} = 2(-4)1 + 0 + 0 + 0 + 3(2)1 + 0 + 0 + 0 + 1(-1)1 = -8 + 6 - 1 = -3 = \vec{A}\cdot\vec{B}$$

$\vec{A}\cdot\vec{B} > 0$

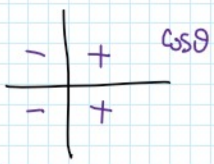
$AB\cos\theta$

$+ \quad + \quad + \quad 0 \quad -$

$+$

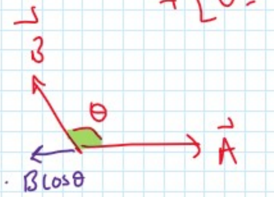
$\Downarrow$ $\quad \theta = 90 \quad$; $\quad \theta > 90° ; \theta < 270°$

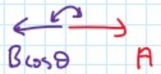
$+ \; [\theta = -90 ; 90]$



$\Downarrow$; $\theta = 90°$; $\rightarrow \theta > 90°$; $\theta < 270°$

$+ [\theta = -90 ; 90]$





what's the angle θ between A & B ?

$$\vec{A}\cdot\vec{B} = AB\cos\theta = -3$$

$$A = [2^2 + 3^2 + 1^2]^{1/2} = \sqrt{14}$$

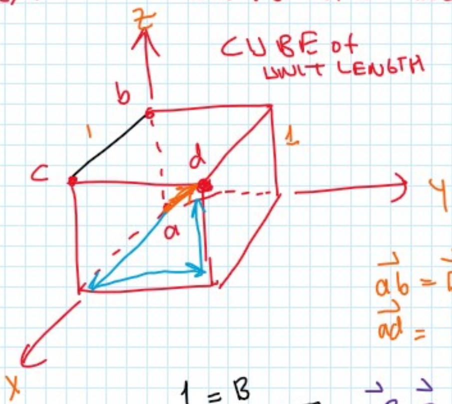
$$B = [4^2 + 2^2 + 1^2]^{1/2} = \sqrt{21}$$

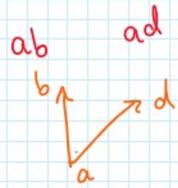
$$AB\cos\theta = -3$$

$$\sqrt{14}\sqrt{21}\cos\theta = -3$$

$$\cos\theta = \frac{-3}{7\sqrt{6}} \quad ; \quad \theta = \cos^{-1}\left(\frac{-3}{7\sqrt{6}}\right) \cong 100°$$

ex: WHAT'S THE ANGLE BETWEEN $\vec{ab}$ and $\vec{ad}$?





$ab = \vec{B}$

$ad = \vec{D} \equiv$ diagonal

$$\vec{ab} = \vec{B} = [0, 0, 1] = 1\hat{k}$$

$$\vec{ad} = \vec{D} = 1, 1, 1 = 1\hat{i} + 1\hat{j} + 1\hat{k}$$

$$1 = B$$

$$D = \sqrt{3} = \sqrt{1^2 + 1^2 + 1^2}$$

$$\vec{B}\cdot\vec{D} = BD\cos\theta = (\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 + 0 + 0$$

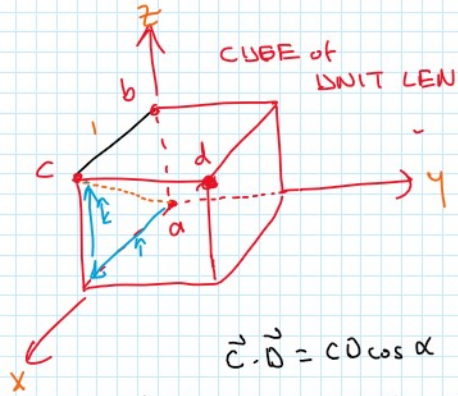
$\hat{k}\cdot\hat{i} = 0$

$\hat{k}\cdot\hat{j} = 0$

0 0 1

$$1\sqrt{3}\cos\theta = 1$$

$$\theta = \cos^{-1}\left(1/\sqrt{3}\right) \cong 55°$$



angle between $\vec{ac}$ & $\vec{ad}$

$$\vec{ad} = \vec{D} = \hat{i} + \hat{j} + \hat{k} \quad ; \quad |\vec{D}| = \sqrt{3}$$

$$\vec{ac} = \vec{C} = \hat{i} + \hat{k} \quad ; \quad |\vec{C}| = \sqrt{2}$$

$$\vec{C}\cdot\vec{D} = CD\cos\alpha$$

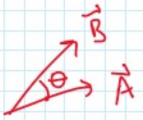
$$1 + 1 + 0 + 0 + 0 + 0 = \sqrt{2}\sqrt{3}\cos\alpha$$

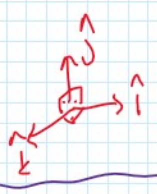
$$2 = \sqrt{2}\sqrt{3}\cos\alpha$$

$$\alpha = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right) \cong 35°$$

summarije

scalar dot product $\vec{A}\cdot\vec{B}$





θ	$\vec{A}\circ\vec{B}$ / AB
0°	AB
90°	0
180°	$-AB$

VECTOR PRODUCT CROSS $\Rightarrow$ $\vec{A}\times\vec{B} = \vec{C}$ $\Rightarrow$ * $|\vec{C}| = |AB\sin\theta|$

$\vec{B}$

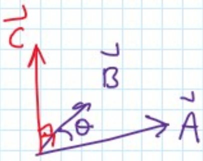
VECTOR PRODUCT (CROSS) ⇒ $\vec{A}\times\vec{B}=\vec{C}$ ⇒ * $|\vec{C}| = |AB\sin\theta|$

cross

vector → direction: Right Hand Rule

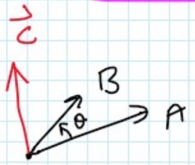
magnitude $= |AB\sin\theta|$

RIGHT HAND RULE
(baş) THUMB $\vec{A} = 1st$
(işaret) INDEX $\vec{B} = 2nd$
(orta) MIDDLE $\vec{C} = 3rd$



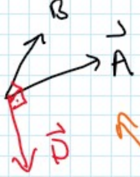
$\vec{A}\times\vec{B}=\vec{C}$

$\theta = 90° = |\vec{C}| = max$

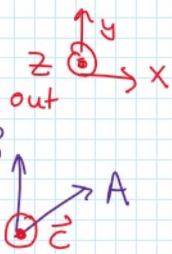


$\vec{A}\times\vec{B}=\vec{C}$ (1st, 2nd)

$[\vec{C} = -\vec{D}]$



$\vec{B}\times\vec{A}=\vec{D}$ (1st, 2nd)



$\vec{A}\times\vec{B}=\vec{C}$

$\vec{B}\times\vec{A}=\vec{D}$ into

A=

$$\vec{A}\times\vec{B} = A_xB_x\,\cancel{\hat{i}\times\hat{i}}^{\,0} + A_xB_y(\hat{i}\times\hat{j})_{\hat{k}} + A_xB_z(\hat{i}\times\hat{k})_{-\hat{j}}$$
$$+ A_yB_x(\hat{j}\times\hat{i})_{-\hat{k}} + A_yB_y\,\cancel{\hat{j}\times\hat{j}}^{\,0} + A_yB_z(\hat{j}\times\hat{k})_{\hat{i}}$$
$$+ A_zB_x(\hat{k}\times\hat{i})_{+\hat{j}} + A_zB_y(\hat{k}\times\hat{j})_{-\hat{i}} + A_zB_z\,\cancel{\hat{k}\times\hat{k}}^{\,0}$$

9 terms, 3 = 0

1) $\theta = 90°$, $\sin\theta$, $\vec{A}\times\vec{B} = max$

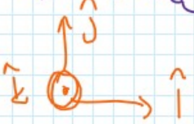
2) $\theta = 0°$, $\sin\theta = 0$ ⟹ $\vec{A}\times\vec{B} = 0$

$\theta = -\theta$, $\sin\theta = -\sin\theta$, changes the direction

$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = \vec{A}$

$B_x\hat{i} + B_y\hat{j} + B_z\hat{k} = \vec{B}$

$0 = \hat{k}\times\hat{k} = \hat{j}\times\hat{j} = \hat{i}\times\hat{i} = |\hat{i}||\hat{i}|\sin 0° = 0$

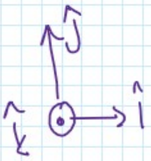


$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{i} = -\hat{k}$

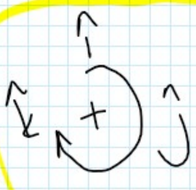
$\hat{i}\times\hat{k} = -\hat{j}$, $\hat{k}\times\hat{i} = +\hat{j}$

$\hat{j}\times\hat{k} = +\hat{i}$, $\hat{k}\times\hat{j} = -\hat{i}$

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k} = \vec{C}$$
$$\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k} = \vec{C}$$



$\vec{A}\times\vec{B} = \vec{C}$ → (middle) result; $\vec{A}$ → 1st (thumb); $\vec{B}$ → 2nd (index)



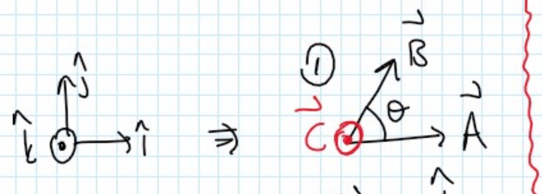
$\hat{i}\times\hat{j} = +\hat{k}$
$\hat{j}\times\hat{k} = +\hat{i}$
$\hat{k}\times\hat{i} = +\hat{j}$

$\hat{i}\times\hat{k} = -\hat{j}$
$\hat{k}\times\hat{j} = -\hat{i}$
$\hat{j}\times\hat{i} = -\hat{k}$

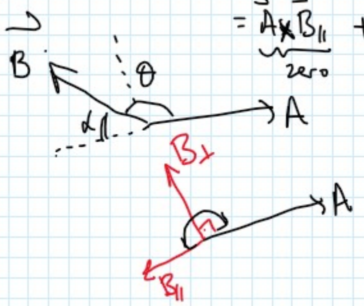
②

$\vec{A}\times\vec{B} = \vec{A}\times(\vec{B}_{\parallel} + \vec{B}_{\perp})$

(thumb)



① $\vec{C} = \hat{k}$

② 

$\vec{A}\times\vec{B} = \vec{A}\times(\vec{B}_{\parallel} + \vec{B}_{\perp})$
$= \underbrace{\vec{A}\times\vec{B}_{\parallel}}_{zero} + \vec{A}\times\vec{B}_{\perp}$

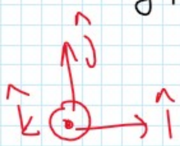
$B_{\perp} = B\sin\alpha$
$= B\sin\theta$

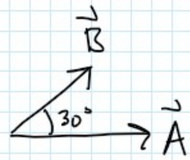
$\sin\theta = \sin\alpha$

$\vec{A}\times\vec{B}_{\parallel} \Rightarrow AB_{\parallel}\sin 180° = 0$; $|\vec{A}\times\vec{B}| = AB\sin\theta = AB_{\perp}$

ex) $\vec{A} = 6\hat{i}$
$|\vec{B}| = 4$ $\vec{B}$ lies on x-y plane with 30° with +x axis

$\vec{A}\times\vec{B} = ?$





$|\vec{A}\times\vec{B}| = |AB\sin\theta|$ $\quad \vec{C} = \vec{A}\times\vec{B}$; $C = AB\sin\theta$
$= 6(4)\sin 30 = 12$

$\vec{C} = 12\hat{k}$ $\quad$ RHR $\Rightarrow$ C $+\hat{k}$

1.96) $|\vec{A}| = 3$ $|\vec{B}| = 3$ $\quad \vec{A}\times\vec{B} = 2\hat{i} - 5\hat{k}$ $\quad \theta = ?$ between $\vec{A}$ & $\vec{B}$??

