Physics I D1 (I) - week

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Pzt - Çarş 1330 - 1530

40-45 / 60 dk 40-45 / 60 dk

— Not alarak dersi takip ediniz

FİZİK I
Newton mekaniği
⇒ teslimi olayarak
$g = 9.8\ m/s^2$

FİZİK II
Elektrik & manyetizma

İngilizce
English ✓
soru?
Scientific calculator
App

⇒ Kitap University Physics *
Pearson Young & Friedman
13th ed. ? TL
Türkçe ⇏ Serway Physics *

aves.akdeniz.edu.tr/denizkaya/dokumanlar
Physics I
%90

⇒ $\boxed{2 \text{ gün} \times 2 \text{ saat}}$ → teorik / → uygulama

⇒ Teams %85-90 ⇒ 1 midterm (vize) / 1 final } canlı

yoklama ~~X~~ 15-10 % ⇒ / ödev / kısasoru quiz / ... /

1 ~~unit~~ chapter

Standart Units

(birim)

SI units

measurments ✓ — experimental — worldwide

theory

1790's

French revolution

time? mass; length

"arsın"

18th – 19th ✓ — neetops

Ottoman empire ✓ — 12 number

"inch" = "queen's finger"

meter kg

SI units 7

- meter
- second
- kg

} physics 1

- mole → chemistry
- Kelvin → thermodynamic
- Amper → physics 2
- Candela → astrophysics / astronomy

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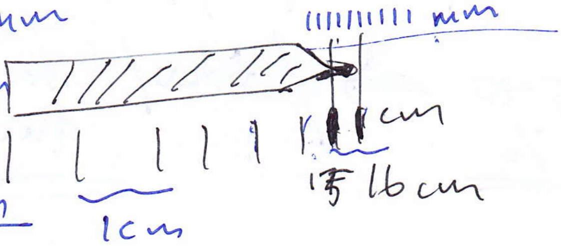
what you measure has always uncertainty!

156 mm ± 1mm

156.0 ± 0.5mm



15cm ± 1cm

15.6 ± 0.1 cm

± ~~1~~ 1cm / 2 → ± 0.5 cm

Uncertainty with measurement

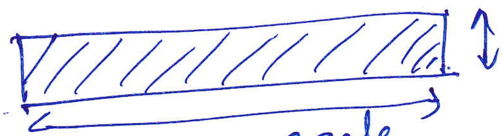
→ write the uncertainty in terms of significant numbers figures

anlamlı sayılar
anlamlı rakamlar

$15.6\,cm \pm 0.1\,cm \equiv \underline{15.6}\,cm$

$\underbrace{15.}_{}$ 3 sf $\qquad$ ∓ 0.1

measure ~~volume~~ area of a rectangle.



mm scale } 2.3 mm

2 sf

cm scale

15cm

2sf

± 1

$Area = 15 \times 2.3$

$= 34.5\ cm^2$

$\cong 35\ cm^2$

2 sf

$\underbrace{0.745}_{3sf} \times \underbrace{2.2}_{2sf} = \underbrace{0.42}_{2sf}$

$\underbrace{3.885}_{4\,sf}$

$0.420 = 42 \times 10^{-2}$ ✓

$\underbrace{0.4020}_{3sf}$ $\quad 402 \times 10^{-3}$

$h = \frac{1}{2} g t^2 = \underbrace{(\ldots)}_{2sf}$

$\frac{1}{2}$ $\quad \underbrace{9.8}_{2}$ $\quad \underbrace{(3.3)^2}_{2}$

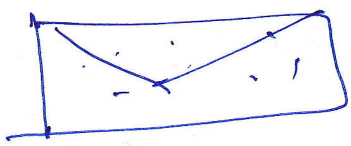
Estimates & orders of magnitude

tahmin

büyüklük

10^{2} 10^{3}

back of envelope calculations!



⇒ rounding (yuvarlama)

number of breaths ~~in~~ you take in your life?

1 breath? in/out = 4 seconds, 5 seconds

$$\text{number of breaths} = \frac{\text{lifetime}}{\text{breath time}} = \frac{70\,\text{yrs}}{5\,\text{seconds}} \simeq 70 \text{ years}$$

1 year = 365 day × ~~60~~ 24 hr × 60 min × 60 sec...

round 400 25

$$= \frac{70 \times 400 \times \cancel{25}^{5} \times 60 \times 60 \text{ sec}}{\cancel{5} \text{ sec}}$$

$$\underbrace{7 \times 20}_{140} \times \underbrace{36}_{40} \times 10^{6}$$

$$5600 \times 10^{6} = 56 \times 10^{8}$$

$$= 5.6 \times 10^{9}$$

⇒ Amount of tax from gasolie of cars in Turkey per year?

~ how many cars ~ (16×10^6)

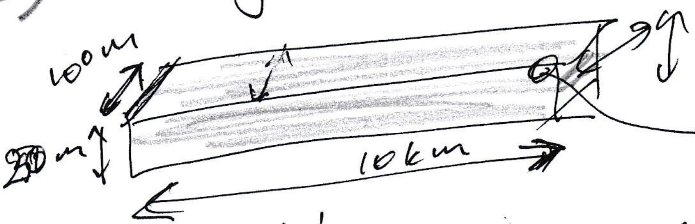
each ~ how many km per year 12000 km

- gas consumption 8 lt 100 km ? lt ⇒

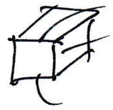
- 1 lt gas ⇒ 7 TL ⇒ %50 tax

⇒ Konyaaltı beach pebble amount?

çakıl.



taş (pebble) volume.



$$\frac{V_{beach}}{V_{pebble}} = \frac{10 \times 10^3 \times 100 \times 20}{(2 \times 10^{-2})^3} = \frac{2}{8} \frac{10^7}{10^{-6}} = 0.25 \times 10^{13}$$

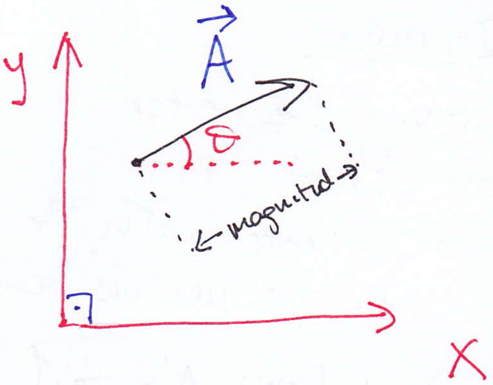
10^9 breath $< 10^{13}$ pebble

Vector ödev ⟶ ☐ ⟶ 3 weeks ☐

canlı sınav ?? Arş 9–11

9 = 17:00

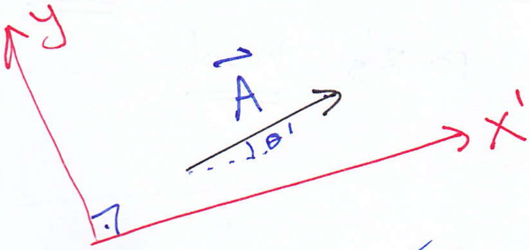
Vectors



Two properties

- magnitude
- direction ≡ angle ⇒ we will need coordinate system

angle = θ (theta) with x axis



$\theta' \neq \theta$

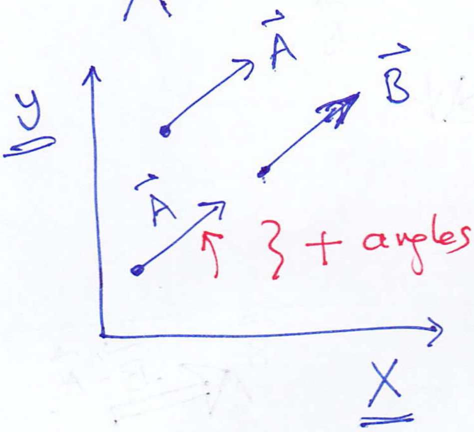
(x', y') (x, y)

direction (θ)

