

6 & 7

work energy & Kinetic E, pot E ...  
energy conservation.

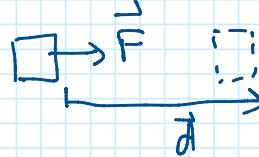
$$\vec{F} \cdot \vec{d} \Rightarrow W = Fd \cos\theta$$

dot product

$$W = Fd \cos\theta$$

d = displacement =  $\Delta x$

$$\text{Work} = \vec{F} \cdot \vec{d} = W$$



$$A \cdot B \cos\theta = \vec{A} \cdot \vec{B}$$

$$W = \vec{F} \cdot \vec{d}$$

$$\left[ \text{Joule} = N \text{ m} = kg \frac{m^2}{s^2} \right]$$

$$\vec{i} \cdot \vec{j} = |\vec{i}| |\vec{j}| \cos\theta$$

| | | |

ex:  $\vec{F} = (160\hat{i} - 40\hat{j}) \text{ N}$      $\vec{d} = (14\hat{i} + 11\hat{j}) \text{ m}$

$$W = \vec{F} \cdot \vec{d} = 160(14) \frac{\hat{i} \cdot \hat{i}}{1} - 40(11) \frac{\hat{j} \cdot \hat{j}}{1} + 0 + 0$$

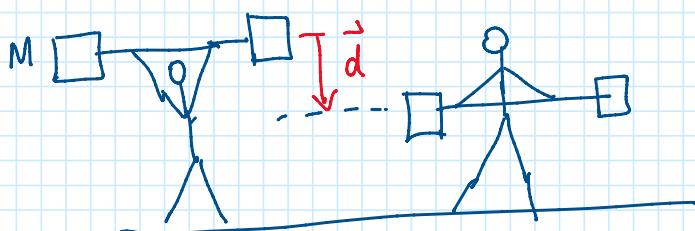
$$= 1800 \text{ J}$$

$$\theta = ?$$

$$W = Fd \cos\theta = 1800$$

$$\sqrt{160^2 + 40^2} \quad \sqrt{14^2 + 11^2} \quad \cos\theta = 1800 \quad \theta = \cos^{-1} \left( \frac{1800}{200(18)} \right) \approx 30^\circ$$

Lowering the weight/dumbbell by a person.



Work done by the person?

$$W = \vec{F} \cdot \vec{d}$$

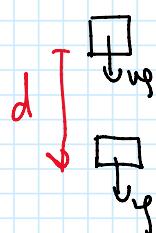
$$= Fd \cos 180^\circ$$

$$= -Fd$$

$$F = mg \quad = -mgd$$

Work done by mg force?

$$W = \vec{mg} \cdot \vec{d} = mgd$$



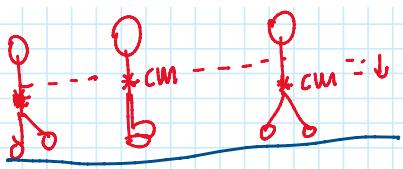
Work done by the person when a person walks on a straight path.



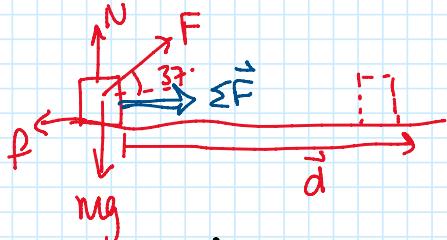
$$\text{cm} \quad \text{running} \quad \text{walking}$$

$$mg = F \quad pd$$

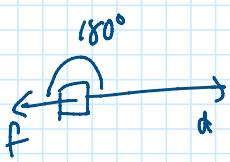
$$\text{displacement} = F_{\text{run}}$$



cm ~~W~~  $\rightarrow \mu mg = F$   $\rightarrow$   
 $80\text{kg} \sim 800\text{N}$   $cm = \text{displacement} = 5\text{cm}$   
 $W = (800)(0.05) = 40\text{J}$



$$F = 5000\text{N} \quad f = 3500\text{N} \quad d = 20\text{m}$$



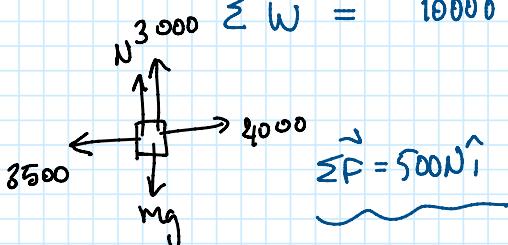
$$W_N = N(20) \cos 90^\circ = 0$$

$$W_{mg} = mg(20) \cos 90^\circ = 0$$

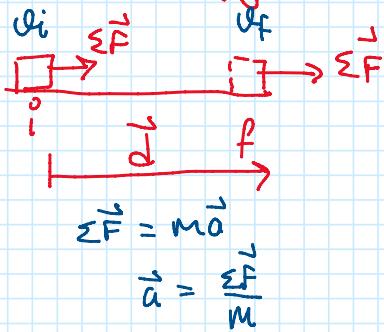
$$W_F = 5000(20) \cos 37^\circ = 80000\text{J}$$

$$W_f = 3500(20) \cos 180^\circ = -70000\text{J}$$

$$\begin{aligned} \sum W &= 10000\text{J} = \vec{\sum F} \cdot \vec{d} \\ &= (500)(20) = \frac{1}{2} P \cdot d \\ \sum W &= 10000\text{J} = \frac{1}{2} P \cdot d \end{aligned}$$



## Kinetic energy & the work-energy theorem.



$\vec{d} = \vec{s} = \Delta \vec{x} = \Delta \vec{r} = \text{displacement}$ .

$v_f > v_i \quad \vec{\sum F} > 0 \quad a > 0$

$$\sum W = W_{\text{tot}} = \vec{\sum F} \cdot \vec{d} > 0$$

$\therefore \Delta x = d$

$$v_f^2 = v_i^2 + 2a \Delta x$$

$$v_f^2 = v_i^2 + 2 \frac{\vec{\sum F}}{m} d \Rightarrow \vec{\sum F} d = \frac{m}{2} (v_f^2 - v_i^2)$$

$$\underbrace{W_{\text{tot}} = \Delta K}_{\text{work - kinetic}}$$

$$\therefore K = m \frac{v^2}{2} = (\text{mass})$$

$$\frac{\sum F d}{m} = \frac{m v_f^2}{2} - \frac{m v_i^2}{2}$$

Energy theorem.

$$W_{\text{tot}} > 0$$

$$W_{\text{tot}} < 0$$

$$\Delta K > 0$$

$$\Delta K < 0$$

$$K_f > K_i$$

$$K_i > K_f$$

$$v_f > v_i$$

$$v_i > v_f$$

$$K = m \frac{v^2}{2}$$

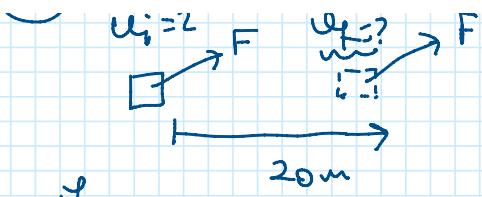
$$\left[ J = kg \frac{m^2}{s^2} \right]$$

(ex)

$$v_i = ? \quad F \quad v_f = ? \quad F$$

$$\sum \vec{F} = 500\text{N} \uparrow$$

$$\begin{array}{c} N \\ \uparrow \\ F \\ \leftarrow \quad \rightarrow \\ s \end{array}$$