$|\vec{C}| = AB\sin\theta = \sqrt{2^2 + (-5)^2} = \sqrt{29} \Rightarrow 3(3)\sin\theta = \sqrt{29}$

$\theta = \sin^{-1}\left(\frac{\sqrt{29}}{9}\right) = 36.8°$

1.95) $\vec{A} = 5\hat{i} - 6.5\hat{j}$ $\quad \vec{A}\times\vec{B} = ?$ $\quad (5\hat{i} - 6.5\hat{j})\times(-3.5\hat{i} + 7\hat{j})$ $= (\ldots)\hat{i} + (\ldots)\hat{j} + (\ldots)\hat{k}$

$\vec{B} = -3.5\hat{i} + 7\hat{j}$

$0 + 35\,\underbrace{\hat{i}\times\hat{j}}_{\hat{k}} - 6.5(-3.5)\underbrace{\hat{j}\times\hat{i}}_{-\hat{k}} + 0$

$\vec{A}\times\vec{B} = 12.25\hat{k}$

END of CH 1

MOTION

ch2 ; ch3

1 dim. motion $\quad$ 2-3 dim

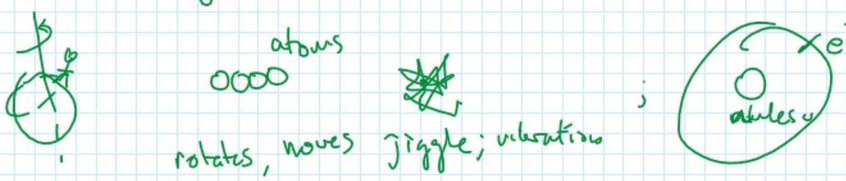
CH2

1 d. motion ≡ MOTION ALONG a STRAIGHT LINE

"motion" ≡ everything moves in the universe ; nothing is stationary

p, atoms, e^-

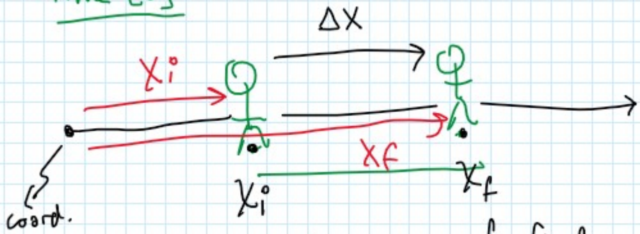
motion = everything moves in the universe; nothing is stationary.



rotates, moves jiggle; vibrations

position [m] → 0.5s → 2s ; time [s]

yerdeğiştirme = Displacement



$$\Delta x = x_f - x_i$$

* ($\Delta \equiv$ final - initial) ; $\Delta t = t_f - t_i$

$\vec{x}_f - \vec{x}_i \Rightarrow$ $-\vec{x}_i$, $\vec{\Delta x}$, $\vec{x}_f$

x position
t time
Δx = displacement

v (velocity) HOW FAST? " SLOW? $\sim \frac{\text{displacement}}{\text{time}}$

AVERAGE VELOCITY

bar $\bar{v} = \frac{\Delta x}{\Delta t}$ → $\bar{v} = $ +, 0, −

velocity ≠ speed
hız ≠ sürat

speed ≥ 0

$$\text{speed} = \frac{\text{total amount of road travelled}}{\text{total time}}$$

$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$; x = 0, −, +

i → f: 10m, 30m; 1s, 4s

$\bar{v} = \frac{30-10}{4-1} = \frac{20}{3}\ \frac{m}{s}$ → $\vec{v}$

f ← i: 10m, 30m; 4s, 1s

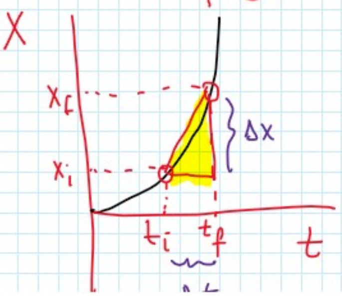
$\bar{v} = \frac{10-30}{4-1} = -\frac{20}{3}\ m/s$ ⇐

instantaneous velocity ≡ anı (anlık) hız

$v \equiv \frac{dx}{dt} \equiv$ derivative of x w.r.t. t   DERIVATIVE (TÜREV)

$$\left\{ \frac{dx}{dt} = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right\}$$

Newton had to invent derivation integration to explain motion



$\bar{v} = \frac{\Delta x}{\Delta t}$ = slope (eğim)

t

$\frac{dx}{dt} = \lim \frac{\Delta x}{\Delta t}$

$t_i \quad t_f \quad t$, Δt

t

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$\bar{v}$ vs v

ex) $x = 20 + 5t^2$ unit analysis $\Rightarrow [m] = [m] + [m]$, $5 \to \frac{m}{s^2}$, $t^2 \to s^2$

a) $\bar{v}$ between $t = 1s$ & $2s$? $\bar{v} = \dfrac{x_f - x_i}{t_f - t_i} = \dfrac{20 + 5(2^2) - [20 + 5(1^2)]}{2-1}$

$\bar{v} = 15\ m/s$

$\bar{v}$ between $t = 1s$ & $1.1s$

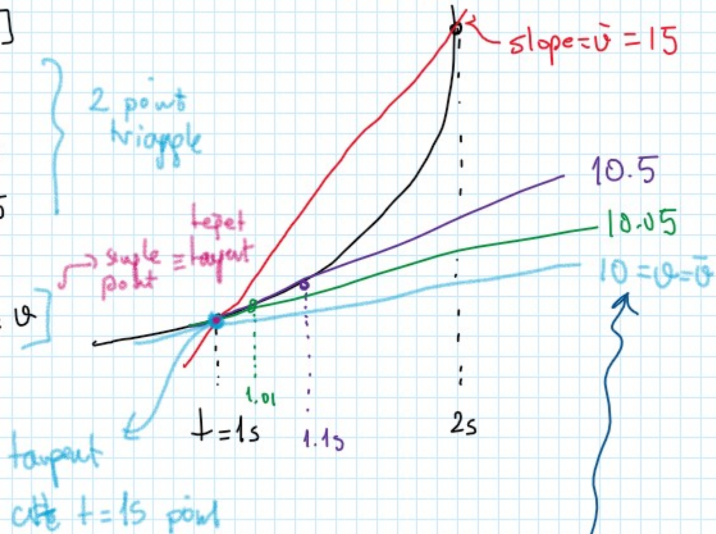
$$\bar{v} = \frac{20 + 5(1.1)^2 - [20 + 5(1^2)]}{1.1 - 1} = 10.5\ m/s$$

$\bar{v} = ?$ $t_i = 1s$ $t_f = 1.01s$ $\Rightarrow$ $\bar{v} = \dfrac{20 + 5(1.01)^2 - [20 + 5 \cdot 1^2]}{0.01} = 10.05$

t_i	t_f	Δt	$\bar{v}$ [m/s]
1	2	1	15
1	1.1	0.1	10.5
1	1.01	0.01	10.05
1	1.001	0.001	10.005
		⋮	⋮
1	1.000...	0	10 $= v$

2 point triangle

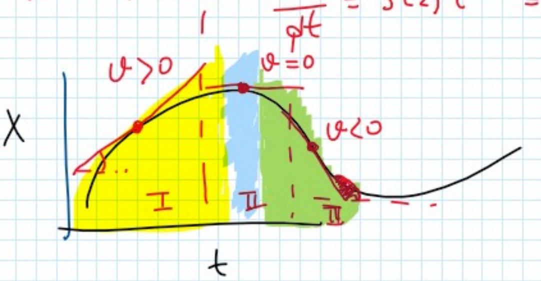
$\lim_{\Delta t \to 0}$



ALSO

$$x = At^n \quad ; A = \text{const},\ n = \text{integer} \qquad \frac{dx}{dt} = A n t^{n-1}$$

$x = 20 + 5t^2$ $\quad \dfrac{dx}{dt} = 5(2)t^{2-1} = 10t = v(t)$ $\quad v(t = 1s) = 10(1) = 10\ m/s$



slope = tangent = $\dfrac{\text{across}}{\text{neighbor}}$

	I	II	III
v	+	0	−

ACCELERATION (ivme) $\to$ $\bar{a} = \dfrac{\Delta v}{\Delta t}$ (average acc.)

$\to$ $a = \dfrac{dv}{dt}$ (ins. acc.)

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{m/s}{s} = \left[\frac{m}{s^2}\right]$$

$x \xrightarrow{d/dt} v \xrightarrow{d/dt} a \xrightarrow{d/dt}$ name ??

$\Delta t \quad s \quad [s^2]$

$$x \xrightarrow{d/dt} v \xrightarrow{d/dt} a \xrightarrow{d/dt} \text{name ??}$$

ex): $v = 60 + 0.5t^2 \longrightarrow 0.5t^2 = \left[\frac{m}{s}\right] \Rightarrow \left[0.5\,\frac{m}{s^3}\right] t^2 = \left[\frac{m}{s}\right]$

a) $t_i = 1s \quad t_f = 3s \qquad \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{60 + 0.5(3^2) - [60 + 0.5(1^2)]}{2} = 2\ m/s^2$

t_i	t_f	Δt	$\bar{a}$
1	3	2	2
1	2	1	1.5
1	1.1	0.1	1.05
		0.01	1.005

$\} \ \bar{a}$

$\lim_{\Delta t \to 0} \ldots\ldots\ldots 1 = a = \bar{a}$

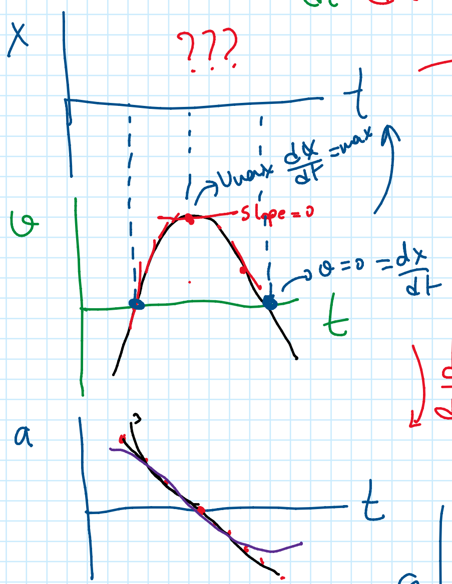
most of the time functions are given derivative of function

$$a = \frac{dv}{dt} = 0.5(2)t' = 1t$$

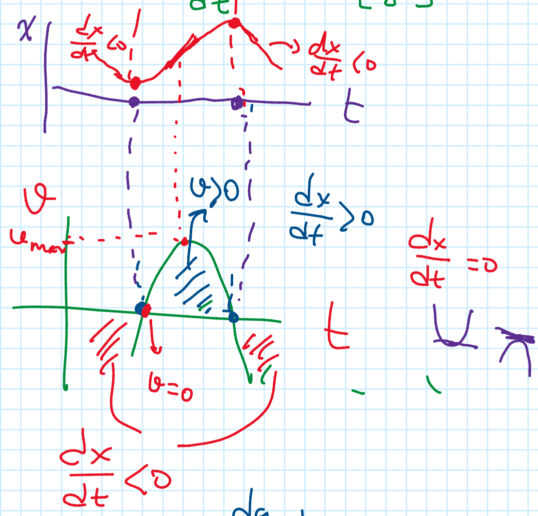
$$\left[\frac{m}{s^2}\right] = \left[1\,\frac{m}{s^3}\right] t$$