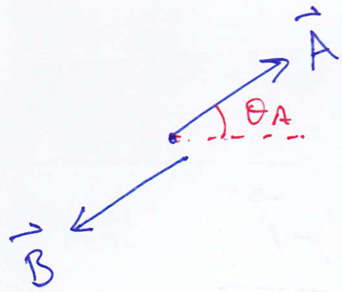
start → end

tail head

$\vec{A}$ } arrow ✓ (must)



$\vec{A} = \vec{B}$ (same magnitude & angle)

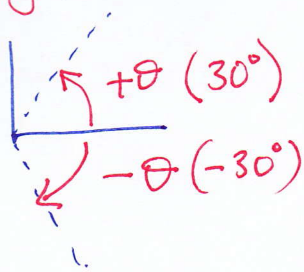


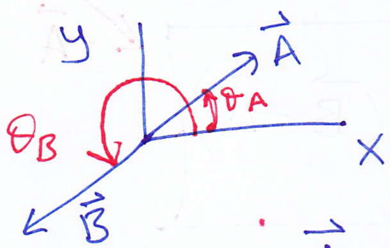
same magnitude

$|\vec{A}| = A = |\vec{B}| = B$

$\theta_A \neq \theta_B$

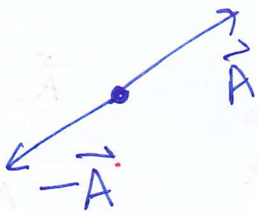
θ (angle) definition has rotational direction to itself.





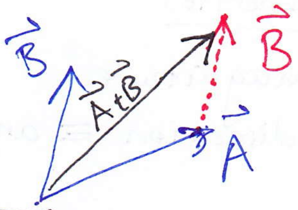
$\theta_B = 180 + \theta_A$

$\vec{A} = -\vec{B}$ → 180° rotation



Vector Addition & Subtraction

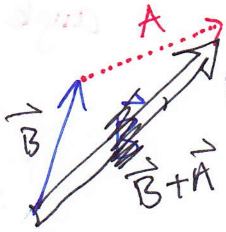
$\vec{A}+\vec{B}$



$$\boxed{\vec{A}+\vec{B}=\vec{B}+\vec{A}}$$

Drawing

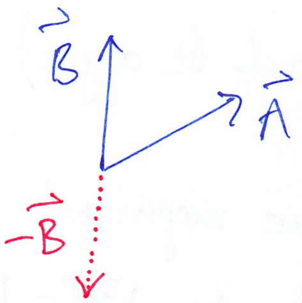
① carry the vector so that its tail meets with the other's vectors head

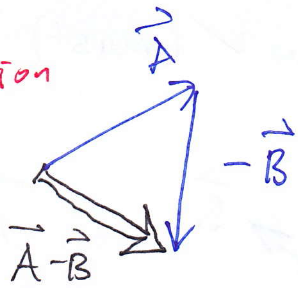


② Draw from A's tail to B's head

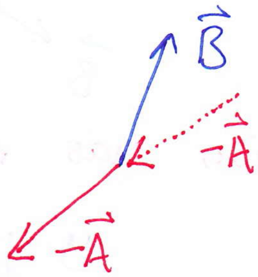
$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$$

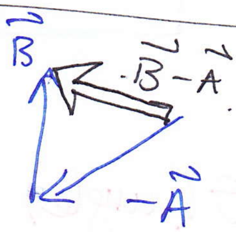
① 180° rotation
② add





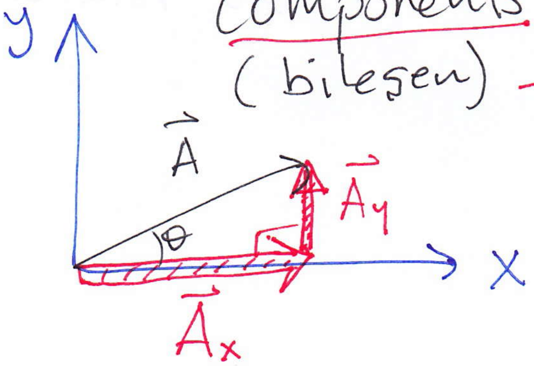
$$\vec{B}-\vec{A}=\vec{B}+(-\vec{A})$$





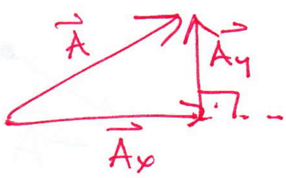
$$\boxed{\vec{A}-\vec{B}=-(\vec{B}-\vec{A})}$$

Components of a vector
(bileşen) ⊥



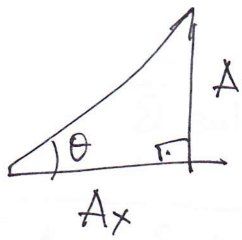
$$\vec{A}_x+\vec{A}_y=\vec{A}$$

perpendicular to each other



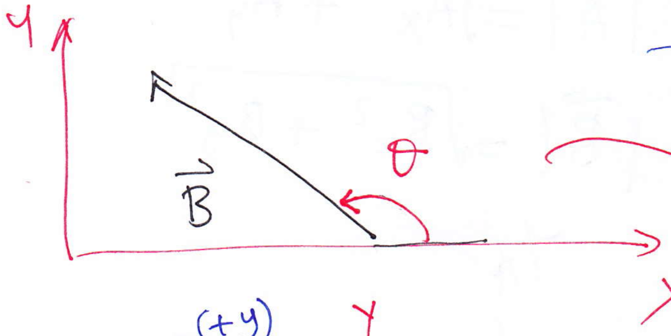
07.10.20 (2)

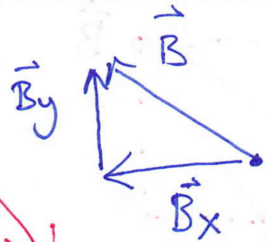
θ?



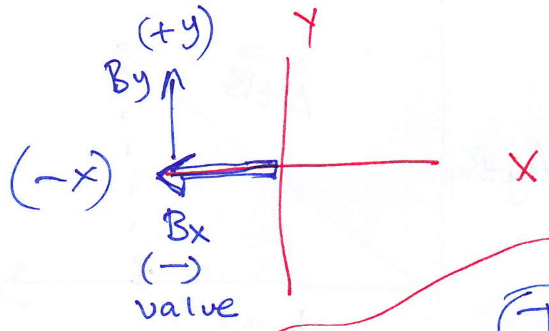
$$\tan\theta = \frac{A_y}{A_x} = \frac{\text{across to the } \theta}{\text{neighbor '' } \theta}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \arctan\left(\frac{A_y}{A_x}\right)$$

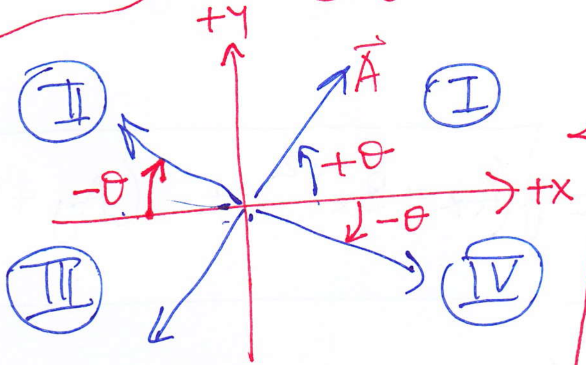




$$\vec{B} = \vec{B}_y + \vec{B}_x$$



$$\theta = \tan^{-1}\left(\frac{B_y}{-B_x}\right)$$



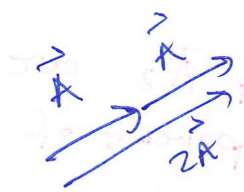
	sign of $\tan^{-1}\left(\frac{?}{?}\right)$
I	$\left(\frac{+A_y}{+A_x}\right)$
II	$\left(\frac{+A_y}{-A_x}\right)$
III	$\left(\frac{-A_y}{-A_x}\right)$
IV	$\left(\frac{-A_y}{+A_x}\right)$

I ; III	$+\theta$
II ; IV	$-\theta$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

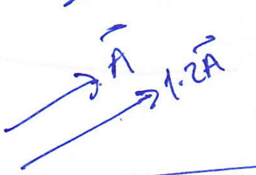
Direction of the vector
need to know components!