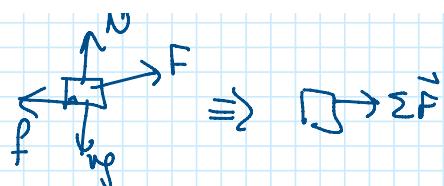


$$mg = 14700N$$

$$m = \frac{14700}{9.8} = 1500\text{kg}$$

$$\sum \vec{F} = 500N \uparrow$$

$$\sum \vec{F} \cdot d = W_{\text{tot}}$$

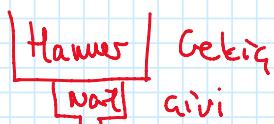


$$= 10000J$$

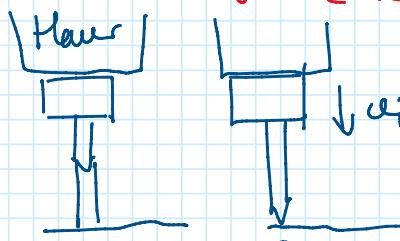
$$= \frac{m v_f^2}{2} - \frac{m v_i^2}{2} = 10000$$

$$v_f = 4.2 \text{ m/s}$$

ex)



after Hammer hits the nail



$F$  = friction

$$W_{\text{tot}} = \vec{F} \cdot \vec{d} ; mg < F$$

$$= -Fd = \Delta K$$

$$-Fd = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$$

$$-Fd = -\frac{mv_i^2}{2}$$

$$F = \frac{mv_i^2}{2d}$$

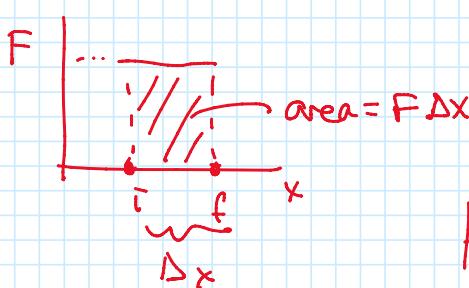
$$v_i = 5 \text{ m/s}$$

$$m = 0.01 \text{ kg}$$

$$d = 1 \text{ cm}$$

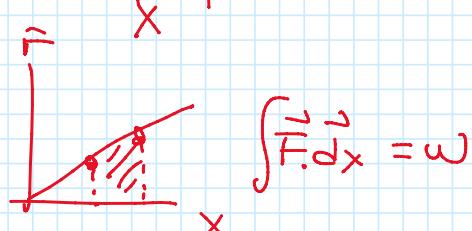
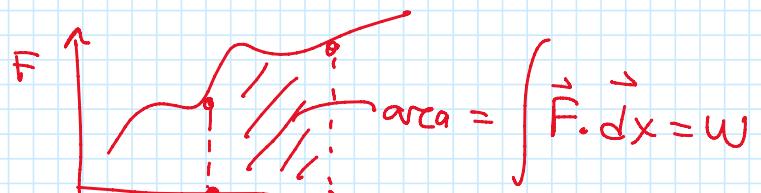
$$F = \frac{(0.01) 5^2}{2 (0.01)} = 12.5 \text{ N}$$

$$W = \vec{F} \cdot \vec{\Delta x} = \text{when } \vec{F} \text{ is const.}$$



$$\vec{F} ; d$$

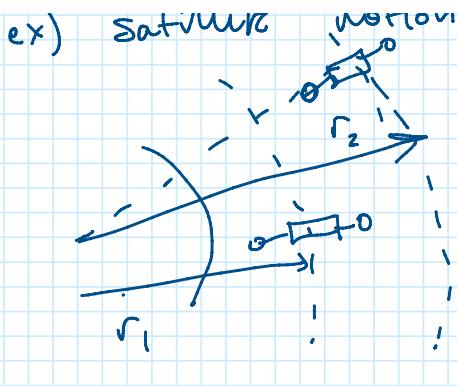
$$\vec{F} ; \vec{d} \quad W < 0 \quad W = \int \vec{F} \cdot \vec{\Delta x}$$



ex) satellite motion

$$r_2 > r_1$$

$$F_{\text{satellite}} = -\frac{1.3 \times 10^{22}}{x^2}$$



$$F_{\text{saturation}} = -\frac{1.3 \times 10^{-1}}{r^2}$$

$$\vec{F} = -\frac{1.3 \times 10^{22}}{r^2} \hat{r}$$

what's the work done by  $\vec{F}$  if satellite moves from  $r_1 = 1.5 \times 10^6 \text{ m}$  to  $r_2 = 2.3 \times 10^6 \text{ m}$

$$W = \int \vec{F} \cdot d\vec{x}$$

(-)

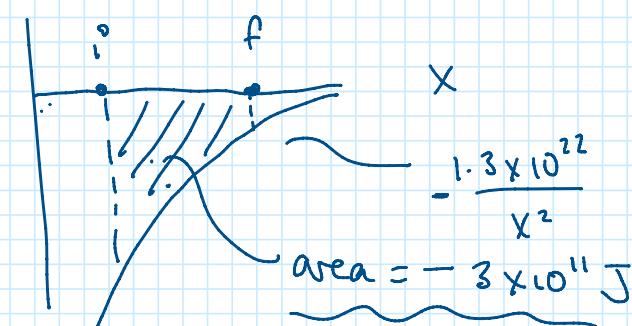
$W < 0$

$$= \int \vec{F} \cdot d\vec{x} = \int |\vec{F}| \cdot |\vec{d}x| \cos \theta = \int |F| dx (-1)$$

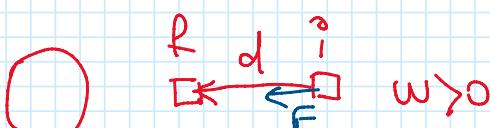
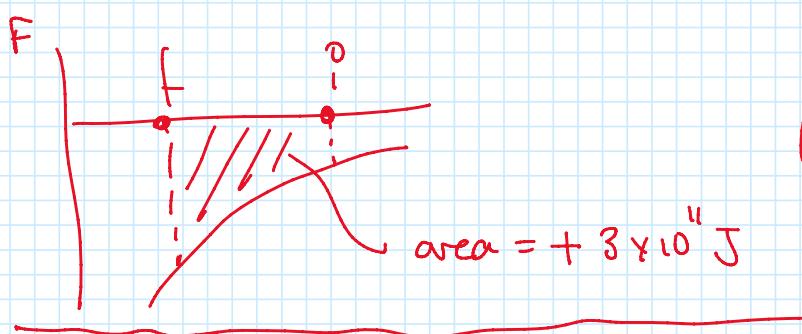
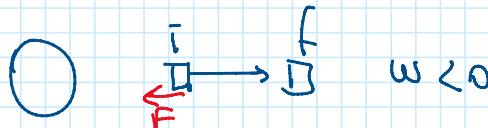
$$= -1.3 \times 10^{22} \int \frac{dx}{r^2}$$

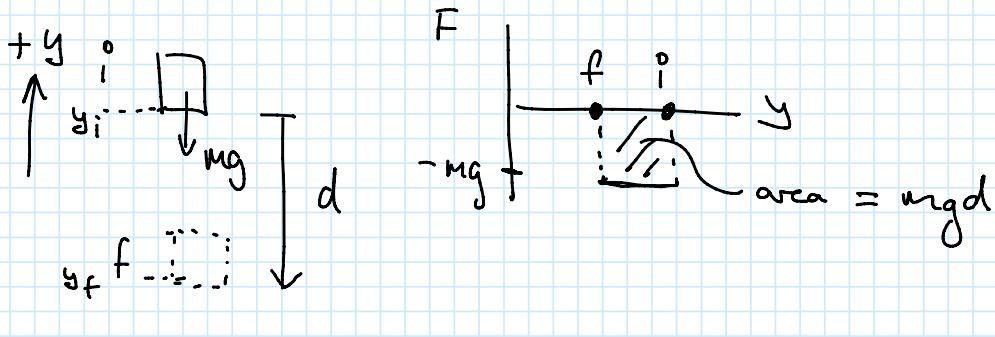
$$= -1.3 \times 10^{22} \left[ -\frac{1}{r} \right]_{r_i}^{r_f} = -1.3 \times 10^{22} \left[ -\frac{1}{2.3 \times 10^6} + \frac{1}{1.5 \times 10^6} \right]$$

$$= 1.3 \times 10^{22} \left[ \frac{1}{2.3 \times 10^6} - \frac{1}{1.5 \times 10^6} \right] = \left[ \frac{1.3}{2.3} - \frac{1.3}{1.5} \right] \times 10^{10}$$

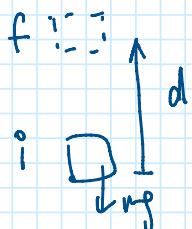


$$W_{\text{tot}} = -3 \times 10^{11} \text{ J} < 0$$

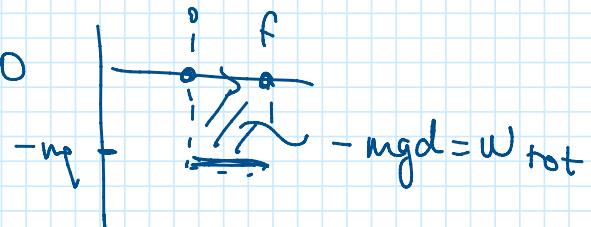




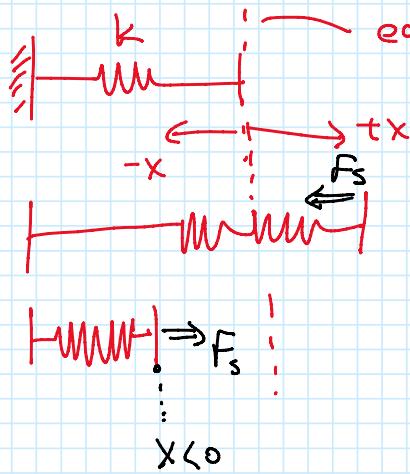
$$\vec{F} = -mg(\hat{j})$$



$$W_g = -mgd < 0$$



### SPRING FORCE



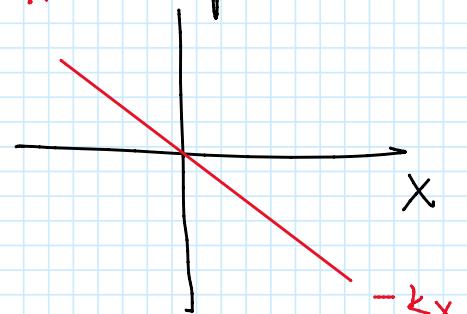
equilibrium position = dege. when  $F_s = -kx = 0 = x$