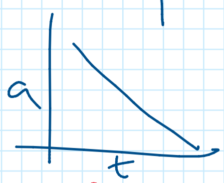
$$\left\{ a = \frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \bar{a} \right\}$$

$$\left\{ \cancel{\bar{a} \dashrightarrow a} \right\} \qquad \frac{d}{dt}\frac{dx}{dt} = \frac{dv}{dt} = a = \frac{d^2x}{dt^2} = \left[\frac{m}{s^2}\right]$$

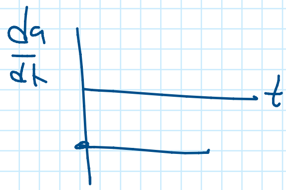


$\frac{d}{dt}$





$\frac{da}{dt} = \text{const}$



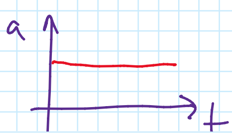
$$\left\{ x \longrightarrow \frac{dx}{dt} = v \qquad \frac{dv}{dt} = a \right.$$
$$\left. \frac{\Delta x}{\Delta t} = \bar{v} \qquad \frac{\Delta v}{\Delta t} = \bar{a} \right\}$$

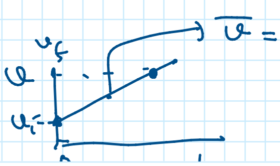
$v_i + at = v_f$

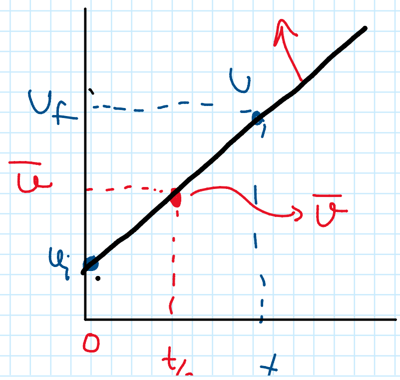
MOTION WITH CONSTANT ACCELERATION

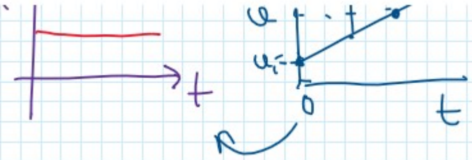
why $a = \text{const}$?! (b/c apple falls with const. acc.)

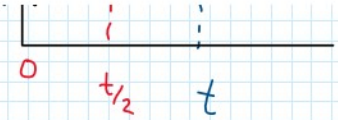
$a = \text{const} \neq 0$











$$\bar{v} = \frac{v_i + v_f}{2}$$

$$\frac{dv}{dt} = \text{const} = a \qquad v = At' + B$$

$$\bar{v} = \frac{v_i + v_f}{2} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t - 0} \Rightarrow x_f - x_i = \frac{t}{2}\left(v_i + v_f\right)$$

$$[v_f = v_i + at]$$

$$x_f - x_i = \frac{t}{2}\left(v_i + v_i + at\right)$$

$$x_f = x_i + v_i t + \frac{at^2}{2}$$

$$\boxed{x_f = x_i + v_i t + \frac{1}{2}at^2} \quad (2)$$

$$\boxed{v_f = v_i + at} \quad (1)$$

(1) & (2) $\Rightarrow$ (3) without "t" term !!

$$t = \frac{v_f - v_i}{a} \to (2)$$

$$x_f = x_i + v_i\frac{(v_f - v_i)}{a} + \frac{1}{2}\cancel{a}\left(\frac{v_f^2 + v_i^2 - 2v_f v_i}{a^{\cancel{2}}}\right)$$

$$= x_i + \frac{v_i v_f - v_i^2}{a} + \frac{v_f^2}{2a} + \frac{v_i^2}{2a} - \frac{v_i v_f}{a}$$

$$x_f = x_i - \frac{v_i^2}{a} + \frac{v_f^2}{2a} + \frac{v_i^2}{2a} = x_i + \frac{v_i^2}{2a} - \frac{v_f^2}{2a}$$

$$(3) \quad \boxed{2a(x_f - x_i) + v_i^2 = v_f^2} \Rightarrow \boxed{v_f^2 = v_i^2 + 2a\Delta x}$$

ex) $\to a = 4\,m/s^2$

$t = 0s \quad x_i = 5m$

$v_i = 15\,m/s$

a) $x_f = ?$ $v_f = ?$ @ $t = 2s$

$$x_f = 5 + 15(2) + \frac{1}{2}4(2^2)$$

$$= 43m$$

$$v_f = 15 + 4(2) = 23\,m/s$$

1. $v_f = v_i + at$
2. $x_f = x_i + v_i t + \frac{1}{2}at^2$
3. $v_f^2 = v_i^2 + 2a\Delta x$

b) when $v = 25\,m/s$ what's its location?

$x_f = ?$

$v_f = \checkmark$

$v_i = \checkmark$

$a = \checkmark$

$x_i = 0$

$$25^2 = 15^2 + 2(4)[x_f - 5]$$

$$\frac{400}{8} + 5 = x_f = 55m$$

ex) a motorist traveling at const. velocity of 15 m/s. passes a police car ex) a motorist traveling at const. velocity of 15 m/s. passes a police car which is stationary. (at rest). After passing the police, police car tries to catch motorist with const $a = 3\,m/s^2$.

1) when does the police catch the motorist?

$$\begin{cases} 1 & v_f = v_i + at \\ 2 & x_f = x_i + v_i t + \frac{1}{2}at^2 \\ 3 & v_f^2 = v_i^2 + 2a\Delta x \end{cases}$$

$X_{pf} = X_{mf} = X_f$

$$x_{mf} = \cancel{x_{mi}}^{0} + v_{M} t + \frac{1}{2}\cancel{a_m}^{0} t^2$$

motorist $X_f = 15t$

$$x_{pf} = \cancel{x_{pi}}^{0} + \cancel{v_{pi}}^{0} t + \frac{1}{2}a_p t^2 = \frac{1}{2}3t^2 = X = 15t$$

$$\frac{3}{2}t^2 = 15t \Rightarrow t = 10s$$

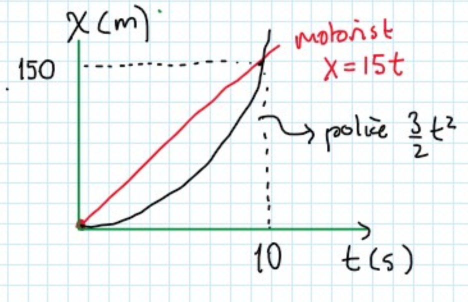
b) what's the police cars' velocity when he catches the motorist?
$v_{pf} = ?$

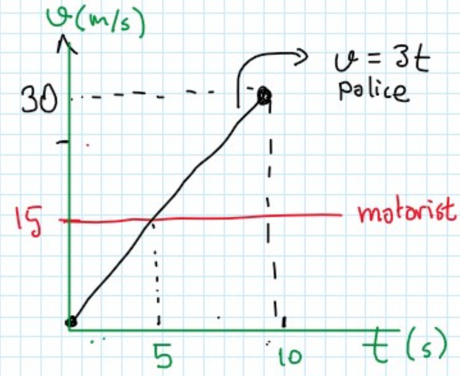
$$v_{pf} = v_{pi} + at = 0 + 3(10) = 30\,m/s$$

c) How far does each vehicle travelled?

$$x_{pf} = X = \frac{3}{2}t^2 = X_{mf} = 15t \qquad t = 10s$$
$$a = 3\,m/s^2$$
$$\frac{3}{2}10^2 = 15(10) = 150\,m$$

d) Sketch X vs t and v vs t graphs for both vehicles?

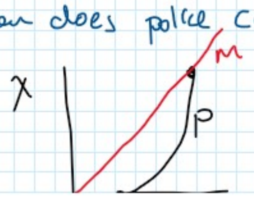


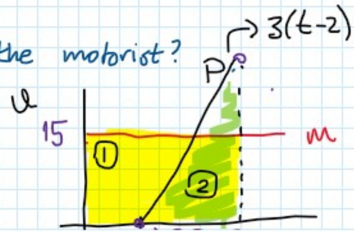


(ex) SAME problem BUT police starts to move 2s AFTER motorist passes him?

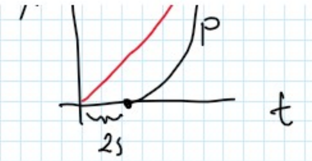
$$t_p \neq t_M$$
$$\downarrow \qquad \downarrow$$
$$t-2 \qquad t$$

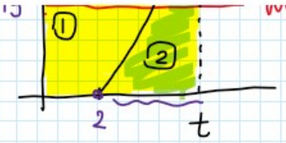
when does police catch the motorist?





$t-2$ $\quad$ t





$X_{mf} = X_{pf}$

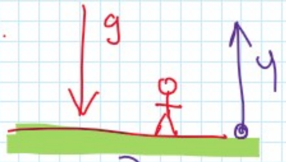
$area_1 = area_2$

$$15(t) = \frac{1}{2} 3 (t-2)^2 \Rightarrow \text{SAME EQN.} \Leftarrow 15t = (t-2)\frac{3(t-2)}{2}$$

$$\left\{ At^2 + Bt + C = 0 \;\to\; t = \frac{-B \mp \sqrt{B^2 - 4AC}}{2A} \right\} \to 7+3\sqrt{5} \cong 13.7s$$

FREELY FALLING OBJECT

const. acc.

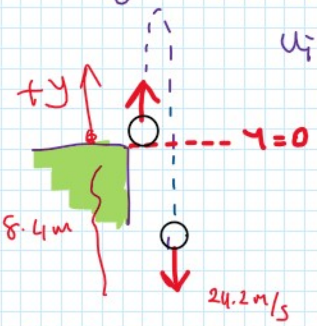


$$\begin{cases} v_f = v_i - gt \\ y_f = y_i + v_i t - \frac{1}{2} g t^2 \\ v_f^2 = v_i^2 - 2g(y_f - y_i) \end{cases}$$

$X \Rightarrow Y$
$a \Rightarrow -g$
$v \Rightarrow v$

$$\boxed{g = 9.8\, m/s^2}$$

ex):

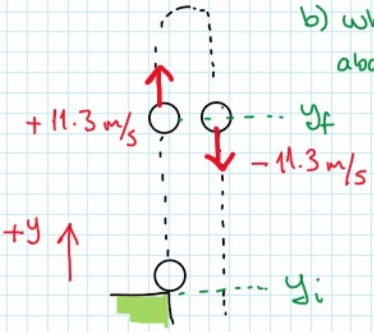


$v_i = 15\, m/s$

a) what's the final position and velocity at $t = 4s$?

$$y_f = y_i + v_i t - \frac{1}{2} g t^2$$
$$= 0 + 15(4) - \frac{1}{2}(9.8)4^2 = -18.4m$$
$$v_f = 15 - 9.8(8) = -24.2\, m/s$$

b) what's the objects velocity when its 5m above the initial position?



$$v_f^2 = v_i^2 - 2g(y_f - y_i)$$
$$v_f^2 = 15^2 - 2(9.8)5 = \sqrt{127} \to +11.3\, m/s \checkmark, \; -11.3\, m/s \checkmark$$

what's the objects max height?