⇒ multiplying a vector by a scalar = number
(sbt sayı)



$\Rightarrow 2\vec{A} = \vec{A} + \vec{A}$

$\underbrace{(1.2)\vec{A}}_{\frac{12}{10}\vec{A}} = \vec{A} + \underbrace{0.2\vec{A}}_{\frac{2}{10}A}$



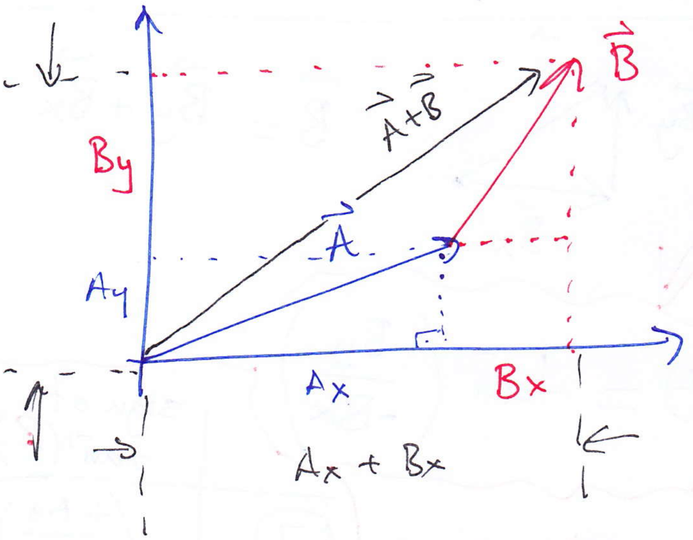
american syst. ⇒ turkish syst.
0.2 ≡ 0,2

5.3 ≡ 5 point 2

5x3

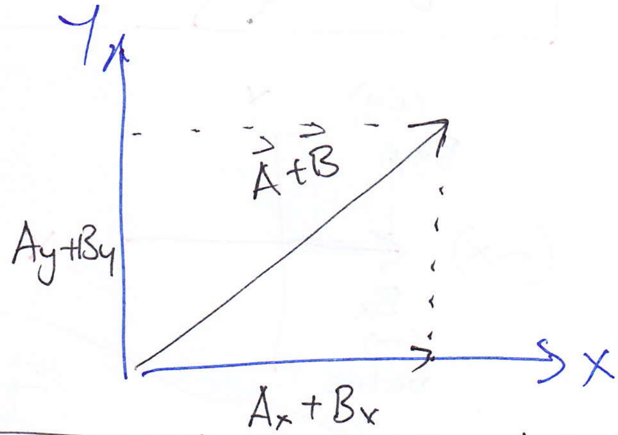
$-\vec{A} = (-1)\vec{A} \Rightarrow$ rot. by 180°

$|\vec{A}+\vec{B}| \Rightarrow$ magnitude of $\vec{A}$ plus $\vec{B}$



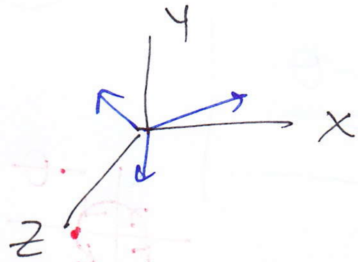
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2}$$



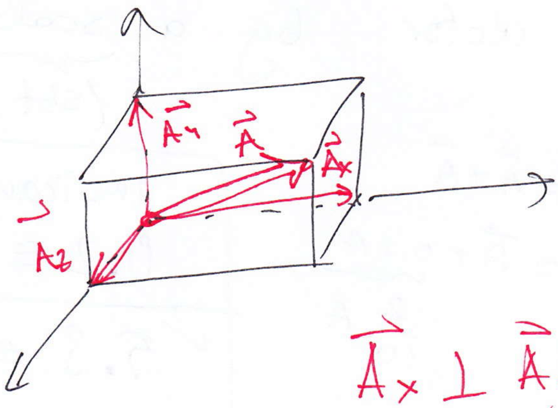
$$|\vec{A}+\vec{B}| = \sqrt{(A_x+B_x)^2 + (A_y+B_y)^2} \neq |\vec{A}| + |\vec{B}|$$

Add vectors in 3D
subtract



$\vec{A}+\vec{B}+\vec{C}+\cdots$

$\vec{A}$ is in 3D $\Rightarrow$ $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$

A_x, A_y, A_z are components of $\vec{A}$

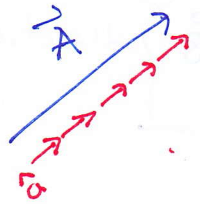
$\vec{A_x} \perp \vec{A_y} \perp \vec{A_z}$

Unit Vectors

→ direction
→ magnitude $= 1$
→ draw hat instead of arrow

"a hat"

$\hat{a}$; $|\hat{a}| = 1$



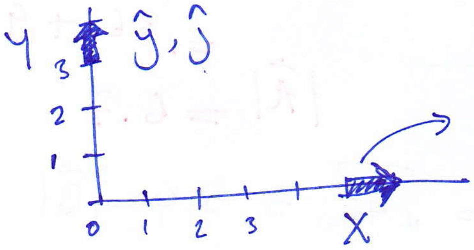
$$\vec{A} = A\,\hat{a}$$

A → number (magnitude)

$$= 5\,\hat{a}$$

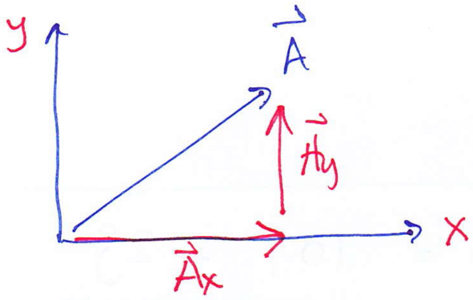
Define axis direction as unit vectors.