$$F_s = -\hat{i}$$

$$F_s = -kx \quad ; \quad x > 0$$

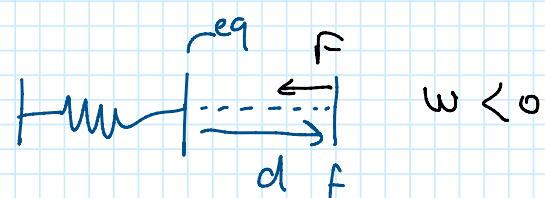
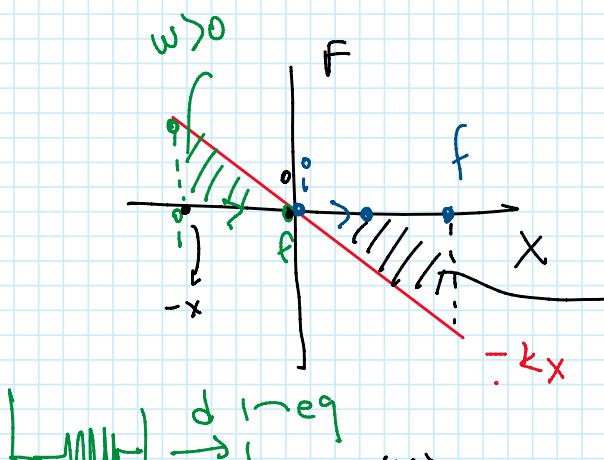
$$F_s = -kx \quad ; \quad x < 0$$

$$F_s + \hat{i}$$



$$x_{eq} = 0 \quad F_s = -kx$$

$$x_{eq} = x_0 \quad F_s = -k\Delta x = -k(x_f - x_0)$$



$$x_i = 0$$

$$x_f = x$$

$$\int_{0}^{x} -kx \, dx = -k \frac{x^2}{2}$$

$$W = -k \frac{x^2}{2} < 0$$

$$\omega = -k \frac{x^2}{2} < 0$$

$$\omega = \int_{-x}^0 -kx \, dx = -k \frac{x^2}{2} \Big|_{-x}^0 = -\left[ k \frac{0^2}{2} - k \frac{(-x)^2}{2} \right]$$

$$\omega = -\left[ k \frac{x_f^2}{2} - k \frac{x_i^2}{2} \right] = k \frac{x_i^2}{2} - k \frac{x_f^2}{2}$$

$$|x_f| < |x_i|$$

$$x_i = -x \quad x_f = 0$$

$$\omega = +k \frac{x^2}{2} > 0$$

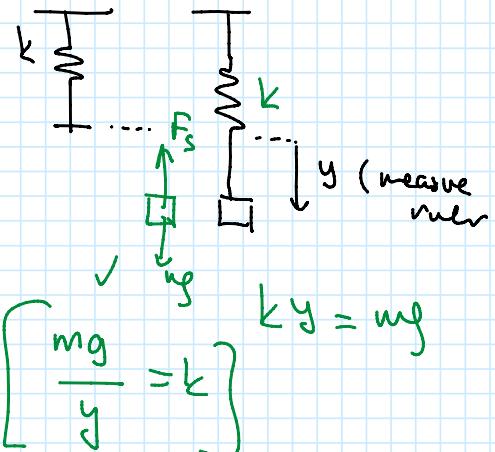
$$F_s = -kx$$

$$k = \frac{\text{spring const}}{m} = \frac{N}{kg \cdot m/s^2} = \frac{kg}{m \cdot s^2}$$

$m$  is known  $\Rightarrow$  measure  $k$   $\leq$  spring

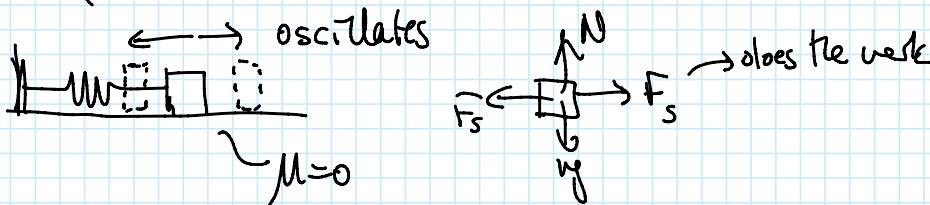
$$m_1, l \text{ is unknown} \quad m_1 = \frac{k y_1}{g}$$

$$F_s = -kx \quad (x_{eq} = 0)$$



$$W_{tot} = \Delta K$$

If only spring force does the work



$$W_{tot} = W_{spring} = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

~~W<sub>tot</sub>~~ ~~W<sub>spring</sub>~~ ~~M=0~~ ~~work by spring~~ = change in K

$$\int_{x_i}^{x_f} -kx \, dx = -k \frac{x^2}{2} \Big|_{x_i}^{x_f} = -\left[ k \frac{x_f^2}{2} - k \frac{x_i^2}{2} \right]$$

$$= k \frac{x_i^2}{2} - k \frac{x_f^2}{2}$$

POWER (giga)

$$P = \frac{\text{work}}{\text{time}} = \frac{W}{t} = \int \frac{F}{s} = \text{watt}$$

$$1 \text{hp} = 1 \text{horsepower} = 746 \text{W}$$

$$P_{ave} = \frac{\text{work}}{\text{time}} = \frac{W}{t} = \left[ \frac{J}{s} = \text{watt} \right]$$

$1 \text{hp} = 1 \text{horsepower} = 746 \text{W}$

$100 \text{hp, or} = 74600 \text{W}$

$\approx 75 \text{kW}$

$$P = \frac{dW}{dt} \quad (\text{instantaneous power})$$

Electric bills (elektrik fatura) = energy =  $[ \text{Joules} ] = 100 \text{ kW} \cdot \text{hr}$

$$\begin{aligned} P &= \frac{dW}{dt} \quad W = \vec{F} \cdot \vec{s} \quad (\vec{F} \text{ const}) \\ &\quad W = \vec{F} \cdot \vec{x} = \int \vec{F} d\vec{x} = \int dW \\ &\quad dW = \vec{F} \cdot \vec{dx} = F dx \\ \Rightarrow P &= \vec{F} \cdot \frac{\vec{dx}}{dt} = \vec{F} \cdot \vec{v} = \vec{F} \cdot \vec{v} \end{aligned}$$

100 1000W 3600S

100  $3.6 \times 10^6$  Joules

ex) Airbus A380 plane  $F = 322 \text{ 000 N}$   
 $v = 250 \text{ m/s}$

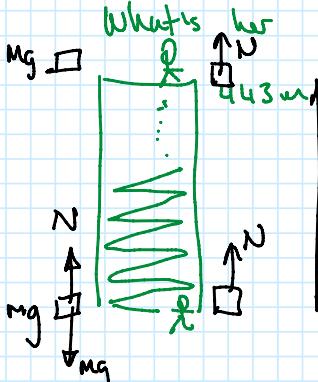
$$1 \text{hp} = 746 \text{W}$$

$$\begin{aligned} P &= \vec{F} \cdot \vec{v} \\ &= F v \cos \theta \\ &= 8.05 \times 10^7 \text{W} \end{aligned}$$

$$P = \underline{108 \text{ 000 hp}} \quad (\text{plane})$$

ex) A 50kg mass of runner, runs up 443m tall tower in 15 minutes.

What's her average power?

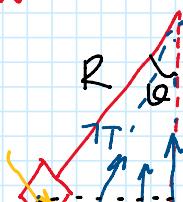


$$\begin{aligned} W &= \vec{F} \cdot \vec{x} \\ &= \frac{W}{t} = P_{ave} \\ &= \frac{N \cdot h}{\text{time}} = \frac{50(9.8)(443)}{15 \times 60} \text{ Joules} = 241 \text{ Watt} \approx 0.32 \text{ hp} \end{aligned}$$

Work-Energy Theorem along a curve path.

$$W = \vec{F} \cdot \vec{s}$$

swing / pendulum



curve path

$$\sum W = W_T + W_{mg}$$

?