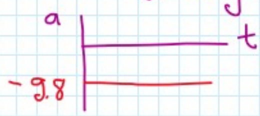
$$v_f^2 = v_i^2 - 2g\underbrace{(y_f - y_i)}_{h = max}$$

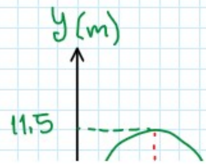
$$0 = 15^2 - 2(9.8)h \Rightarrow h = 11.5m$$

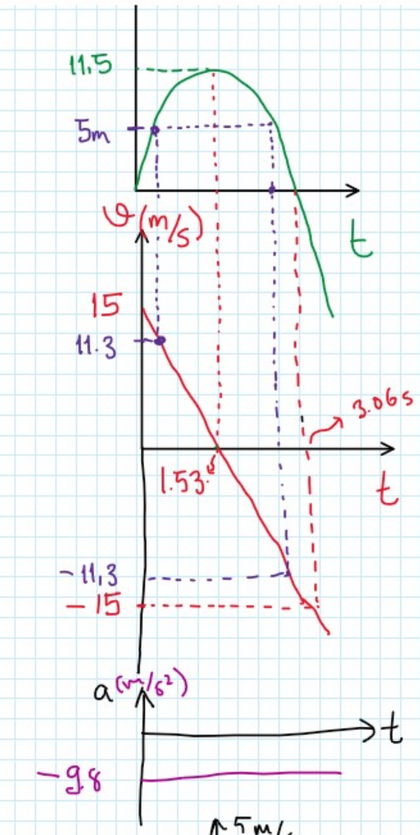
whats the obj acceleration at max hight?

the acceleration of the object is CONSTANT at all times!!

$$a = -g = -9.8\, m/s^2$$







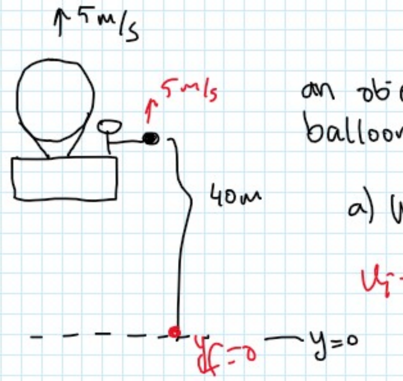
How long does it take to reach max height?

$v_f = v_i - gt$

$0 = 15 - 9.8(t)$

$t = \frac{15}{9.8} = 1.53s$

⇒ ex) 2.44)



an object is released from a hot air balloon which is going up with 5m/s velocity

a) When does the object hit the ground?

$v_i = 5m/s$; $y_i = 40m$ $g = 9.8m/s^2$

1 $v_f = v_i - gt$

2 $y_f = y_i + v_i t - \frac{1}{2}gt^2$

3 $v_f^2 = v_i^2 - 2g(y_f - y_i)$ ↓g ↑y+

$0 = 40 + 5t - 4.9t^2$

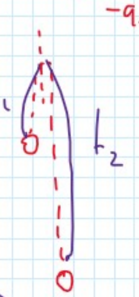
$At^2 + Bt + C = 0$

$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\frac{-5 \pm \sqrt{25 - 4(-4.9)40}}{2(-4.9)} = \frac{-5 \mp 28.4}{-9.8}$

$t \to -2.39s$ ✗ $t<0$

$\to 3.41s$ ✓



$t_1 + t_2 = 3.41s$

b) whats the objects final velocity just before it hits the ground?

I $v_f^2 = 5^2 - 2(9.8)[0 - 40]$

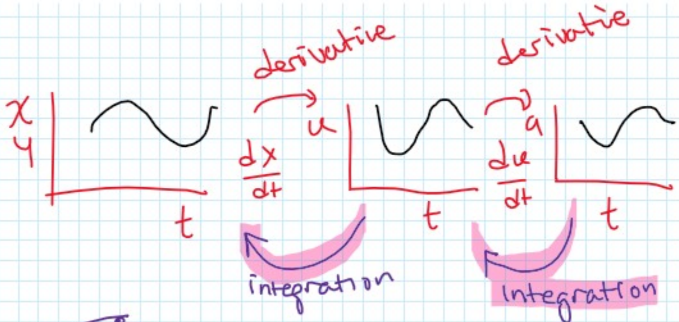
$= 809 \Rightarrow v_f = \sqrt{809} = -28.4m/s$ ~~+28.4m/s~~ b/c ↓ v_f is in ⊖ direction

II $t = 3.41s$ (time of flight)

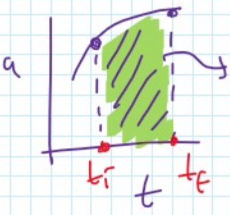
$v_f = 5 - 9.8(3.41) = 5 - 33.4 = -28.4m/s$

finding VELOCITY and POSITION by INTEGRATION

derivative derivative



$\int dx = \Delta X$

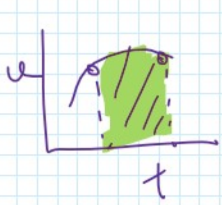


→ area ≡ integration

$\left\{ a, v \text{ are always instantaneous} \right\}$

$a = \frac{dv}{dt} \Rightarrow \int a\,dt = \underbrace{\int dv}$

$\text{area} = \int_{t_i}^{t_f} a\,dt = \Delta v = v_f - v_i$



$\Rightarrow \text{area} = \int_{t_i}^{t_f} v\,dt = \Delta X = X_f - X_i$

ex) at $t = 0s$ $v = 10\ m/s$ in $+X$ direction $X = 50m$

$\boxed{a = 2 - 0.1t}$

$\int t\,dt = \frac{t^2}{2}$

a) find v, X as a function of time?

$v(t) = ?$ $\Delta v = \int a(t)\,dt = \int_0^t (2 - 0.1t)\,dt = \int 2dt - \int 0.1t\,dt$

$= \left[2t\right]_0^t - \left[0.1\frac{t^2}{2}\right]_0^t$

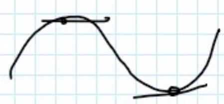
$\Delta v = 2t - \frac{0.1}{2}t^2 = v_f - v_i = v(t) - \underbrace{v(0)}_{(+10)}$

$$\boxed{v(t) = 10 + 2t - \frac{0.1}{2}t^2}$$

$x(t) = ?$ $\int v\,dt = \Delta X = \int \left(10 + 2t - \frac{0.1}{2}t^2\right)dt = \left[10t + \frac{2t^2}{2} - \frac{0.1}{2}\frac{t^3}{3}\right]$

$X(t) - \underbrace{X(0)}_{50} =$ $\left[X(t) = 50 + 10t + t^2 - \frac{0.1}{6}t^3\right]$

b) $v_{max} = ?$ $\frac{dv}{dt} = 0$



$\frac{dv}{dt} = a = 0 = 2 - 0.1t = 0$

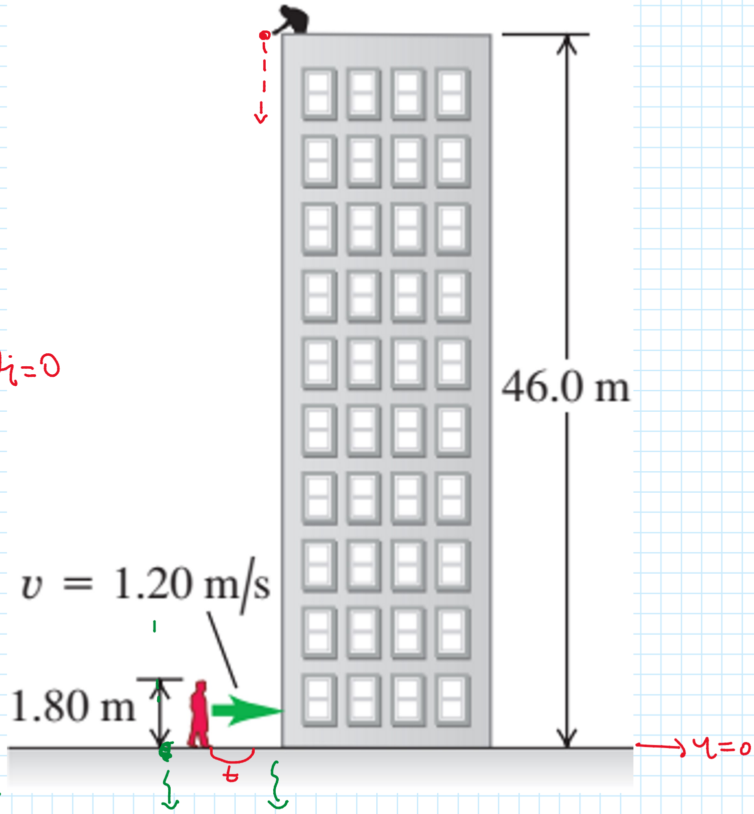
$t = \frac{2}{0.1} = 20s$ → $v(t) = \left[10 + 2(20) - \frac{0.1}{2}(20)^2\right]$

$= 30\ m/s$

c) what is the position of the car @ maximum velocity?

$X(t = 20s) = 50 + 10(20) + 20^2 - \frac{0.1}{6}(20)^3 = 516.67 \simeq 517m$

2.80 • Egg Drop. You are on the roof of the physics building, 46.0 m above the ground (Fig. P2.80). Your physics professor, who is 1.80 m tall, is walking alongside the building at a constant speed of 1.20 m/s. If you wish to drop an egg on your professor's head, where should the professor be when you release the egg? Assume that the egg is in free fall. → $v_i = 0$



$y_f = y_i + v_i t - \frac{1}{2} g t^2$

$1.8 = 46 + 0 - 4.9 t^2$

$t^2 = \frac{44.2}{4.9} \Rightarrow t = 3\,s$

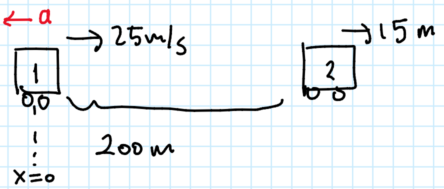
prof. should be X distance away $\{X = vt\}$ before the egg drop

$X = (1.2)(3) = 3.6\,m$ away

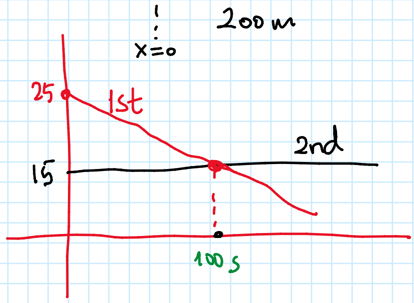
$x_f = x_i + v_i t + \frac{1}{2} a t^2$ → 0

$x_f = 0 + v_i t$

2.70)



The 1st car slows down with $a = -0.1\,m/s^2$ not to hit the 2nd car. Will there be a collision?