$\hat{z}, \hat{k}$



$\hat{y}, \hat{j}$

$\hat{x} ; \hat{i}$



$$\vec{A} = \vec{A_x} + \vec{A_y}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{A} + \vec{B} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j}) = \vec{C}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{C} = C_x\hat{i} + C_y\hat{j}$$

$$|\vec{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

$$|\vec{A} - \vec{B}| = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$$

Ex:

$\vec{A} = 6\hat{i} + 3\hat{j}$

$\vec{B} = 4\hat{i} - 5\hat{j}$

ⓐ $|\vec{A}| = ?$ ⓑ $|\vec{B}| = ?$

ⓒ $\vec{A} + \vec{B} = ?$

$|\vec{A} + \vec{B}| = ?$

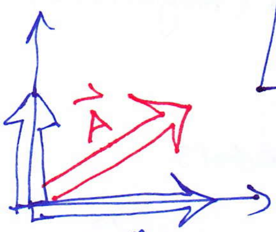
direction of $\vec{A} + \vec{B}$

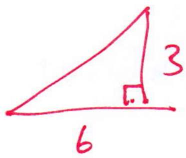
ⓓ $\vec{A} - \vec{B} = ?$

$|\vec{A} - \vec{B}| = ?$

direction of $\vec{A} - \vec{B}$

ⓐ $|\vec{A}| = ?$



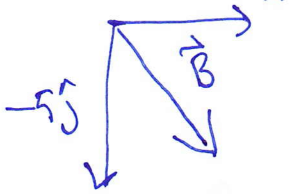


$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$= \sqrt{36 + 9} = \sqrt{45}$

$|\vec{A}| = 6.7$

ⓑ $|\vec{B}| = ?$ $\vec{B} = 4\hat{i} - 5\hat{j}$

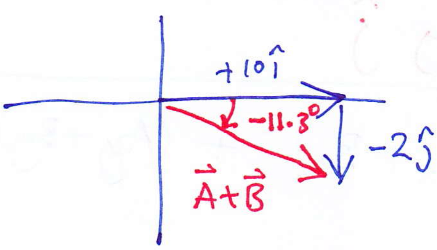


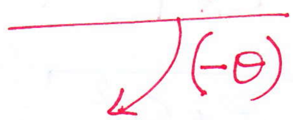
$|\vec{B}| = \sqrt{4^2 + (-5)^2} = \sqrt{41} = 6.4 = |\vec{B}|$

ⓒ $\vec{A} + \vec{B} = (6\hat{i} + 3\hat{j}) + (4\hat{i} - 5\hat{j}) = \underline{\underline{10\hat{i} - 2\hat{j}}}$

$|\vec{A} + \vec{B}| = \sqrt{10^2 + 2^2} = \sqrt{104} = 10.2$

direction of $\vec{A} + \vec{B} = ?$ $\theta = \tan^{-1}\left(\frac{-2}{10}\right) = \underline{\underline{-11.3°}}$



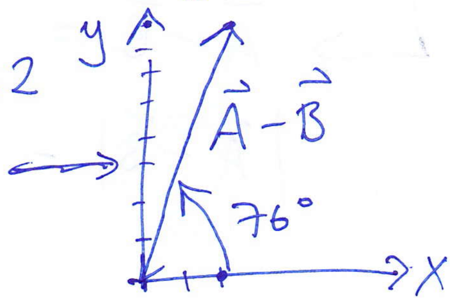


ⓓ $\vec{A} - \vec{B} = (6\hat{i} + 3\hat{j}) - (4\hat{i} - 5\hat{j}) = 2\hat{i} + 8\hat{j}$

$|\vec{A} - \vec{B}| = \sqrt{2^2 + 8^2} = \sqrt{68} = 8.2$

direction $\theta = \tan^{-1}\left(\frac{8}{2}\right)$

$\theta = 76°$



0 1 2 3 4 5 6 7 8 9 10 } 14 s

071020 (4)

Ex: $\vec{D} = (6\hat{i} + 3\hat{j} - 1\hat{k})\ m$ ← meter

$\vec{E} = (4\hat{i} - 5\hat{j} + 8\hat{k})\ m$

$\vec{D}$; $\vec{E}$ are displacement vectors. (yerdeğiştirme)

$|2\vec{D} - \vec{E}| = ?$

$$2\vec{D} - \vec{E} = [12\hat{i} + 6\hat{j} - 2\hat{k} - (4\hat{i} - 5\hat{j} - 8\hat{k})]\ m$$

$$= [8\hat{i} + 11\hat{j} + 6\hat{k}]\ m$$

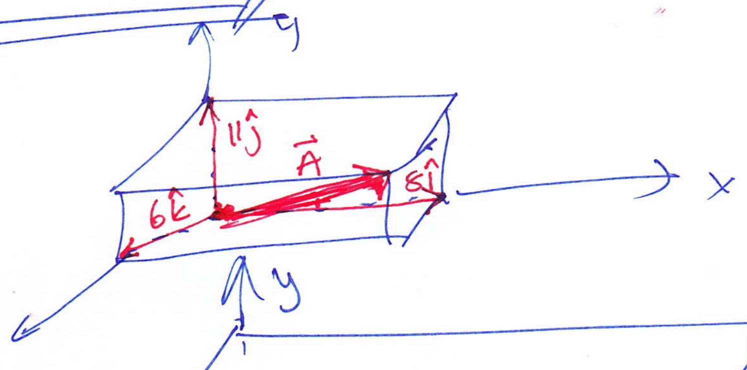
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$|\vec{A}| = |2\vec{D} - \vec{E}| = \sqrt{8^2 + 11^2 + 6^2}$$

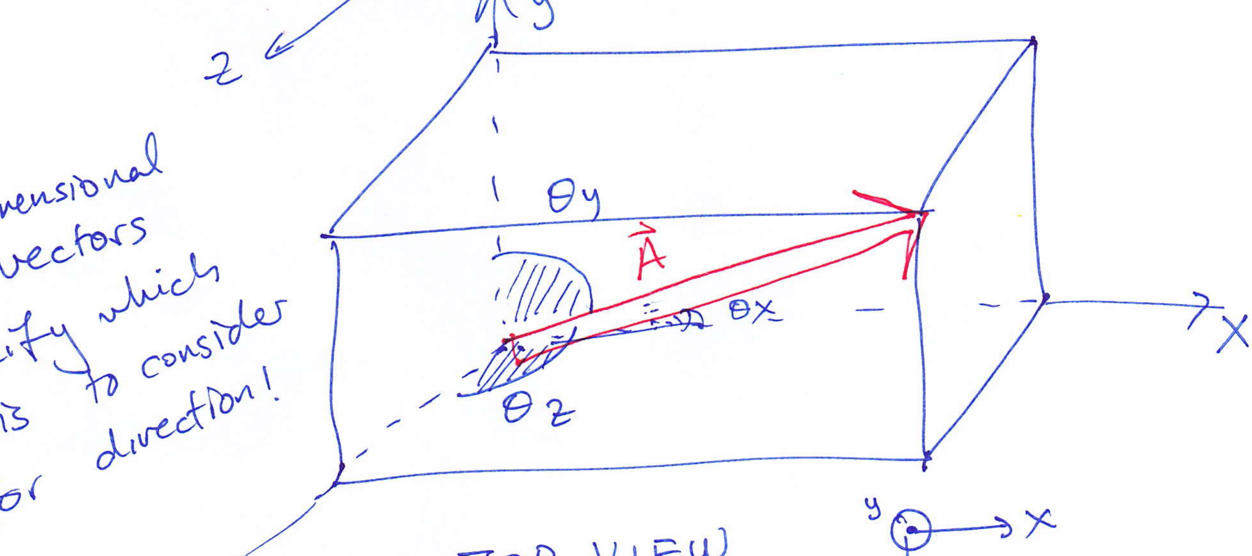
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= 14.9$$

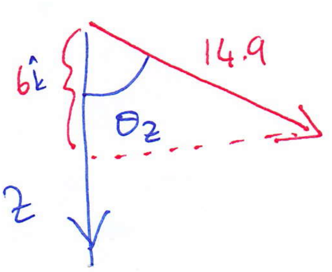
$2\vec{D} - \vec{E} = \vec{A}$



3 dimensional vectors specify which axis to consider for direction!



TOP VIEW



$\vec{A} = 8\hat{i} + 11\hat{j} + 6\hat{k}$

$\cos\theta_z = \dfrac{6}{14.9}$; $\theta_z = \cos^{-1}\left(\dfrac{6}{14.9}\right)$

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significant figures:

121 cm : 3 s.f.

121.02 cm = 5 s.f.

$\boxed{121.00}$ ⇒ 5 s.f.