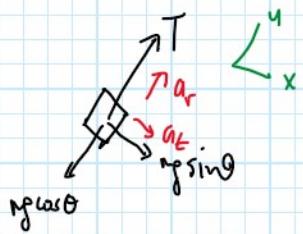
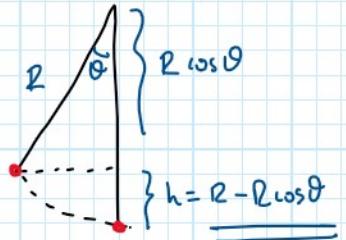
$$\begin{aligned} h &\{ \} s_{\parallel} \quad \{ \} s_{\perp} \\ \sum s_{\parallel} &= T s \cos 90^\circ = 0 \end{aligned}$$

$\vec{N} \cdot \vec{s} = mg \sin \theta \cdot s$

$\theta = \text{change in } \theta_f$

$\sum W = W_{\text{wg}} + 0$

$$W = \int F \, dx = \int mg \sin \theta \frac{R d\theta}{\text{path length}} = mg \Delta y = mg h = \Delta K$$



{Chapter 5}

$$\sum F_y = ma_y$$

$$T - mg \cos \theta = m \frac{v^2}{R}$$

$$\sum F_x = ma_x = ma_t$$

$$mg \sin \theta = m a_t$$

$$[a_t = g \sin \theta]$$

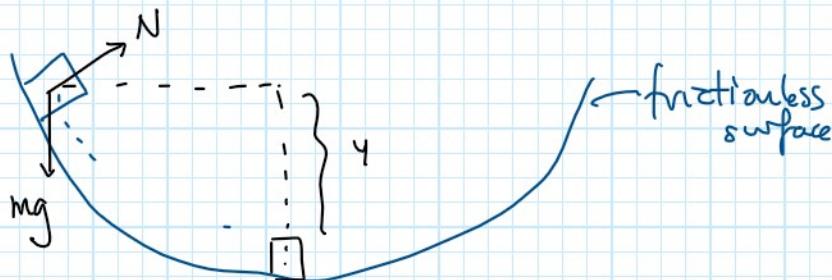
$a_t = \text{tangential acc.}$

$$\theta \uparrow a_t \uparrow a_r \downarrow a_r = \frac{v^2}{R}$$

} NLLCM

$$\sum W = W_N + W_{\text{wg}}$$

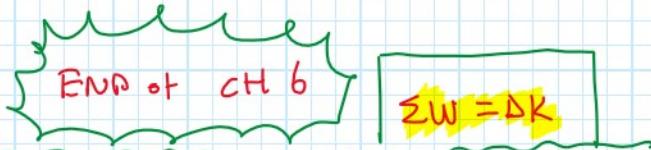
$N \perp \text{curve path}$



$$\sum W = W_{\text{wg}} + W_N = W_{\text{wg}} = mg y = \Delta K = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$

$$W_N = \int \vec{N} \cdot d\vec{s}; \vec{N} \perp d\vec{s}$$

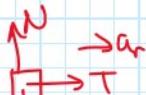
$$N ds \cos 90^\circ = 0$$



6.75) Block is rotating at a distance of 0.4m with a speed of 0.7m/s

If the rope is pulled from below and the new radius is 0.1m  $v_{\text{new}} = 2.8 \text{ m/s}$   $m = 0.09 \text{ kg}$

a) what's the tension of the rope initially?



$$\sum F_x = ma_x$$



$$\sum F_x = ma$$

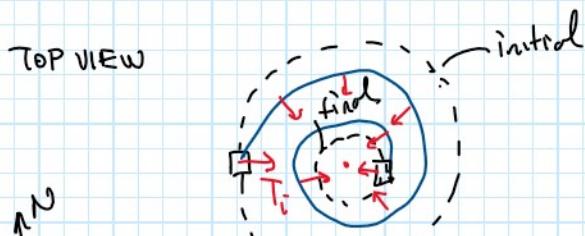
$$T = m \frac{v^2}{R} = (0.09) \left( \frac{0.7^2}{0.4} \right) = \underline{\underline{0.11 \text{ N}}}$$

b) What's the final tension of the rope?

$$T = m \frac{v_f^2}{R_f} = 0.09 \frac{2.8^2}{0.1} = 7.04 \text{ N}$$

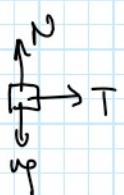
c) How much work is done by the person who pulled the rope?

TOP VIEW



$$T_i = 0.11 \text{ N}$$

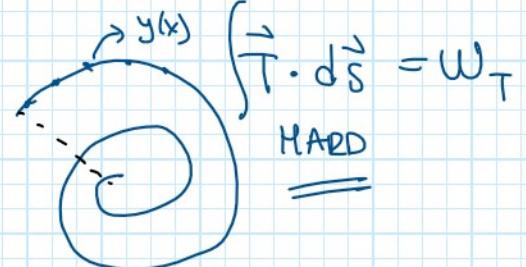
$$T_f = 7.04 \text{ N}$$



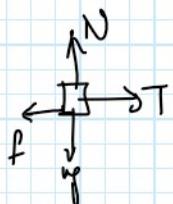
$$\sum w = \underbrace{W_T}_{\text{?}} + W_{mg} + W_N = \underline{\underline{\Delta K}}$$

$\vec{N} \perp \vec{s}$      $\vec{mg} \perp \vec{s}$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_T$$

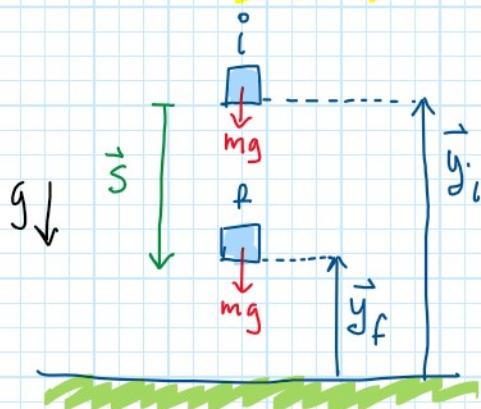


$$\frac{1}{2}0.09 (2.8^2 - 0.7^2) = W_T = \underline{\underline{0.33 \text{ Joules}}}$$



$$\sum w = \underbrace{W_T}_{\text{?}} + W_f + \underbrace{W_{mg}}_0 + \underbrace{W_N}_0$$

## CHAPTER 7 POTENTIAL ENERGY



mass is displaced S closer to ground.

$$\begin{aligned} W_{mg} &= \vec{F} \cdot \vec{s} \\ &= \vec{mg} \cdot \vec{s} > 0 \\ &= mg s \cos 0 \\ &= mg s \end{aligned}$$

$$\begin{aligned} \vec{y}_i - \vec{y}_f &= -\vec{s} \\ \vec{s} &= \vec{y}_f - \vec{y}_i \end{aligned}$$

$$\begin{aligned} W_{mg} &= \vec{mg} \cdot \vec{s} = \vec{mg} \cdot (\vec{y}_f - \vec{y}_i) = \vec{mg} \cdot \vec{y}_f - \vec{mg} \cdot \vec{y}_i \\ &= mg y_f \cos 180 - mg y_i \cos 180 \end{aligned}$$

$\square = \text{potential energy}$

$\underline{U}$  = potential energy

$$\underline{U} \equiv mg y$$



$$= mg y_f \cos 180 - mg y_i \cos 180$$

$$= -mg y_f + mg y_i$$

$$W_{\text{ng}} = mg y_i - mg y_f$$

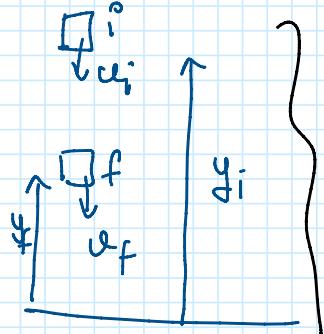
$$W_{\text{ng}} = k_i - k_f = -\Delta U = W_{\text{ng}}$$

$$\text{Ch 6} \quad \sum W = \Delta K$$

If only force (does work) is  $mg$

$$\sum W = W_{\text{ng}} = \Delta K$$

$$-\Delta U = \Delta K \Rightarrow U_i - U_f = k_f - k_i$$



$$\sum E_i = \sum E_f$$

$$mg y_i + \frac{mv_i^2}{2} = mg y_f + \frac{mv_f^2}{2}$$

$$*\left\{ U_i + k_i = U_f + k_f \right\} \quad \underbrace{K + U}_{\text{mechanical energy}} \equiv E$$