$v_f = v_i + at \Rightarrow 15 = 25 - 0.1t$

$t = \frac{25-15}{0.1} = 100\,s$

it takes 100s for the 1st car to slow down to 15 m/s.

1st car: $x_f = x_i + v_i t + \frac{1}{2} a t^2$

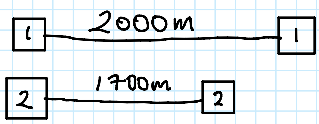
$x_{1f} = 0 + 25(100) - \frac{1}{2}(0.1)(100)^2$

$x_{1f} = 2000\,m$ @ 100s

2nd car: $x_f = x_i + v_i t + \frac{1}{2} a t^2$ → 0

$x_{2f} = 200 + 15(100)$

$x_{2f} = 1700\,m$ 2nd car @ 100s

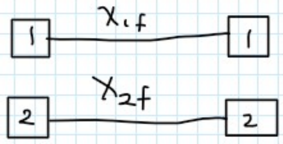


$x_{1f} > x_{2f}$

there will be a COLLISION!

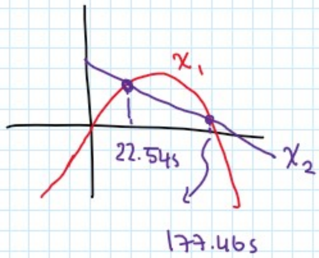
b) When will the 1st car hit the 2nd car?

b) When will the 1st car hit the 2nd car?



$$x_{1f} = 0 + 25t - \frac{1}{2}(0.1)t^2$$

$$x_{2f} = 200 + 15t$$



$x_{1f} = x_{2f}$ the moment for collision

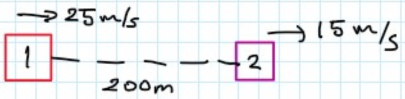
$$25t - 0.05t^2 = 200 + 15t$$

$$0 = 200 - 10t + 0.05t^2$$

$$\frac{+10 \pm \sqrt{10^2 - 4(0.05)(200)}}{2(0.05)} = \frac{10 \pm \sqrt{60}}{0.1} \begin{cases} 22.54\text{ s} \ \checkmark \\ 177.46\text{ s} \ (\times) \end{cases}$$

collision will happen @ $t = 22.54s$

c) What would be the acceleration of the 1st car so that there will be no collision? $a = ?$



1st car should have final velocity of 15 m/s

$$15 = 25 - at$$

$$a = \frac{10}{t}$$

$$x_{2f} = x_{1f}$$

$$200 + 15t = 25t - \frac{1}{2}at^2$$

$$200 + 15t = 25t - \frac{1}{2}\frac{10}{\cancel{t}}t^{\cancel{2}} \Rightarrow 200 = 5t \Rightarrow t = 40s$$

$$a = \frac{10}{t} = \frac{10}{40} = 0.25 \text{ m/s}^2$$

CHAPTER 3
MOTION in 2D & 3D

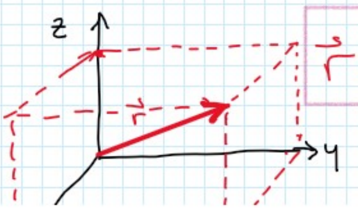
1D we defined x, v, a, Δ

* x, v, a are vectors; they were in 1D
  + values → direction
  − values ← direction

$\left\{ \begin{matrix} x \\ v \\ a \end{matrix} \right\}$ → + / → − these are vectors.

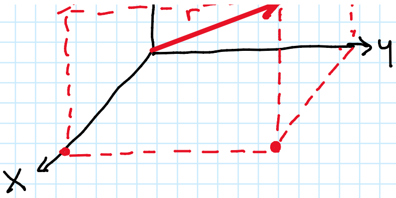
1D	2D & 3D
$x \rightarrow$	$\vec{r}$
$v \rightarrow$	$\vec{v}$
$a \rightarrow$	$\vec{a}$

for 2D & 3D we will use the same concepts and we use arrow for vectors.



$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$ position is 3D vector!

instantaneous velocity = derivative of position



instantaneous velocity = derivative of position

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}}$$

average velocity $= \dfrac{\text{Displacement} = \Delta x}{\Delta\, \text{time}}$

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$\boxed{\vec{\bar{v}} = \bar{v}_x\hat{i} + \bar{v}_y\hat{j} + \bar{v}_z\hat{k}}$$

Δ = final − initial

instantaneous acceleration = derivative of velocity

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$$

average acceleration $= \dfrac{\Delta\,\text{velocity}}{\Delta\, \text{time}}$

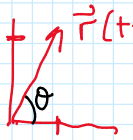
$$\vec{\bar{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \frac{\Delta v_z}{\Delta t}\hat{k}$$

$$\boxed{\vec{\bar{a}} = \bar{a}_x\hat{i} + \bar{a}_y\hat{j} + \bar{a}_z\hat{k}}$$

ex) $x = 2 - 0.25t^2$
$y = t + 0.025t^3$
$z = 0$
} position is given as these formula

a) What's the location of the object at $t = 2s$?
b) " " average velocity between 0s and 2s?
c) " " instantaneous velocity expression?
d) " " average acceleration between 0s and 2s?
e) " " instantaneous acceleration at $t = 2s$?

a) $\vec{r}(t=2) = (2-1)\hat{i} + (2+0.2)\hat{j} = \hat{i} + 2.2\hat{j}$



$|\vec{r}| = \sqrt{1^2 + 2.2^2} = 2.4\,m$

$\theta = \tan^{-1}\left(\frac{2.2}{1}\right) = 66°$

b) " " average velocity between 0s and 2s?

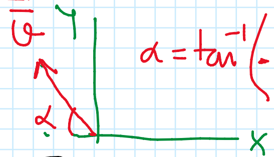
$$\vec{v} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \cancel{\frac{\Delta z}{\Delta t}}\hat{k}$$

$x = 2 - 0.25t^2$
$y = t + 0.025t^3$
$z = 0$

$$\vec{v} = \frac{x(2) - x(0)}{2-0}\hat{i} + \frac{y(2) - y(0)}{2-0}\hat{j} = \frac{\vec{r}(2) - \vec{r}(0)}{2}$$

$$= \frac{(1\hat{i}) - (2\hat{i})}{2} + \frac{2.2\hat{j} - 0\hat{j}}{2} = \left[-\frac{1}{2}\hat{i} + 1.1\hat{j}\right] m/s$$

$\hookrightarrow \sqrt{1.1^2 + 0.5^2}$



$\alpha = \tan^{-1}\left(\frac{1.1}{0.5}\right)$

c) $\vec{v}(2) = ?$ $\quad \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \vec{v}(t)$

c) $\vec{v}(2) = ?$ $\quad \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \vec{v}(t)$

$$-0.25(2)t\,\hat{i} + 1 + 0.025(3)t^2\,\hat{j} = -0.5t\,\hat{i} + (1+0.075t^2)\hat{j}$$

$$\vec{v}(2) = -0.5(2)\hat{i} + 1.3\hat{j} = \left[-\hat{i} + 1.3\hat{j}\right] m/s$$

$$\vec{v}(2) \neq \vec{v}(0,2s) \qquad \sqrt{1^2 + 1.3^2} = |\vec{v}(2)|$$

d) $\vec{a} = ?$ between 0s, 2s?

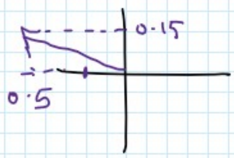
$$\vec{v} = \underbrace{-0.5t\,\hat{i}}_{v_x} + (\underbrace{1+0.075t^2}_{v_y})\hat{j}$$

$$\vec{a} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$= \frac{-0.5(2) - (\cancel{-0.5(0)})}{2-0}\hat{i} + \frac{1+0.075(4) - [1 + \cancel{0.075(0)}]}{2-0}$$

$$\vec{a} = -\frac{1}{2}\hat{i} + \frac{1.3-1}{2} = \left[-0.5\hat{i} + 0.15\hat{j}\right] m/s^2$$

$$|\vec{a}| = \sqrt{0.5^2 + 0.15^2}\ m/s^2$$



e) $\vec{a}(t=2) = ?$

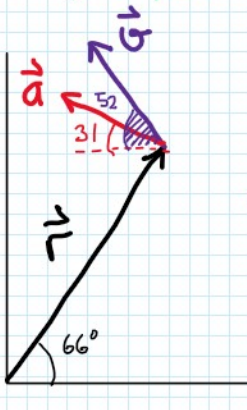
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = \left[-0.5\hat{i} + 0.15t\,\hat{j}\right] = \vec{a}(t)$$

$$\vec{v} = -0.5t\,\hat{i} + (1+0.075t^2)\hat{j} \qquad \vec{a}(2) = \left[-0.5\hat{i} + 0.3\hat{j}\right] m/s^2$$

$$\sqrt{0.5^2 + 0.3^2} = 0.58$$

$$\theta = \tan^{-1}\left(-\frac{0.3}{0.5}\right) = -31°$$

sketch $\vec{r}, \vec{v}, \vec{a}$ at $t = 2s$



$\vec{r}(2) = \hat{i} + 2.2\hat{j}$ $\quad |\vec{r}| = 2.42\ m$ $\quad \theta = 66°$

$\vec{v}(2) = -\hat{i} + 1.3\hat{j}$ $\quad |\vec{v}| = 1.64\ m/s$ $\quad \theta = -52°$

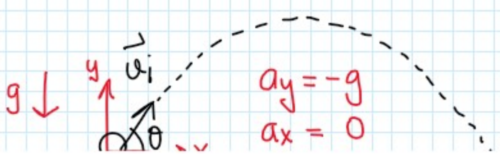
$\vec{a}(2) = -0.5\hat{i} + 0.3\hat{j}$ $\quad |\vec{a}| = 0.58$ $\quad \theta = -31°$

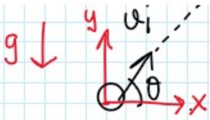
$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$

{$\vec{a}$ has parallel components to $\vec{v}$ / perpendicular "}

PROJECTILE MOTION (EĞİK ATIŞ)

In a projectile motion
if v_i, θ, g are given
you can find everything about the motion (range; time of flight h_{max}; impact velocity)





$a_y = -g$
$a_x = 0$

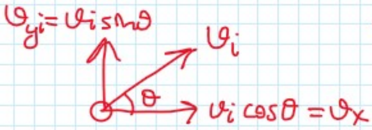
the motion (range; time of flight, h_{max}; impact velocity)

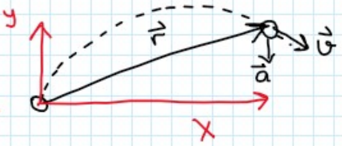
$$x_f = x_i + v_{xi}t + \cancel{\tfrac{1}{2}a_x t^2} \Rightarrow x = v_{xi}t \Rightarrow \text{RANGE (how far it goes)} \text{ (menzil)}$$

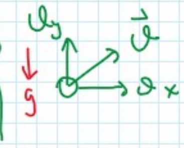
since $a_x = 0$: $v_x = v_{xi} = v_{xf} = \text{constant}$

$$\left\{\begin{aligned} y_f &= y_i + v_{yi}t - \tfrac{1}{2}gt^2 \\ v_{yf} &= v_{yi} - gt \\ v_{yf}^2 &= v_{yi}^2 - 2g(y_f - y_i) \end{aligned}\right\}$$

* time of flight (how long does it stay in the air?)
* how high it can go up? (h_{max})

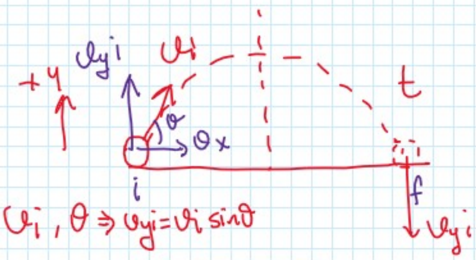






$\vec{v} = v_x\hat{i} + v_y\hat{j}$
$\vec{r} = x\hat{i} + y\hat{j}$
$\vec{a} = 0\hat{i} - g\hat{j}$

TIME of FLIGHT (How long does it stay in the air?)