1,2305

1,2305~~000~~

s.f

multiplication / division ⇒ en küçük a.s. s.f. sonucu yönetir.

$$\frac{3.52 \times 2.307}{1.8} = (4.5)\ 2sf$$

$f(x) = \ldots$

↳ s.f

$\sin(x) = 0.707106\ldots$

↑ 45 ↑ 2s.f

7.07106×10^{-1}

0.71 ✓

$\sin(45.0) = 0.707$ ✓

$15.2 \times \sin(45) = (11.)\ 2sf.$

$15.2 \times \sin(45.0) = 10.7\ 3sf$

addition / subtraction

ondalık sayıların noktanın sağında kalan s.f / a.s.

15.2 → (1.sf) → rough / kaba

+ 0.278

15.478

15.5 → 1.sf.

15.2
− 0.278

14.922 ⇒ 14.9 ✓

14.981 ≃ 15.0

Vector product
(çarpım)

Scalar pr. (dot pr) — (sбt sayı number)

$\vec{A} \cdot \vec{B} = C$

vector pr. (cross pr.)

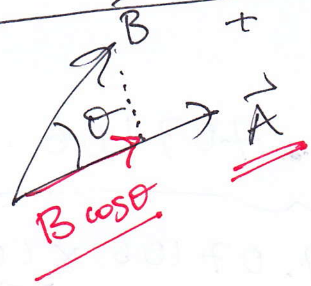
$\vec{A} \times \vec{B} = \vec{C}$

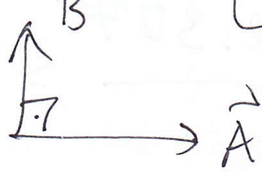
$|\vec{A}| = A$

magnitude

$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\theta}$$

+ (+, −, 0)



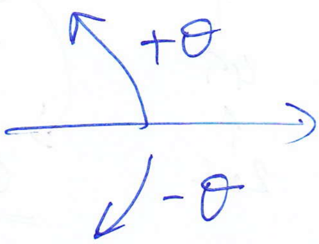


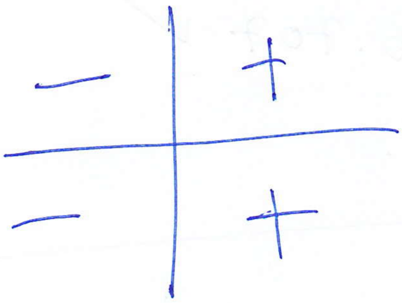
$AB\cos\cancel{90}^{\,0} = 0$

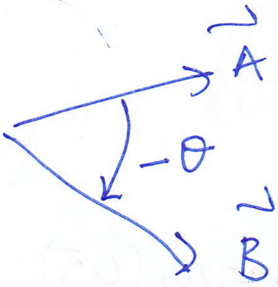
scalar pr. projection of two vector

(izdüşüm)

$\cos\theta$



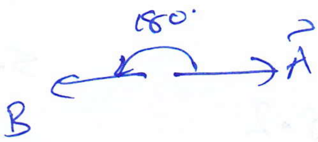




$\vec{A} \cdot \vec{B} = AB\cos(-\theta)$ ✓ $= \cos\theta$ ✓

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\Longrightarrow^{B}_{A}$ $AB(+)$ $\theta = 0°$



$AB(-)$ $\theta = 180°$

$AB\underbrace{\cos 180}_{(-)} = -AB$

unit vectors ✓

$|\hat{\imath}| = 1$

$\hat{\imath}, \hat{\jmath}, \hat{k}$

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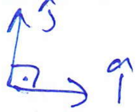
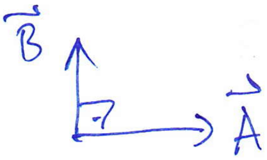
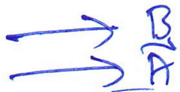
②

$$\vec{A}\circ\vec{B} = A\hat{\imath}\cdot B\hat{\jmath} = AB\,\hat{\imath}\cdot\hat{\jmath}$$



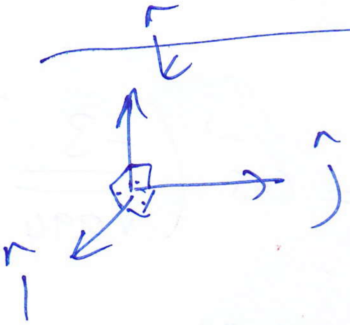


$$|\hat{\imath}|\,|\hat{\jmath}|\cos\cancel{90^\circ} = 0$$
$$1 \quad 1 \quad 0$$



$$\vec{A}\circ\vec{B} = A\hat{\imath}\cdot B\hat{\imath} = AB\,\hat{\imath}\cdot\hat{\imath}$$

$$= AB\left\{|\hat{\imath}|\,|\hat{\imath}|\cos 0\right.$$
$$1 \quad 1 \quad 1$$



$$\hat{\imath}\circ\hat{\jmath} = \hat{\jmath}\circ\hat{k} = \hat{\imath}\cdot\hat{k} = 0$$
$$\hat{\jmath}\cdot\hat{\imath} = \hat{k}\cdot\hat{\jmath} = \hat{k}\cdot\hat{\imath} = \quad = 0$$
$$\hat{\imath}\cdot\hat{\imath} = 1 = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k}$$

$$\vec{A}\circ\vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right)\cdot\left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right)$$

(3d vectors)

$$= A_xB_x\,\hat{\imath}\cdot\hat{\imath}^{\,1} + A_xB_y\,\cancel{\hat{\imath}\cdot\hat{\jmath}}^{\,0} + A_xB_z\,\cancel{\hat{\imath}\cdot\hat{k}}^{\,0}$$
$$+ A_yB_x\,\cancel{\hat{\jmath}\cdot\hat{\imath}}^{\,0} + A_yB_y\,\hat{\jmath}\cdot\hat{\jmath}^{\,1} + A_yB_z\,\cancel{\hat{\jmath}\cdot\hat{k}}^{\,0}$$
$$+ A_zB_x\,\cancel{\hat{k}\cdot\hat{\imath}}^{\,0} + A_zB_y\,\cancel{\hat{k}\cdot\hat{\jmath}}^{\,0} + A_zB_z\,\hat{k}\cdot\hat{k}^{\,1}$$

$$\boxed{\vec{A}\circ\vec{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta}$$

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}\ ; \quad B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

θ = angle from $\vec{A}$ to $\vec{B}$

$\vec{A} = 2\hat{i} + 3\hat{j} + 1\hat{k}$

$\vec{B} = -4\hat{i} + 2\hat{j} - 1\hat{k}$

a) $\vec{A} \cdot \vec{B} = ?$

b) $A = ?$ $B = ?$

c) θ between A & B ?

$$\vec{A} \cdot \vec{B} = \left(2\hat{i} + 3\hat{j} + 1\hat{k}\right) \cdot \left(-4\hat{i} + 2\hat{j} - 1\hat{k}\right)$$

$$\vec{A} \cdot \vec{B} = 2(-4) + 3(2) + (1)(-1) = \underline{\underline{-3}}$$

$$A = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14} \qquad B = \sqrt{4^2 + 2^2 + 1^2} = \sqrt{21}$$

$$\vec{A} \cdot \vec{B} = AB\cos\theta = -3$$

?

$$\sqrt{14}\,\sqrt{21}\,\cos\theta = -3 \qquad \theta = \cos^{-1}\left(\frac{-3}{\sqrt{294}}\right)$$

$\theta > 90^\circ$

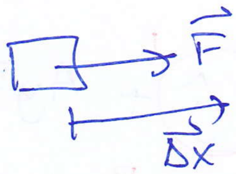
$\underline{-3 = \vec{A} \cdot \vec{B} < 0}$

$\theta = 100.\cancel{0}^\circ$

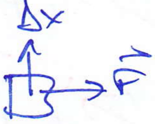
$\underline{\underline{\theta = 100^\circ}}$ ✓