$$\sum E_i = \sum E_f$$

energy conservation

$$\sum E_i = \sum E_f \quad \left\{ \sum W = W_{\text{ng}} \right\}$$

If there are other forces than  $mg$

$$\sum W = \Delta K$$

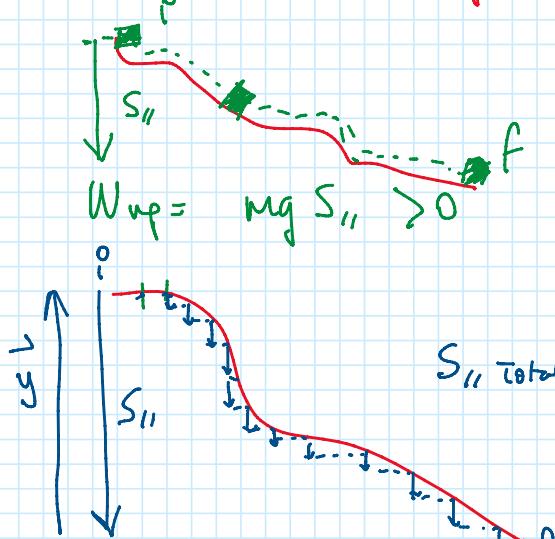
$$W_{\text{other}} + W_{\text{ng}} = \Delta K$$

$W_{\text{other}} = \bar{F}$ , frictional...

gravitational potential energy  
for motion along a curved path.

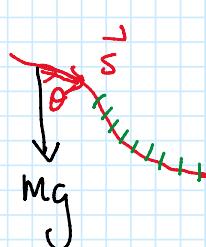
$$W_{\text{other}} + (-\Delta U) = \Delta K$$

$$W_{\text{other}} + U_i + k_i = U_f + k_f$$



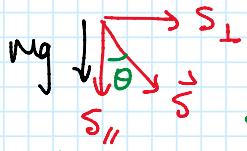
$$S_{11, \text{total}} = \sum S_{11}$$

$$[W_{\text{ng}} = mg S_{11} = -\Delta U] \vec{S}_{11} = -\vec{\Delta y}$$



$$W_{\text{ng}} = \vec{mg} \cdot \vec{s} =$$

$\theta$  changes



$$\vec{mg} \cdot \vec{s} = mg s_{\parallel} + mg s_{\perp}$$

$$s_{\parallel} = s \cos \theta$$

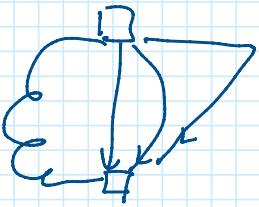
$$\sum s_{\parallel} = \underline{s}_{\parallel}$$

|| ... u

$| \downarrow$

$$\cancel{W_{\text{up}} = mg(y_f - y_i)} \quad \Delta U = -W$$

$W_{\text{mg}}$  is independent of the path !!

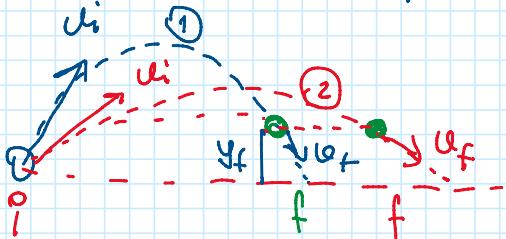


$$W_{\text{mg}} = \vec{mg} \cdot \vec{s}$$

1st path

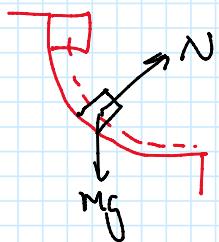
$$\underline{U} = mg y$$

$$\Delta U = U_f - U_i = mg(y_f - y_i)$$



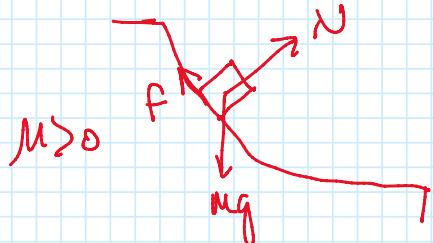
$$U_i + k_i = U_f + k_f$$

$$\theta + \frac{1}{2}mv_i^2 = mg y_f + \frac{1}{2}mv_f^2 = mg y_f + \frac{1}{2}mv_f^2$$



$$\sum E_i = \sum E_f$$

$$\sum W = W_{\text{up}} + W_N$$



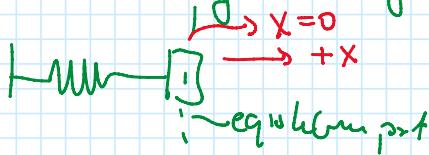
$$\sum W = W_{\text{up}} + W_N + W_f = \Delta K$$

$$\downarrow -\Delta U$$

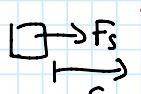
$$\underline{W_f + U_i + k_i = U_f + k_f}$$

$$W_f < 0$$

other = spring force (yay kuveti)



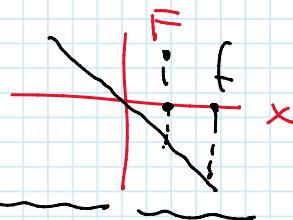
$$W_{F_s} > 0$$



$$U_{\text{el}} = \text{elastic potential energy} = \text{spring energy}$$

$$W_{\text{el}} = \frac{1}{2}kx^2$$

$F = -kx$  → pulls toward equilibrium point



$$W_s = \int_{x_i}^{x_f} -kx \, dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_s = 0 \quad W_s < 0 \quad W_s > 0$$

$$W_{\text{el}} = W_s = -\Delta U_{\text{el}}$$

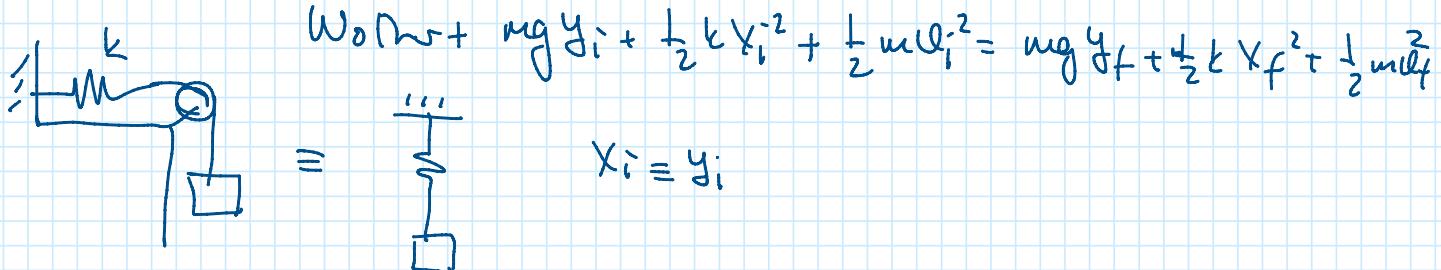
$$\sum W = \Delta K$$

$$\sum W = \Delta K$$

$$W_{up} + W_s + W_{work} = \Delta K$$

$$(-\Delta U_g) + (-\Delta U_{el}) + W_{work} = \Delta K$$

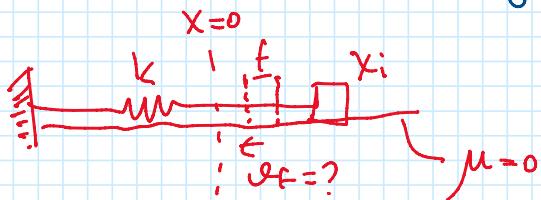
$$mg y_i - mg y_f + \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 + W_{work} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$



$$\left. \begin{array}{l} * \\ * \\ * \end{array} \right\} W_{0,initial} + U_{g,i} + U_{el,i} + K_i = K_f + U_{el,f} + U_{g,f}$$

$$U_g = mg y \quad U_{el} = \frac{1}{2} k x^2 = \frac{1}{2} k y^2 \dots$$

ex



spring is stretched 0.1 m  
mass is released ( $v_i = 0$ )

no displacement in y

$$K = 5 \text{ N/m} \quad m = 0.2 \text{ kg}$$

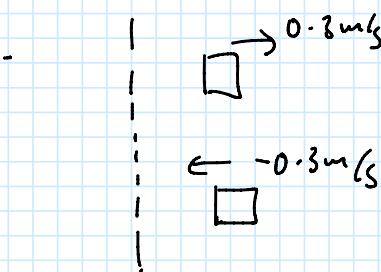
$$\text{when } x_f = 0.08 \text{ m} \quad v_f = ?$$

$$\underline{U_g = 0}$$

$$\underline{U_g = 0}$$

$$W_{0,initial} = 0$$

$$\frac{1}{2} k x_i^2 + \frac{1}{2} m v_i^2 = \frac{1}{2} k x_f^2 + \frac{1}{2} m v_f^2$$