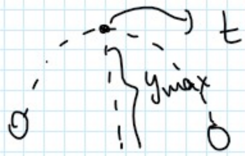
$v_i, \theta \Rightarrow v_{yi} = v_i \sin\theta$

$v_{yi} - gt = v_{yf}$; $y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$

$v_{yi} - gt = -v_{yi}$

$$t^* = \frac{2v_{yi}}{g} = \boxed{\frac{2v_i \sin\theta}{g}}$$

MAXIMUM HEIGHT (How high it can go up)



(1st eqn)

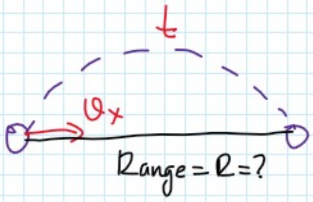
$$t = \frac{t^*}{2} = \frac{v_i \sin\theta}{g} \Rightarrow y_{max} = 0 + \frac{v_i^2 \sin^2\theta}{g} - \frac{1}{2}g\frac{v_i^2 \sin^2\theta}{g^2}$$

$$\boxed{y_{max} = \frac{v_i^2 \sin^2\theta}{2g}}$$

or (3rd eqn)

$$\underbrace{v_{yf}^2}_{0} = v_{yi}^2 - 2g(\underbrace{y_f}_{y_{max}} - \underbrace{y_i}_{0}) \Rightarrow \frac{v_{yi}^2}{2g} = \boxed{\frac{v_i^2 \sin^2\theta}{2g} = y_{max}}$$

RANGE (How far it can go)



time of flight

$$R = v_x t^* = v_x \frac{2v_i \sin\theta}{g}$$

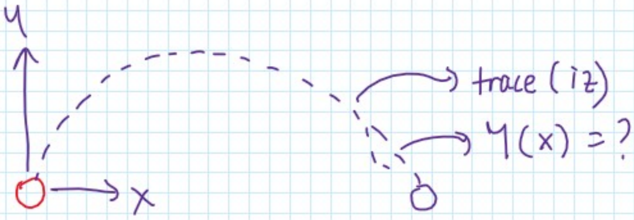
$$= v_i \cos\theta \frac{2v_i \sin\theta}{g} = \frac{2v_i^2 \cos\theta \sin\theta}{g}$$

$$\boxed{R = v_i^2 \frac{\sin 2\theta}{g}}$$

$\frac{dR}{d\theta} = 0$ you will find special θ value to make R max.

$$\frac{dR}{d\theta} = \frac{v_i^2}{g} 2\cos 2\theta = 0 \Rightarrow \cos 2\theta = 0 \Rightarrow 2\theta = 90^\circ$$

$$\underline{\underline{\theta = 45^\circ}}$$



$v_{yi} = v_i \sin\theta$

$v_x = v_i \cos\theta$

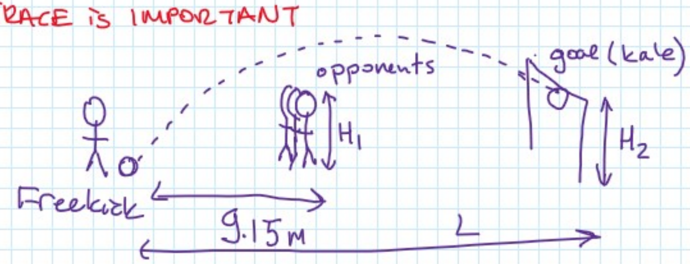
$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2 = y(t) = y_i + v_{yi}\frac{x}{v_x} - \frac{1}{2}g\frac{x^2}{v_x^2} = C + Bx + Ax^2$

$x = v_x t \Rightarrow t = \frac{x}{v_x}$

$$y(x) = y_i + \tan\theta \, x - \frac{g}{2v_i^2\cos^2\theta} x^2$$

This shows you the trace formula for $y(x)$. for any x value you can find y (height) of the ball

TRACE IS IMPORTANT



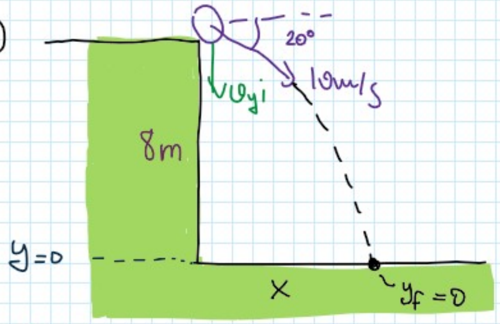
→ you want the ball to go over the oponents (baraj)

$y(x = 9.15\,m) > H_1$

→ you want the ball to go under the top post of goal.

$y(x = L) < H_2$

ex



$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$

$v_{yf} = v_{yi} - gt$

$\left[v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i) \right]$

$x = v_x t$?

?

$v_x = v_i \cos\theta = 10\cos 20 = 9.3\,m/s$

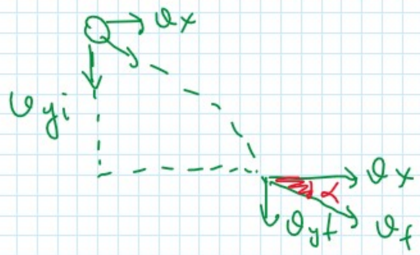
$v_{yi} = 10\sin 20 = 3.42\,m/s$

$0 = 8 - 3.42t - 4.9t^2 \equiv 0$

$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = $ → $-1.7s$ ✗ negative

→ $0.98s$ ✓

$x = v_x t = (9.3)(0.98) \simeq 9.2\,m$



$\vec{v}_f = ?$

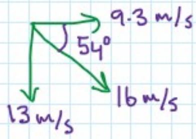
$v_{yf} = v_{yi} - gt$

$v_f = \sqrt{v_x^2 + v_{yf}^2} = \sqrt{9.3^2 + 13^2} = 15.98 = 16\,m/s$

$v_{yf} = -3.42 - 9.8(0.98)$

$v_{yf} = -13.02 \simeq -13\,m/s = v_{yf}$

$\alpha = \tan^{-1}\left(\frac{-13}{9.3}\right) = -54.4° = -54°$

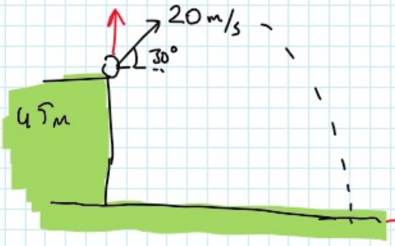


ex

$20\sin 30 = 10\,m/s$

a) what's time of flight? $\left(y_f = y_i + v_{yi}t - \frac{1}{2}gt^2 \right)$

(ex)



a) what's time of flight?

$$0 = 45 + 10t - \frac{1}{2}(9.8)t^2$$

$$4.9t^2 - 10t - 45 = 0$$

$$\frac{+10 \mp \sqrt{10^2 - 4(-45)(4.9)}}{9.8} = \begin{cases} t < 0 \\ t = 4.22\,s \end{cases}$$

$$\left\{ \begin{array}{l} y_f = y_i + v_{yi}t - \frac{1}{2}gt^2 \\ v_{yf} = v_{yi} - gt \\ v_{yf}^2 = v_{yi}^2 - 2g\Delta y \\ x = v_x t \end{array} \right\}$$

b) what's the range of motion?

range = menzil

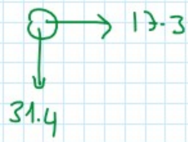
$v_x = 20\cos 30 = 10\sqrt{3}$ m/s

$x = v_x t = 10\sqrt{3}\,(4.22) = 73.1\,m$

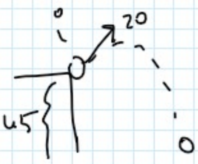
c) what's the final velocity just before it hits the ground?

$v_{yf} = v_{yi} - gt = 10 - (9.8)(4.22) = -31.4$ m/s

$\left[17.3^2 + 31.4^2\right]^{1/2} = v_f = 35.9$ m/s



energy conservation = $\frac{m v_{yf}^2}{2} = \frac{m v_{yi}^2}{2} - 2g\Delta y\, m$



$E_i = E_f$ ✓

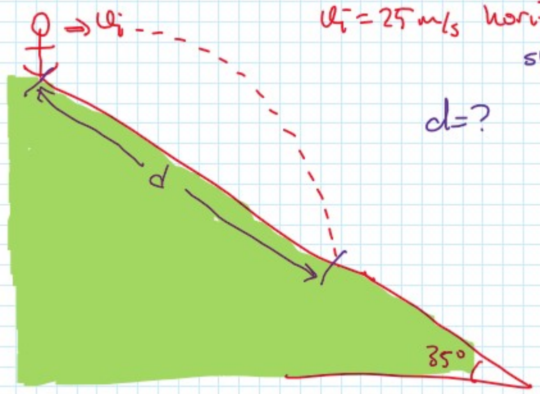
ex

$v_i = 25$ m/s horizontal

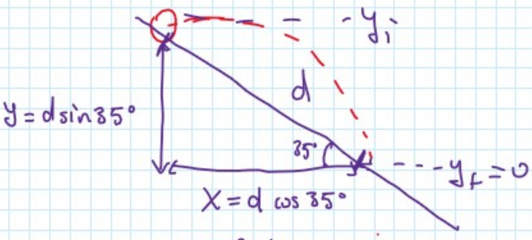
slope of hill is 35°

d = ?

$$\left\{ \begin{array}{l} y_f = y_i + v_{yi}t - \frac{1}{2}gt^2 \\ v_{yf} = v_{yi} - gt \\ v_{yf}^2 = v_{yi}^2 - 2g\Delta y \\ x = v_x t \end{array} \right.$$