---

scalar product will be used in work/energy



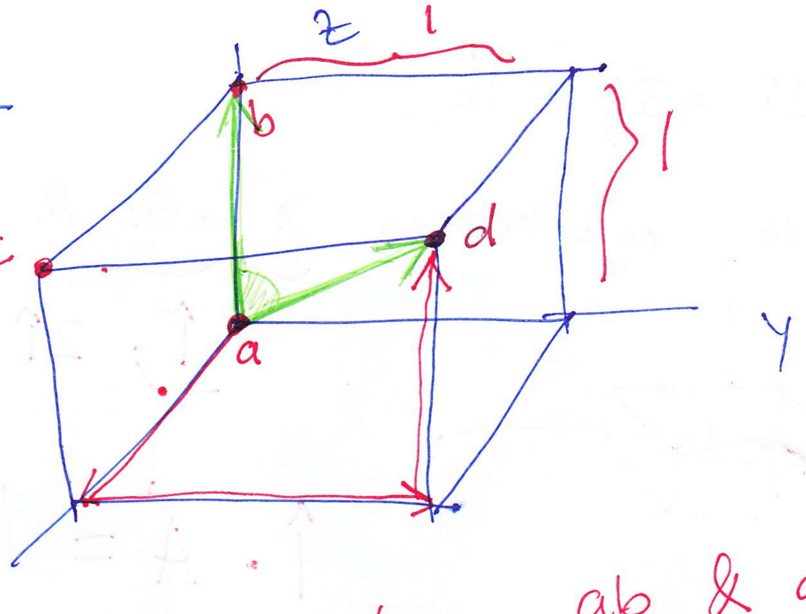
$$\text{Work} = \underbrace{\vec{F} \cdot \vec{\Delta x}}_{\text{scalar pr.}} = F\Delta x \cos\cancel{\theta}$$



$$\underline{W = F\Delta x \cos \cancel{90}^{0} = 0}$$

Dot (scalar) product is very useful to find θ angle between 2 $\underline{\text{3 dim. vector.}}$

cube



what's the angle between ab & ad?

$\vec{ab} = \vec{A} = 1\hat{k} \quad ; \quad \vec{ad} = \vec{D} = 1\hat{i} + 1\hat{j} + 1\hat{k}$

$\vec{A} \cdot \vec{D} = 1\hat{k} \cdot (\hat{i} + \hat{j} + \hat{k}) = A D \underbrace{\cos\theta}_{?}$

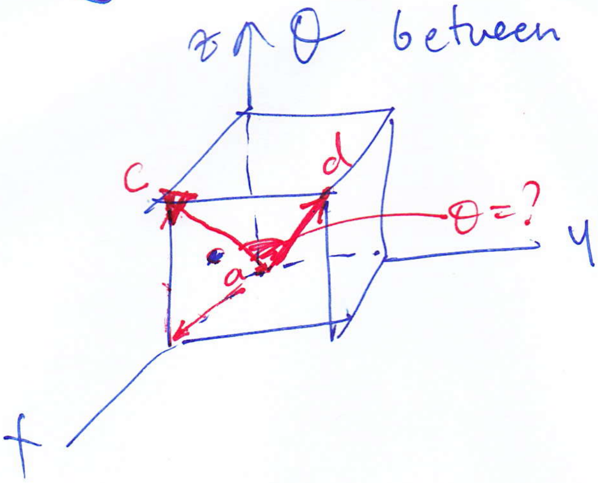
$\underbrace{1}_{} = 1 \overbrace{\sqrt{1^2+1^2+1^2}}^{\sqrt{3}} \cos\theta$

$\cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = 54.7°$

$\Rightarrow$ Try yourself.

θ between $\vec{ac}$; $\vec{ad}$



use the same formulas!

$\vec{ac} \cdot \vec{ad}$

$\vec{C} \cdot \vec{D} = C D \cos\theta$

$(\hat{i}+\hat{k}) \cdot (\hat{i}+\hat{j}+\hat{k}) = \sqrt{1^2+1^2}\sqrt{1^2+1^2+1^2}\cos\theta$

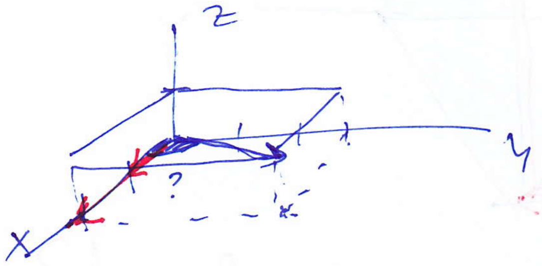
$(1+1) = \sqrt{2}\ \sqrt{3}\ \cos\theta$

$\theta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right) = 35.3°$

$\vec{A} = 2\hat{i} + 3\hat{j} + 1\hat{k}$

whats the angle between X axis & $\vec{A}$ = ?

$\hat{i}$; $2\hat{i}$; $-3\hat{j}$ . .



$\hat{i} \cdot \vec{A} = |\hat{i}|\,|\vec{A}| \cos\theta$

$\underbrace{\hat{i} \cdot (2\hat{i} + 3\hat{j} + 1\hat{k})}_{2 \quad 0 \quad 0} =$

$2 = 1\sqrt{2^2 + 3^2 + 1^2}\cos\theta$

$2 = \sqrt{14}\cos\theta$

$\theta = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$

$\frac{2\hat{i} \cdot \vec{A}}{\cancel{4}\ 2} = \overset{2}{\cancel{2}}\sqrt{14}\cos\theta$

$\theta = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$

$\theta = \underline{58^\circ}$

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(4)

Vector product of 2 vectors.

$$\vec{A} \times \vec{B} = \vec{C}$$

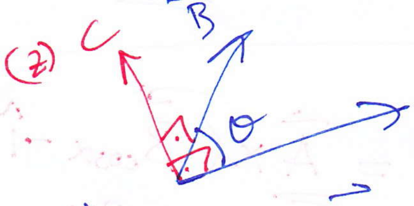
× → cross

$\vec{C}$ is a vector → magnitude ✓ → direction ✓

$$|\vec{C}| = |AB\sin\theta|$$

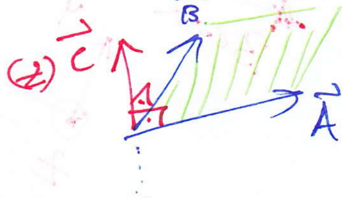
$$\boxed{\begin{aligned}\vec{A}\cdot\vec{B} &= AB\cos\theta \\ \vec{A}\times\vec{B} &= |AB\sin\theta|\,\hat{C}\end{aligned}}$$

direction of $\vec{C}$ is found by right-hand rule (saġ el kerah)



$\vec{C} \perp \vec{A}$ & $\perp \vec{B}$; AB form a surface

$\vec{C} \perp AB$ surface.



$$\vec{A} \times \vec{B} = \vec{C}$$

$\vec{A}$ ↑ thumb (baş) — $\vec{B}$ ↑ index (işaret) — $\vec{C}$ ↑ middle (orta)

right hand rule

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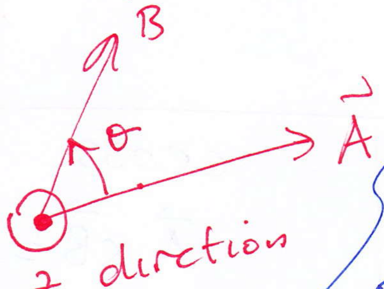
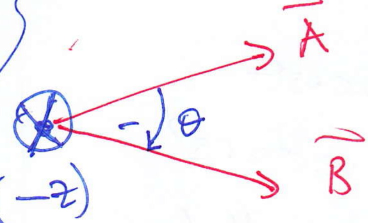
⊗ into " "