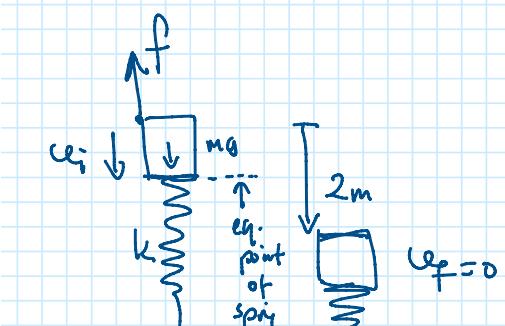
$$\frac{1}{2} (5) 0.1^2 + 0 = \frac{1}{2} 5 (0.08)^2 + \frac{1}{2} 0.2 v_f^2$$

$$v_f^2 = 0.09 \text{ m}^2/\text{s}^2$$

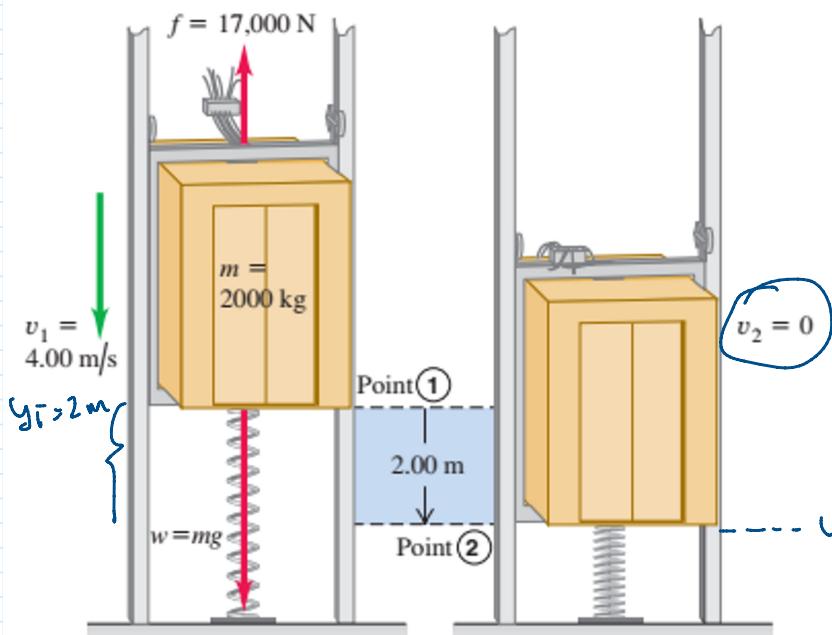
$$W_{0,initial} + U_{g,i} + U_{el,i} + K_i = K_f + U_{el,f} + U_{g,f}$$

A 2000-kg (19,600-N) elevator with broken cables in a test rig is falling at 4.00 m/s when it contacts a cushioning spring at the bottom of the shaft. The spring is intended to stop the elevator, compressing 2.00 m as it does so (Fig. 7.17). During the motion a safety clamp applies a constant 17,000-N frictional force to the elevator. What is the necessary force constant  $k$  for the spring?

**7.17** The fall of an elevator is stopped by a spring and by a constant friction force.



**7.17** The fall of an elevator is stopped by a spring and by a constant friction force.



$$K_i = \frac{1}{2} m v_i^2 \\ = \frac{1}{2} (2000) 4^2 = 16000 \text{ J}$$

$$W_{el,f} = \frac{1}{2} k x_f^2 \\ = \frac{1}{2} k 2^2 = 2k$$

$$U_{g,i} = \frac{1}{2} k x_i^2 = 0 \\ X_i = 0$$

$$K_f = 0 = U_f = 0$$

$$W_{friction} + U_{g,i} + W_{el,i} + K_i = K_f + U_{el,f} + U_{g,f} \\ - 34000 + 39200 + 0 + 16000 = 0 + 2k + 0 \\ 21200 = 2k$$

$$k = 10600 \text{ N/m}$$

What if there was NO FRICTION?

$$\hookrightarrow 39200 + 16000 = 2k' \\ k' = 27600 \text{ N/m}$$

FORCES

Conservative  
(korunum L/U)

energy is conserved  
when the object is  
moved back to initial  
position

nonconservative  
(korunum S/U)

not conserved

( moved back to initial position)

→ gravitational ( $mg$ )

→ spring ( $-kx$ )

→ electrostatic force ( $\frac{kq_1 q_2}{r^2}$ )

→ frictional forces.

( $F_f$  force)

① conservative force

$$W = -\Delta U$$

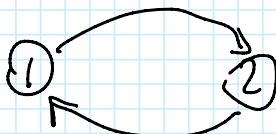
$$W_{mg} = -\Delta U_g$$

$$W_{el} = -\Delta U_{el}$$

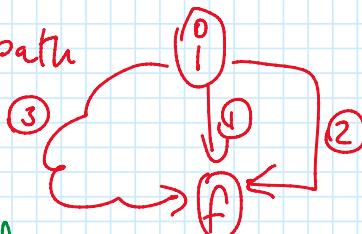
$W \sim \underline{\underline{U}}$   
potential expression.

② It's reversible

$$0 = W_{1 \rightarrow 2} + W_{2 \rightarrow 1}$$



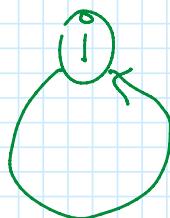
③ Independent of path



$$W_{mg}^{(1)} = W_y^{(2)} = W_y^{(3)}$$

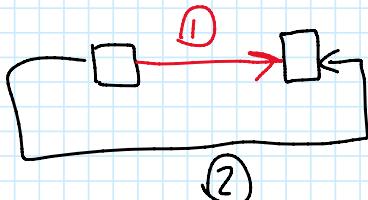
④ When start = end

$$\begin{matrix} \overset{q=f}{\square} \\ \downarrow \\ \overset{q=f}{\square} \end{matrix} \quad W_y = 0$$



$$W_{q \rightarrow f} = 0 = W_{1 \rightarrow 1}$$

nonconservative force (friction)



$W_f$  depend on path

$$W^{(2)} \neq W^{(1)}$$

$$W = -\Delta U = F_x \Delta x \Rightarrow F_x = -\frac{\Delta U}{\Delta x} = -\frac{dU}{dx}$$

$|U| =$

$$-\frac{dU}{dx} = F_x$$

$$-\frac{dU}{dy} = F_y$$

$$-\frac{dU}{dz} = F_z$$

$$F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad F_z = -\frac{\partial U}{\partial z} \quad (\text{partielle Differenziale})$$

$$U = 5x^2 y$$

$$\frac{\partial U}{\partial x} = 10xy + 5x^2 \frac{\partial y}{\partial x}$$

$$\frac{\partial U}{\partial x} = 10xy$$

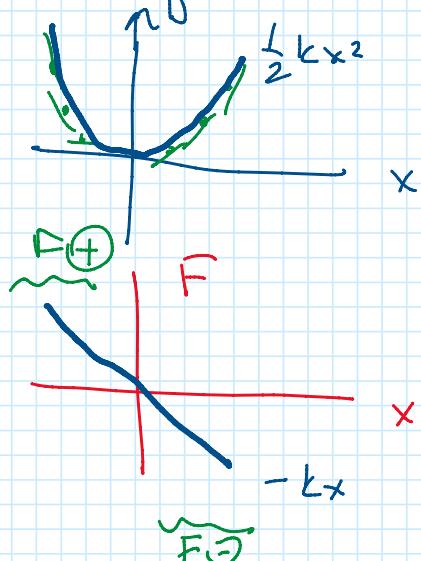
$$U = mg y$$

$$-\frac{\partial U}{\partial x} = 0 \quad -\frac{\partial U}{\partial z} = 0 \quad -\frac{\partial U}{\partial y} = -mg$$

$$U = \frac{1}{2} k x^2$$

$$-\frac{\partial U}{\partial z} = -\frac{\partial U}{\partial y} = 0$$

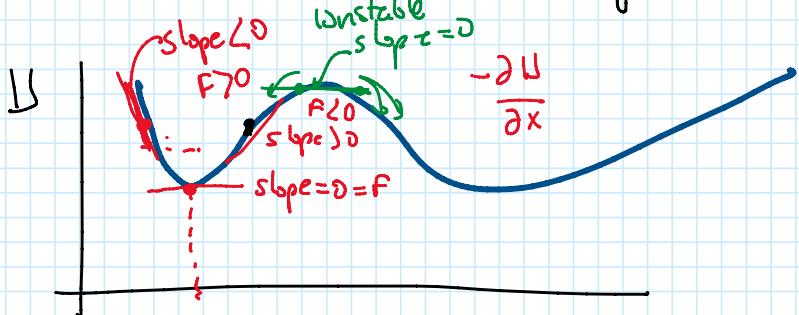
$$-\frac{\partial U}{\partial x} = -\frac{1}{2} k 2x = -kx$$



$$-\frac{\partial U}{\partial x} = -(egim)$$

POTENSIJEL FUNK.  
→ EGIM O ANDAKI  
KUUVETI VERO!!