① $d\sin 35 + 0 - \frac{1}{2}(9.8)t^2 = 0$



① $\boxed{d\sin 35 = 4.9t^2}$

$x = v_x t$

$\boxed{d\cos 35 = 25t = x}$ ②

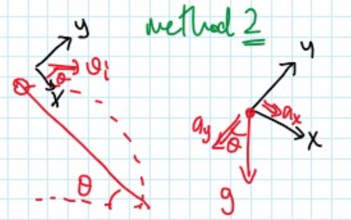
$\frac{①}{②} = \frac{\sin 35}{\cos 35} = \tan 35 = \frac{4.9t^2}{25t} = \frac{4.9}{25}t$

$0.7 = 0.196t \Rightarrow t = 3.57\,s$

$d = \frac{25t}{\cos 35} = \frac{25(3.57)}{0.82}$

$d = 108.8 \simeq 109\,m$

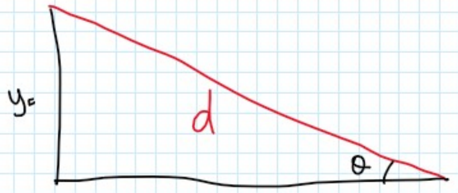
method 2



$\cancel{x = v_x t}$

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

$d = 0 + 25\cos 35\, t + \frac{1}{2} g\sin 35\, t^2$



$x = v_x t$

① $d = \sqrt{x^2 + y^2} = \left[(v_x t)^2 + (4.9 t^2)^2 \right]^{1/2}$

② $\tan\theta = \dfrac{4.9 t^2}{v_x t} \rightarrow \dfrac{\frac{g}{2} t^2}{v_x t} = \dfrac{g}{2 v_x} t$

$$\boxed{t = \frac{2 v_x \tan\theta}{g}}$$

$$d = \left[\frac{4 v_x^4 \tan^2\theta}{g^2} + \frac{1}{\cancel{4}} \cancel{g^2} \, \frac{\cancel{4}16 v_x^4 \tan^4\theta}{g^{\cancel{4}2}} \right]^{1/2} = \sqrt{\frac{4 v_x^4 \tan^2\theta}{g^2} \left(1 + \tan^2\theta\right)} = d$$

$\left\{ \begin{array}{l} d(v_x, \theta) \\ \text{is a function} \\ \text{of } v_x \text{ and } \theta \end{array} \right\}$ $\boxed{\dfrac{2 v_x \tan\theta}{g} \sqrt{1 + \tan^2\theta} = d}$

CIRCULAR MOTION (Dairesel Hareket)

2D motion

UNIFORM CIRCULAR MOTION

UCM $\quad v = \text{const} = |\vec{v}|$

$R = \text{const}$

NONUNIFORM CIRCULAR MOTION

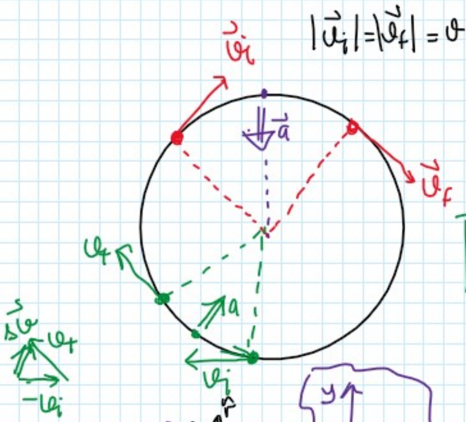
NUCM $\quad v \neq \text{constant}$

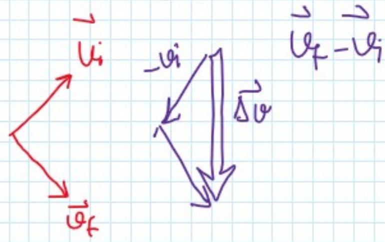
$|\vec{v}_i| = |\vec{v}_f| = v$

$\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$ ⇓

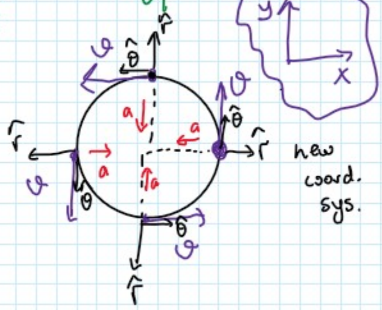
$\vec{a}$ ⇓

$\boxed{\vec{a} \text{ is towards center}}$ *





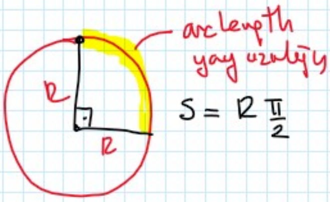
radial acc. $\left\{ a \overset{?}{=} \dfrac{v^2}{R} \right\}$



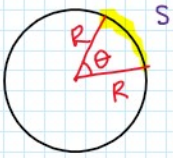
$\hat{\theta} (\perp \hat{r})$

$\hat{r}$ (out of center)

$\left\{ \begin{array}{l} \vec{v} \text{ is } \hat{\theta} \text{ direction} \\ \vec{a} \text{ is } -\hat{r} \text{ direction} \end{array} \right\}$



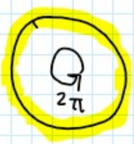
$S = R \dfrac{\pi}{2}$



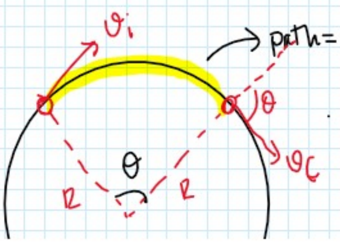
$S = R\theta$

θ = radians

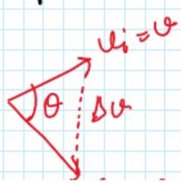
$\pi = 3.14$ rad



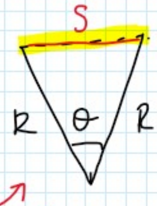
$S = R \, 2\pi$

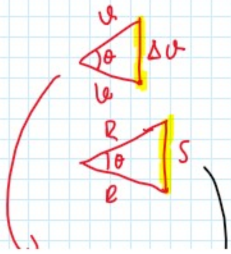


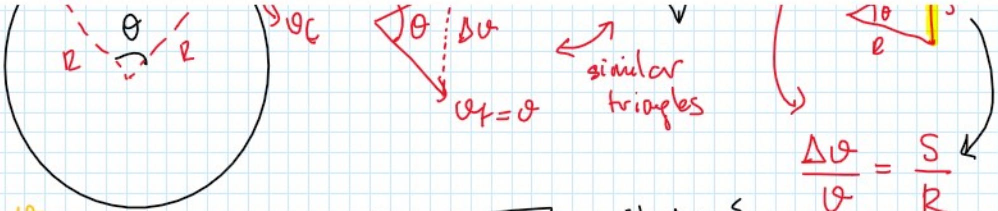
→ path = arclength = $R\theta$



similar triangles







$$\frac{\Delta v}{v} = \frac{S}{R}$$

$$\frac{1}{\Delta t}\frac{\Delta v}{v} = \frac{S}{R}\frac{1}{\Delta t} \Rightarrow \frac{a}{v} = \frac{v}{R} \Rightarrow \boxed{a = \frac{v^2}{R}}$$ magnitude of acceleration

$$\frac{1}{\Delta t}\frac{\Delta v}{v} = \frac{\cancel{R}\theta}{\cancel{R}}\frac{1}{\Delta t}$$

$$\frac{a}{v} = \omega$$

$$a = \omega \underset{\omega R}{v}$$

$$\boxed{a = \omega^2 R}$$

1 full revolution = 1 tam tur = $2\pi R$

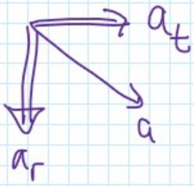
$$\left\{ T = \frac{2\pi R}{v} \equiv \text{period} \right\} \qquad a = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2} = a$$

RADIAL ACCELERATION

$$\boxed{\vec{a}_r = \frac{v^2}{R}\left(-\hat{r}\right)}$$ * * *

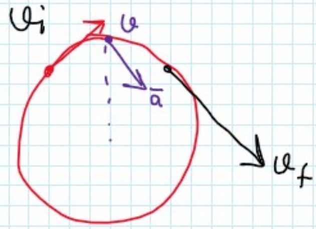
WHAT IF YOU SPEED UP or SLOW DOWN ON A CIRCULAR PATH? $\Longrightarrow$ NUCM

$v \neq$ const $\quad \frac{|d\vec{v}|}{dt} \neq 0$

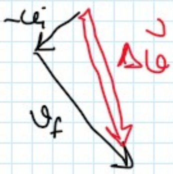


$$\vec{a}_t = \frac{|d\vec{v}|}{dt}$$

$$a_r = \frac{v^2}{R}$$

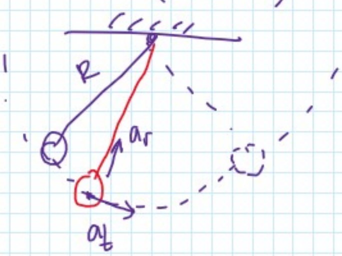


$$|\vec{v}_f| > |\vec{v}_i| \qquad \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

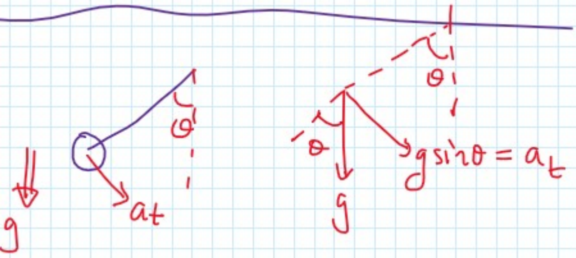


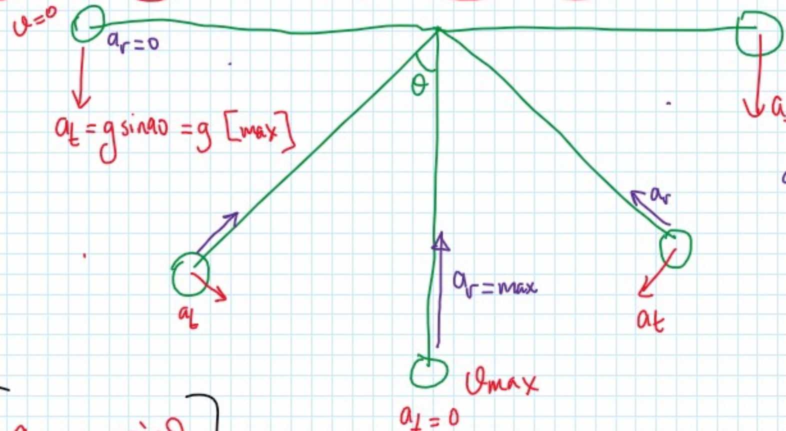
a_t = tangential acceleration (teğetsel ivme)

PENDULUM (sarkaç) (NUCM)



$$a_r = \frac{v^2}{R}$$ ✓ always TRUE!