$\vec{A}\times\vec{B} = C(+\hat{k})$

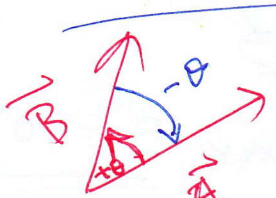


$AB\sin\theta > 0$
$+\ +\ \ +$



$\vec{A}\times\vec{B} = C(-\hat{k})$

$AB\sin\theta$
$+\ +\ \underbrace{\sin(-\theta)}_{(-)} < 0$



$\vec{A}\times\vec{B}$ $+\theta$

$\vec{B}\times\vec{A}$ $-\theta$

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

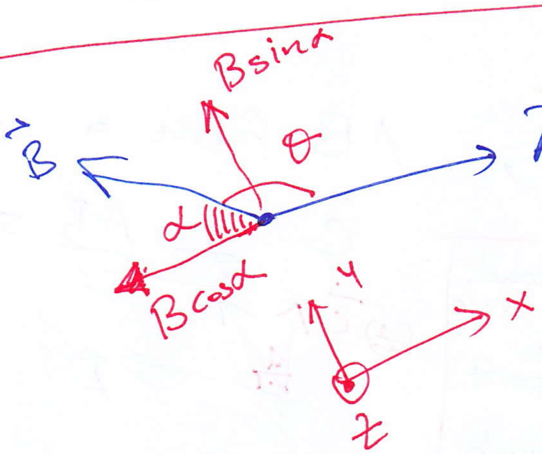
$\vec{A}$ ↑ baş — $\vec{B}$ ↑ baş

$$|\vec{C}| = |AB\sin\theta|$$

$$AB\sin\theta\,\hat{C} = -BA\sin(-\theta)\,\hat{C}$$

$$AB\sin\theta\,\hat{C} = -(-BA\sin\theta\,\hat{C})$$

θ		$\sin\theta$	
0	$\vec{B}$, $\vec{A}$	0	$\vec{A}\times\vec{B}=0$
180	$\vec{B}$ 180° $\vec{A}$	0	$\vec{A}\times\vec{B}=0$
90 →	B, A	+1	$\hat{j}$, $\hat{k}$, $\hat{i}$ — $\vec{A}\times\vec{B}=\vec{C}=AB\sin 90\,\hat{k}$
270	270, −90, $\vec{A}$, $\vec{B}$	−1	$\vec{A}\times\vec{B}=\vec{C}=AB(-\hat{k})$; $\lvert\vec{C}\rvert=\lvert AB\sin 90\rvert=AB$



$$\vec{A}\times\vec{B}=\vec{A}\times\left(B\cos\alpha\,(-\hat{i})+B\sin\alpha\,\hat{j}\right)$$

$$=\underbrace{\vec{A}\times B\cos\alpha(-\hat{i})}_{\theta=180}\Rightarrow 0$$

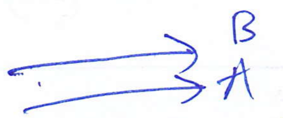
$$+\underbrace{\vec{A}\times\vec{B}\sin\alpha\,\hat{j}}_{\vec{A}\perp\vec{B}\sin\alpha}$$

$$\vec{A}\times\vec{B}=AB\sin\alpha\,\hat{c}$$

DOT PRODUCT

$\vec{A}\cdot\vec{B}=AB\cos\theta$

- projection of vectors?
- shows how parallel two vectors are!



$AB\cos\theta=\max$

$\underline{\theta=0}$

CROSS PRODUCT

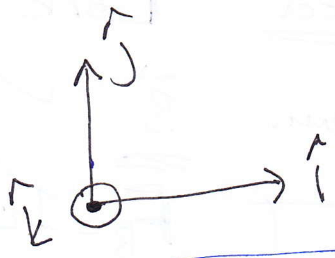
$\vec{A}\times\vec{B}=|AB\sin\theta|\,\hat{c}$

- show how perpendicular two vectors are...
- $\vec{A}\times\vec{B}$ ⇒ forms a surface.

$\vec{A}\times\vec{B}$ max $\underline{\theta=90°}$

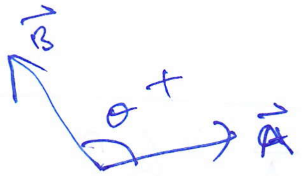
Phys I - Vector cont'd

14.10.20 ①



$\vec{A} \times \vec{B} = \vec{C}$ .... $|\vec{C}| = |AB\sin\theta|$

(+ −)



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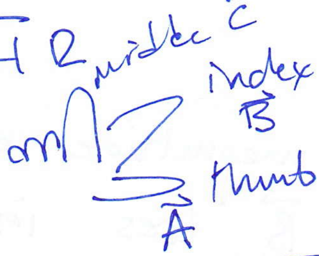
direction

RHR



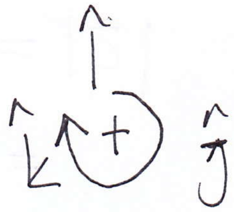
$\sin\theta < 0$

RHR

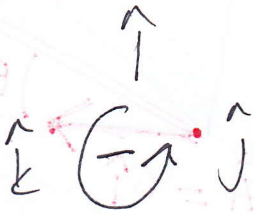
1st way

$\hat{i} \times \hat{j} = \hat{k}$	$\hat{j} \times \hat{k} = \hat{i}$	$\hat{k} \times \hat{i} = \hat{j}$
$\hat{j} \times \hat{i} = -\hat{k}$	$\hat{k} \times \hat{j} = -\hat{i}$	$\hat{i} \times \hat{k} = -\hat{j}$

2nd way



$\hat{i} \times \hat{j} = \hat{k}$
$\hat{j} \times \hat{k} = \hat{i}$
$k \times \hat{i} = \hat{j}$



$\hat{i} \times \hat{k} = -\hat{j}$
$\hat{k} \times \hat{j} = -\hat{i}$
$\hat{j} \times \hat{i} = -\hat{k}$

$$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x \cancel{(\hat{i} \times \hat{i})}^{0} + A_xB_y (\hat{i} \times \hat{j})^{\hat{k}} + A_xB_z (\hat{i} \times \hat{k})^{-\hat{j}}$$

$$+ A_yB_x (\hat{j} \times \hat{i})^{-\hat{k}} + A_yB_y \cancel{(\hat{j} \times \hat{j})}^{0} + A_yB_z (\hat{j} \times \hat{k}) \Rightarrow +\hat{i}$$

$$+ A_zB_x (\hat{k} \times \hat{i})^{+\hat{j}} + A_zB_y \underbrace{\hat{k} \times \hat{j}}_{-\hat{i}} + A_zB_z \cancel{(\hat{k} \times \hat{k})}^{0}$$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

+ − + don't memorize.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \Rightarrow \hat{i}(A_yB_z - A_zB_y) + \ldots\ldots$$

Vector products will be used $\vec{\tau}$ork ✓

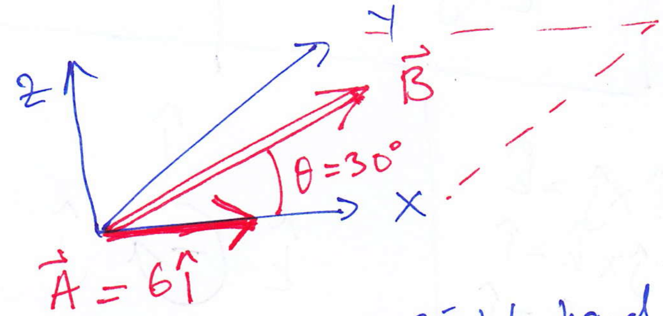
2nd sem. $\vec{B}$ ✓

$\vec{F}_B = I\vec{L} \times \vec{B}$

ex: $\vec{A} = 6\hat{i}$

$\vec{B}$ has magnitude of 4 units and $\vec{B}$ lies in x-y plane; with angle of 30° with +x axis