$$\vec{F} = -\nabla U \quad \nabla \equiv \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right)$$

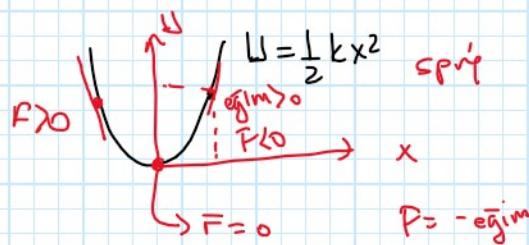


$$U = -\frac{A}{r}$$

$$\begin{aligned} \vec{F} &= -\frac{\partial}{\partial r} \left( -\frac{A}{r} \right) \hat{r} \\ &= +A \frac{\partial}{\partial r} \left( \frac{1}{r} \right) \hat{r} \end{aligned}$$

$$\hat{r} = -A \hat{r}$$

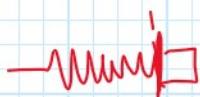
$$\frac{\partial U}{\partial x} = \text{slope}$$



$$\vec{F} = -\frac{A}{r^2} \hat{r}$$



$$\frac{\partial U}{\partial x} = 0 \Rightarrow \text{stable}$$



$$F > 0 \rightarrow x$$

$$F < 0 \rightarrow x$$

end of Ch 7

$$U_A = 0$$

- 7.45 • A 350-kg roller coaster starts from rest at point A and slides down the frictionless loop-the-loop shown in Fig. P7.45. (a) How fast is this roller coaster moving at point B? (b) How hard does it press against the track at point B?

$$W_f = 0$$

$$\left. \begin{array}{l} W_f + E_A = E_B \\ \Rightarrow \end{array} \right\} \text{only force } \downarrow g$$

$$U_A + K_A = U_B + K_B$$

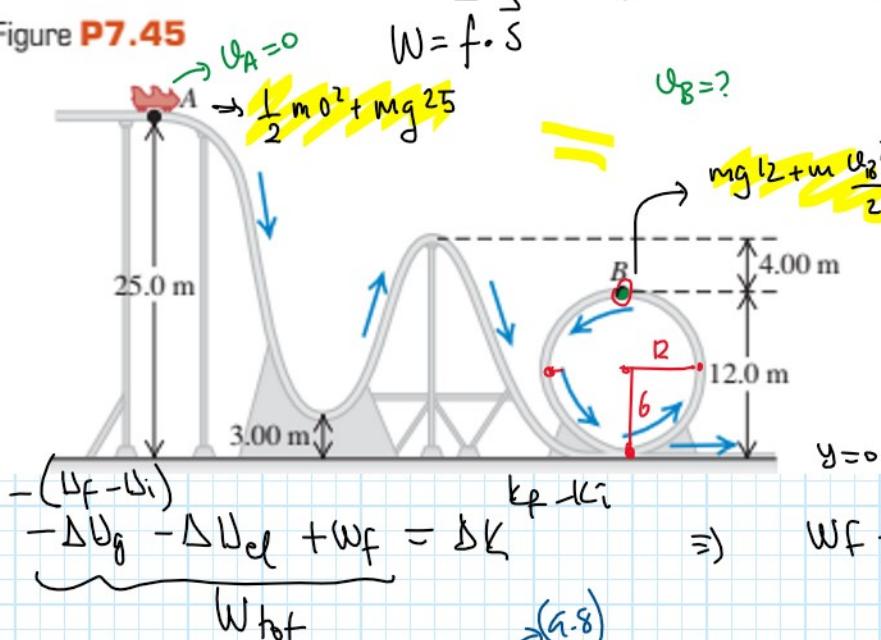
$$W = \Delta K$$

$$-\Delta U_g \text{ grow.}$$

$$-\Delta U_{el} \text{ el.}$$

$$W_f \text{ Friction}$$

Figure P7.45



$$U_B = \sqrt{26g} = 15.96 \text{ m/s}$$

$$v_B^2 = 26g$$

what's the Normal force at B?



$$\sum \vec{F} = m \vec{a}$$

$$N + m g = m a_r = m \theta_B^2 r \Rightarrow N = m \theta_B^2 r - m g$$



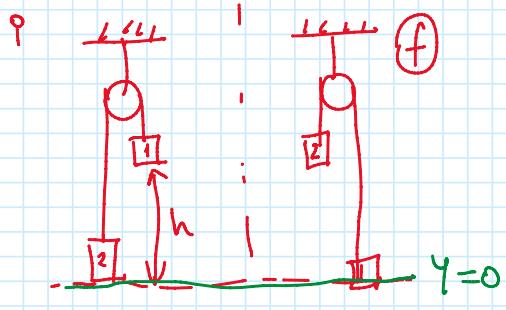
$$\sum \bar{F} = m\bar{a}$$

$$N + mg = m\bar{a}_r = \frac{m\theta_R^2}{R} \Rightarrow N = \frac{m\theta_R^2}{R} - mg$$

$$= \frac{(350)(26g)}{6} - 350g$$

$$\underline{N = 11433.3N}$$

Atwood machine.



$$\begin{aligned} m_1 &= 12 \text{ kg} & \text{what are the velocities} \\ m_2 &= 4 \text{ kg} & \text{just before } m_2 \text{ hits the ground?} \\ h &= 2 \text{ m} \\ \text{system is} & \\ \text{released} & \\ \text{from rest} & \rightarrow v_i = 0 \end{aligned}$$

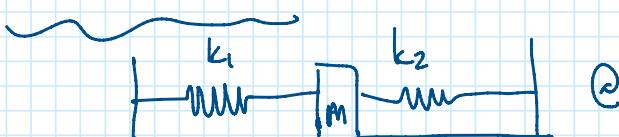
$$E_i^o = E_f$$

$$K_{1i}^o + K_{2i}^o + U_{1i}^o + U_{2i}^o = K_{1f} + K_{2f} + U_{1f} + U_{2f}$$

$$\left. \begin{aligned} &+ \cancel{0} + M_1 g h & \downarrow \\ &+ \cancel{0} & = \frac{m_1 \theta_f^2}{2} + \frac{m_2 \theta_f^2}{2} + \cancel{0} + m_2 g h \end{aligned} \right\}$$

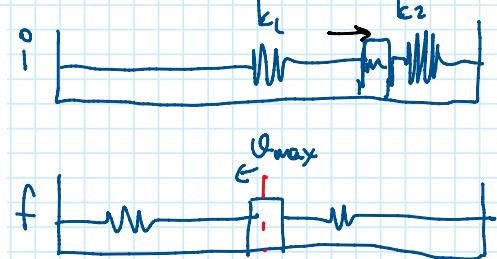
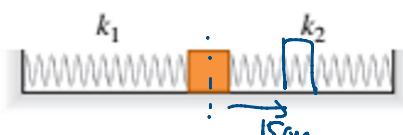
$$\frac{2(12-8)(9.8)2}{12+8} = \theta_f^2 = \frac{2(m_1 - m_2)gh}{m_1 + m_2}$$

$$\downarrow \\ \theta_f = 4.4 \text{ rad/s}$$



**7.70** • A 3.00-kg block is connected to two ideal horizontal springs having force constants  $k_1 = 25.0 \text{ N/cm}$  and  $k_2 = 20.0 \text{ N/cm}$  (Fig. P7.70). The system is initially in equilibrium on a horizontal, frictionless surface. The block is now pushed 15.0 cm to the right and released from rest. (a) What is the maximum speed of the block? Where in the motion does the maximum speed occur? (b) What is the maximum compression of spring 1?