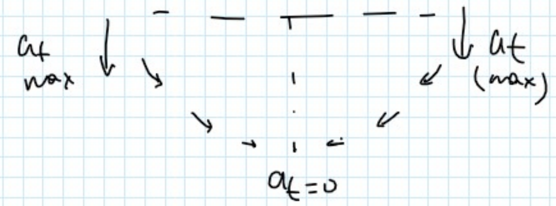
$[a_t = g\sin\theta]$

a_t a_t

$$\boxed{\begin{aligned} a_t &= g\sin\theta \\ a_r &= \frac{v^2}{R} \end{aligned}}$$

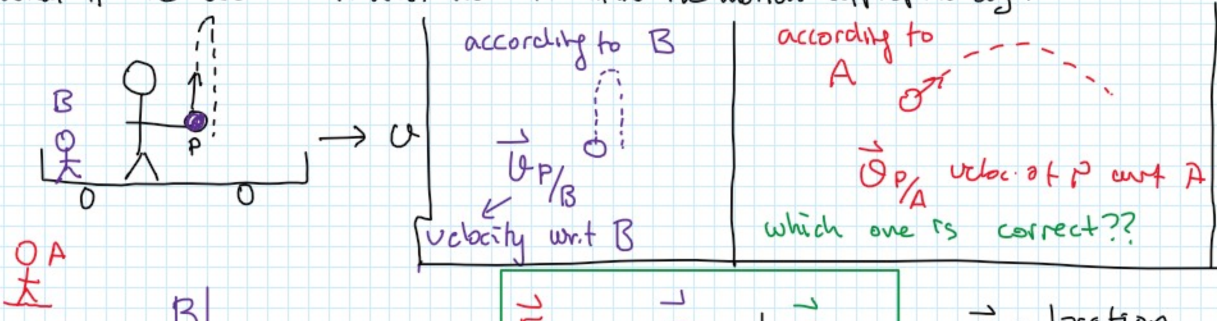
① v_{max}

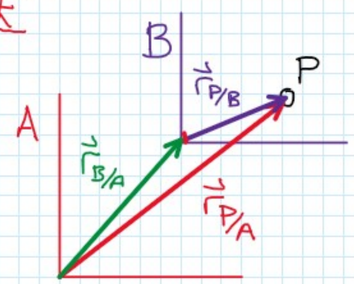
$a_t = 0$



RELATIVE MOTION (Bağıl hareket)

- we define motion with respect to (w.r.t.) an observer.
- what if the observer moves? how to define the motion appropriately?





$$\boxed{\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A}}$$

$\vec{r}$ = location vector $\left\{ \vec{r}_{Head/Tail} \right\}$

$\frac{d}{dt}$

$$\boxed{\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}}$$

$\frac{d}{dt}$

$$\boxed{\vec{a}_{P/A} = \vec{a}_{P/B} + \vec{a}_{B/A}}$$

⇒ we want A & B to measure the same acceleration $a = g$

$$\boxed{\vec{a}_{P/A} = \vec{a}_{P/B}\ ; \quad \underline{\vec{a}_{B/A} = 0}\ ; \quad \underline{\vec{v}_{B/A} = \text{const}}}$$

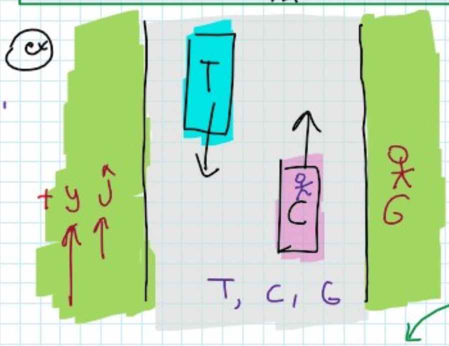
⇒ If not, A & B don't agree on the measurements.

* ANY MEASUREMENTS SHOULD BE DONE IN NON-ACCELERATING FRAME = INERTIAL FRAME of REFERENCE

will discuss it more in Chapter 4

$$\boxed{\text{SO WE WANT } \vec{a}_{B/A} = 0 \text{ ALWAYS}}$$

ex



Speed of the TRUCK is 88 km/hr

" " " CAR " 104 km/hr.

they move towards each other.

What's the velocity of the TRUCK w.r.t. CAR? $\vec{v}_{T/C} = ?$

① $\vec{v}_{T/C} = \vec{v}_{T/G} + \vec{v}_{G/C}$

① $\vec{v}_{T/C} = -88\hat{j} + (-104\hat{j})$

$= -192$ km/...

$\left\{ \vec{v}_{T/G} = \vec{v}_{T/C} + \vec{v}_{C/G} \right.$ ②

$= -88\hat{j}$ ...

(1) $\vartheta_{T/C} = -88\hat{j} + (-104\hat{j})$
$= -192\ km/hr\ \hat{j}$

$\vartheta_{T/G} = \vartheta_{T/C} + \vartheta_{C/G}$ (2)

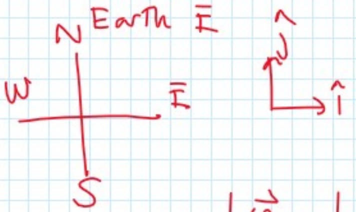
$-88\hat{j} = \vec{\vartheta}_{T/C} + 104\hat{j}$

$\vec{\vartheta}_{T/C} = -192\ km/hr\ \hat{j}$

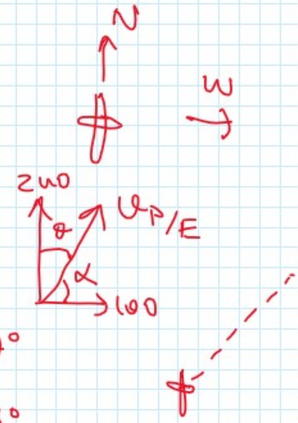
---

ex) A plane's compass (pusula) shows North direction. Its speed is 240 km/hr.
(a) There is a wind from West to East with a speed of 100 km/hr.
What's the velocity of the Plane w.r.t. Earth?

Plane P
Wind W
Earth E



$$\vec{\vartheta}_{P/E} = \vec{\vartheta}_{P/W} + \vec{\vartheta}_{W/E}$$
$$= 240\hat{j} + 100\hat{i}$$
$$= 100\hat{i} + 240\hat{j}$$

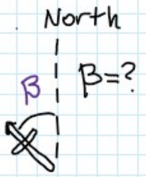


$|\vec{\vartheta}_{P/E}| = \sqrt{100^2 + 240^2}$
$= 260\ km/hr$

$\alpha = \tan^{-1}\left(\frac{240}{100}\right) = 67°$

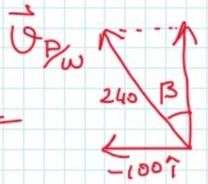
$\theta = \tan^{-1}\left(\frac{100}{240}\right) = 23°$

(b) The wind's velocity is same as above and the speed of the plane is the same.
What would be the direction of plane such that it travels directly to North.



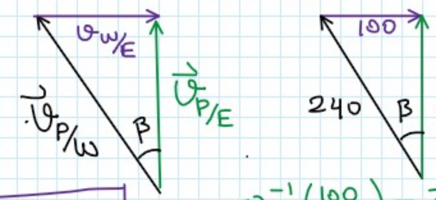
$$\vec{\vartheta}_{P/E} = \vec{\vartheta}_{P/W} + \vec{\vartheta}_{W/E}$$

1st method
$\underbrace{(\dots)\hat{j}}_{\text{North!}} = \underbrace{(\dots)\hat{i} + (\dots)\hat{j}}_{\text{the magnitude is 240km/hr}} + 100\hat{i}$ $\Rightarrow \hat{i}$ terms cancel eachother



$\beta = \sin^{-1}\left(\frac{100}{240}\right) = 24.6° \Leftarrow$
$\beta = 25°$

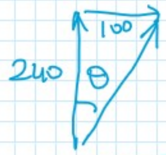
2nd method
$\vec{\vartheta}_{P/E} = \vec{\vartheta}_{P/W} + \vec{\vartheta}_{W/E}$
DRAW THIS



$\sqrt{240^2 - 100^2} = |\vec{\vartheta}_{P/E}|$
$= 218\ km/hr$

$\sin^{-1}\left(\frac{100}{240}\right) = 24.6 = 25°$

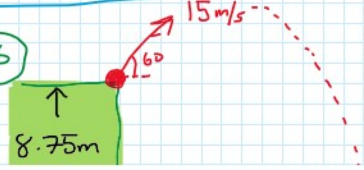
in part (a) we found



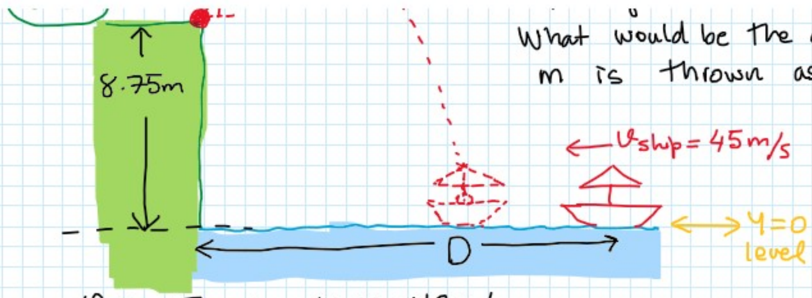
$\theta = \tan^{-1}\left(\frac{100}{240}\right) \simeq 23°$

$\beta \neq \theta$
$25° \neq 23°$

3.56



a package of mass m must be thrown to ship.
What would be the distance D when mass m is thrown as in the figure?



What would be the distance D when mass m is thrown as in the figure?

① $y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$

② $v_{yf} = v_{yi} - gt$

③ $v_{yf}^2 = v_{yi}^2 - 2g\Delta y$

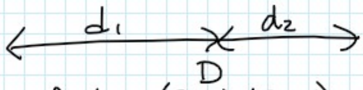
④ $x = v_x t$

$v_{yi} = 15 \sin 60 = 12.99 \simeq 13 m/s$

$v_x = 15 \cos 60 = 7.5 m/s$

$y_f = 0$

① $0 = 8.75 + 13t - 4.9t^2 \rightarrow t = 3.21 s$ (time of flight)



$D = d_1 + d_2$

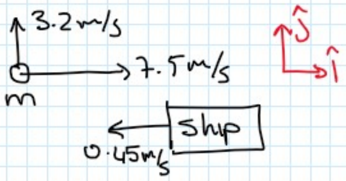
$$\left.\begin{aligned} d_1 &= v_x t = (7.5)(3.21) = 24.08 m \\ d_2 &= v_{ship} t = (0.45)(3.21) = 1.44 m \end{aligned}\right\} D = 25.52 m$$

ⓑ What's the velocity of mass m at 1s w.r.t. ship?

$v(t=1s) = ?$

$v_x = 7.5 m/s$

$v_{yf} = 13 - (9.8) 1 = 3.2 m/s$



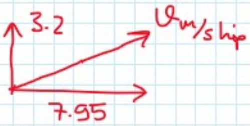
$\vec{v}_{m/ship} = ?$

$\vec{v}_{ship/sea} + \vec{v}_{m/ship} = \vec{v}_{m/sea}$

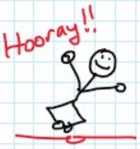
$\vec{v}_{m/ship} \stackrel{?}{=} \vec{v}_{m/sea} - \vec{v}_{ship/sea}$

$= [7.5\hat{i} + 3.2\hat{j}] - [-0.45\hat{i}]$

$\vec{v}_{m/ship} = 7.95\hat{i} + 3.2\hat{j}$



$|\vec{v}_{m/ship}| = 8.57 m/s$



the end of Ch2 & Ch3

I