Figure P7.70



$$E_i^o = E_f$$

$$U = \frac{1}{2} k x^2 = 1/2 k_{\text{eff}} l_{\text{max}}^2$$

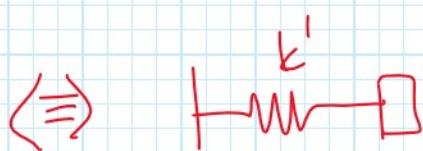
$$U_{1i}^o + U_{2i}^o + \cancel{K_0^o} = K_f + U_{1f} + U_{2f}$$

minimum compression of spring?

$$\frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 + 0 = \frac{1}{2}mv_f^2 + 0 + 0$$

$$\frac{1}{2}(25)15^2 + \frac{1}{2}(20)15^2 = \frac{1}{2}3v_f^2$$

$$\sqrt{\frac{15^2(45)}{3}} = v_f = 15\sqrt{15} \text{ m/s} = 58.1 \text{ cm/s}$$



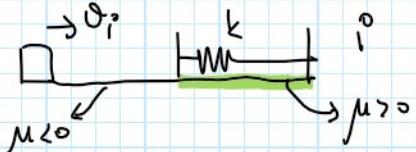
$$E_{total} = U_{if} + U_{2f} + K_f$$

$$\frac{1}{2}k_1x_i^2 + \frac{1}{2}k_2x_i^2 + \frac{1}{2}mv_i^2 = \frac{1}{2}k_1x_f^2 + \frac{1}{2}k_2x_f^2 + \frac{1}{2}mv_f^2$$

$U_i = 0$        $x \Rightarrow \max$  ;  $v_f = 0$

$$\frac{1}{2}(k_1+k_2)x_i^2 = \frac{1}{2}(k_1+k_2)x_f^2 \Rightarrow x_i = x_f = \underline{\underline{15 \text{ cm}}}$$

(2)



$$\mu = 0.5$$

$$m = 0.8 \text{ kg}$$

$$k = 50 \text{ N/m}$$

max compression?

$$v_i = 1.2 \text{ m/s}$$

$$w_f + E_i = E_f$$

$$w_f + U_i + K_i = U_f + K_f$$

$$U = \frac{1}{2}kx^2$$

$$-fx + 0 + \frac{1}{2}mv_i^2 = \frac{1}{2}kx^2 + 0$$

$$f = \mu mg$$

$$0 = \frac{1}{2}kx^2 + fx - \frac{1}{2}mv_i^2$$

$$0 = Ax^2 + Bx + C$$

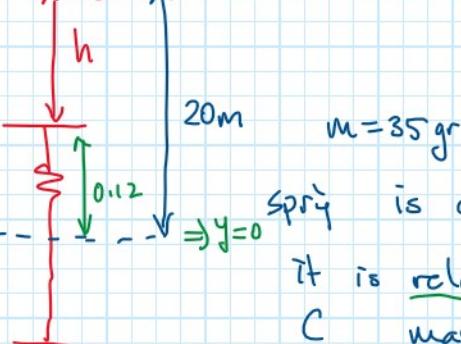
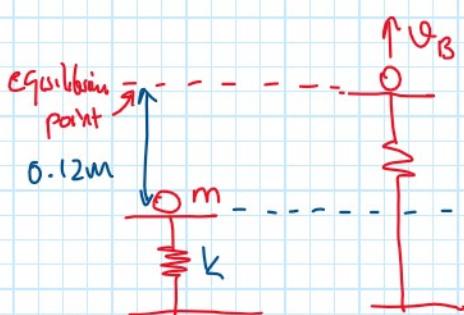
$$0 = \frac{1}{2}50x^2 + (0.5)(0.8)(1.2)x - \frac{1}{2}(0.8)(1.2)^2$$

$$25x^2 + 3.92x - 0.576 = 0$$

$$x = -0.25 \text{ m} ? \quad X \quad w_f > 0 \quad X$$

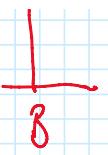
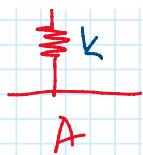
$$x = 0.092 \text{ m} ? \quad \checkmark$$

(ex)



Spry is compressed 0.12m  
it is released  $\rightarrow v_A = 0$

C mass is 20m above train



It is released.  $\Rightarrow v_A = 0$

C mass is 20m above from point @ A.  $v_C = 0$

$$E = U_{\text{el}} + U_g + K$$

$$\frac{1}{2}k(y)^2 + mgY + \frac{1}{2}mv^2$$

$$E_A = E_B = E_C$$

$$K = ? \quad v_B = ?$$

$$y=0 \text{ (establish)}$$

$$K_A + U_{g,A} + U_{\text{el},A} =$$

$$0 + 0 + \frac{1}{2}k(0.12)^2 = \frac{1}{2}m v_B^2 + mg(0.12) + 0 = 0 + mg(20) + 0 \Rightarrow \Delta Y = 0$$

$$\frac{1}{2}k(0.12)^2 = \frac{1}{2}(0.035)v_B^2 + (0.035)(9.8)(0.12) = 0.035(9.8)(20) \\ = 6.86 \text{ J}$$

$$k = \frac{6.86 \times 2}{(0.12)^2} = 953 \text{ N/m}$$

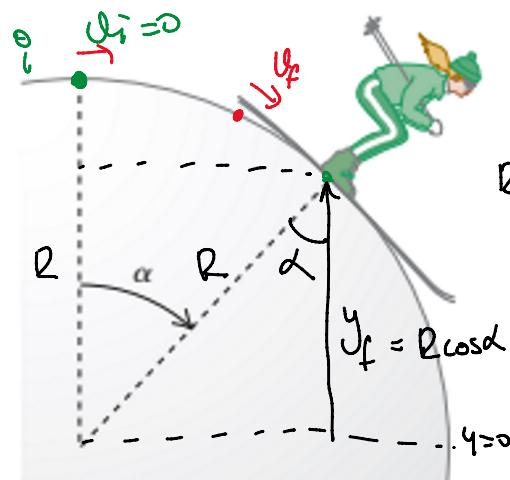
$$v_B = \sqrt{\frac{(6.86 - 0.041)^2}{0.035}}$$

$$= 19.7 \text{ m/s}$$

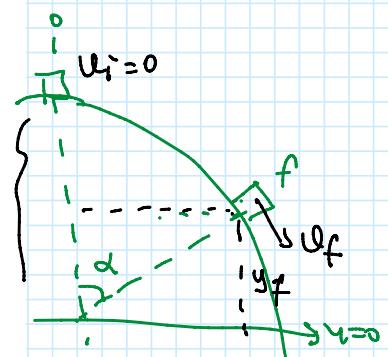
**7.63 • CP** A skier starts at the top of a very large, frictionless snowball, with a very small initial speed, and skis straight down the side (Fig. P7.63). At what point does she lose contact with the snowball and fly off at a tangent? That is, at the instant she loses contact with the snowball, what angle  $\alpha$  does a radial line from the center of the snowball to the skier make with the vertical?

Figure P7.63

$$M=0$$



$$E_i = E_f$$



$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

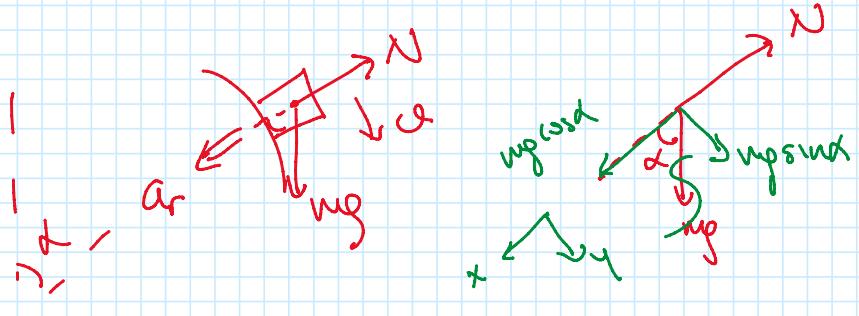
$$0 + mgR = \frac{mv_f^2}{2} + mg y_f$$

$$mgR = \frac{mv_f^2}{2} + mgR \cos \alpha$$

$$\frac{v_f^2}{2} = gR - gR \cos \alpha \Rightarrow \boxed{v_f^2 = 2gR(1 - \cos \alpha)}$$

$$\rightarrow N$$

$$N < r - m \cos \alpha - \lambda r = m a_r$$



$$N = mg \cos d - \frac{m v_f^2}{R} = 0 \quad (\text{we want})$$

$$mg \cos d = \cancel{m} \frac{v_f^2 (1 - \cos d)}{\cancel{R}}$$

$$\sum F_x = mg \cos d - N = m a_r$$

$$mg \cos d - N = m \frac{v_f^2}{R}$$

$$\Rightarrow v_f^2 = g R \cos d \quad \left\{ \begin{array}{l} N=0 \\ \alpha=0 \end{array} \right.$$

$$\cos d = 2(1 - \cos d)$$

$$\cos d = \frac{2}{3} \quad ; \quad d = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\underline{\alpha = 48.2^\circ}$$