$\vec{C} = \vec{A} \times \vec{B}$; $\vec{C} = ?$ $|\vec{C}| = ?$



$\vec{A} \times \vec{B} = \vec{C}$

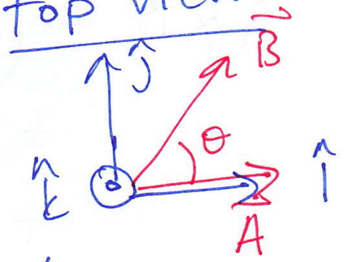
$|\vec{C}| = |AB\sin\theta|$

$= |6(4)\sin 30|$

$= 12$ ✓

Just use Right hand to find the direction of $\vec{C}$

$\vec{A} \times \vec{B}$ they form ~~a plane~~ xy plane $\perp$ $\vec{C}$ $\to \hat{k}$ / $\to -\hat{k}$



$\vec{A} \times \vec{B} \Rightarrow +\hat{k}$ direction with RHRule.

$\vec{C}$ in $+\hat{k}$

$|\vec{C}| = 12$

$\vec{C} = 12\hat{k}$

2nd way $\vec{A} * \vec{B} = 6\hat{i} \times (4\cos 30\,\hat{i} + 4\sin 30\,\hat{j})$

$= 24\cos 30\,(\hat{i} \times \hat{i}) + \underbrace{24\sin 30\,(\hat{i} \times \hat{j})}_{12\ +\hat{k}}$

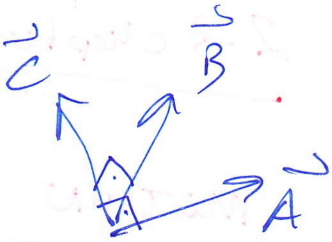
0

12 $+\hat{k}$ ✓

Q 1.96 $|\vec{A}| = 3$ $|\vec{B}| = 3$

$\vec{C} = \vec{A} \times \vec{B} = -5\hat{k} + 2\hat{i}$

angle betwen $\vec{A}$ & $\vec{B}$ = ?



$\vec{A} \times \vec{B} = \vec{C}$; $|\vec{C}| = |AB \sin\theta|$

radyan	degree
$3.14 = \pi$	$180°$

$3.14\ rad = 180°$

$1\ rad = \frac{180°}{3.14}$

$1\ rad = 57.3°$

$0.017\ rad = 1°$

$\vec{C} = -5\hat{k} + 2\hat{i}$

$|\vec{C}| = \sqrt{5^2 + 2^2} = \sqrt{29}$

$\sqrt{29} = \underset{3}{A}\ \underset{3}{B} \sin\theta$

$\sin\theta = \frac{\sqrt{29}}{9}$ $\theta = \sin^{-1}\left(\frac{\sqrt{29}}{9}\right)$

$\theta = 36.8°$

Q 1.95 $\vec{A} = 5.0\hat{i} - 6.5\hat{j}$ $\vec{B} = -3.5\hat{i} + 7.0\hat{j}$

$\vec{A} \times \vec{B} = ?$ $(5\hat{i} - 6.5\hat{j}) \times (-3.5\hat{i} + 7\hat{j})$

$-5(3.5)\cancel{\hat{i} \times \hat{i}}^{\,0} + 35 (\hat{i} \times \hat{j}) \Rightarrow \hat{k}$

$+ 6.5(3.5)(\hat{j} \times \hat{i}) \Rightarrow -\hat{k}$ $+ (-6.5)7\,\cancel{\hat{j} \times \hat{j}} \Rightarrow 0$

$= (35 - 22.75)\hat{k}$

$\vec{A} \times \vec{B} = 12.25\,\hat{k}$; $|\vec{A} \times \vec{B}| = 12.25$

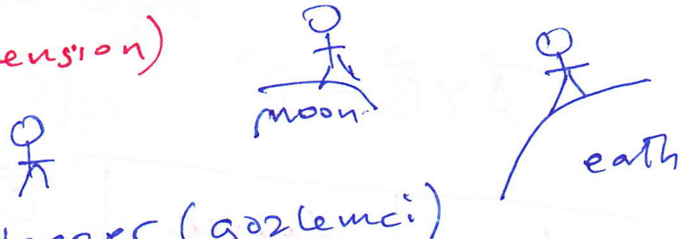
$= 12$

End of 1st chapter

2-3 chapter / { MOTION → 2^{nd} chapter ⇒ 1 dimension
→ 3rd chapter ⇒ 2 dimensional (~3 dimens.)

MOTION along A STRAIGHT LINE
(1 dimension)

"nothing is stationary" in the universe

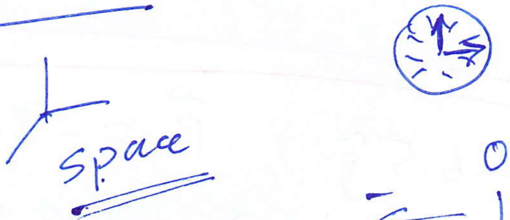


observer (gozlemci)

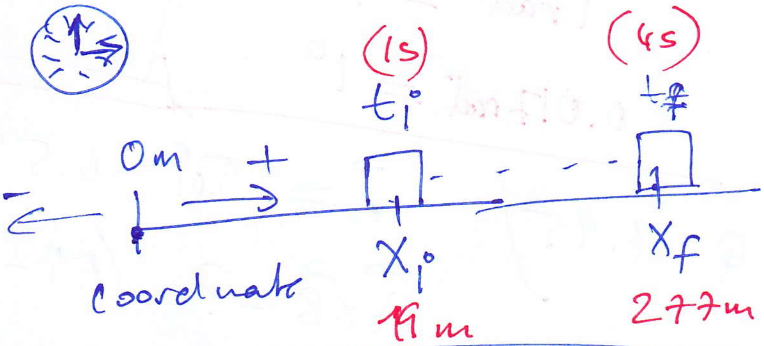
everything moves / revolves in some way.

p, n, e⁻ ... } vibrates / rotates.

position (m) time (s) SI units



1 dimension motion



*Displacement (yer değiştirme) = ΔX (≡ vector)

Delta

$$\boxed{\Delta = \text{final} - \text{initial}}$$
son - ilk

$\Delta X = X_f - X_i$

$\Delta t = t_f - t_i$

$\Delta X = 277 - 19 = 258m$

ΔX → +, → − $\Delta t \rightarrow +$

1d ΔX → ± } vector

Velocity (≡ Hız)

it's a vector.

(velocity ≠ hız ≠ sürat ≠ speed)

velocity

instantaneous velocity

$$v = \frac{d\cancel{x}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

{ derivative of X with respect to t }

(anlık ani hız)

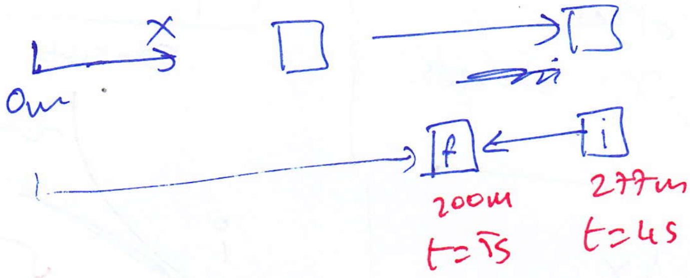
average velocity (ortalama hız)

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\text{meter}}{\text{sec.}} = \left[\frac{m}{s}\right]$$

square brackets are used for units.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{277 - 19}{4 - 1} = 86 \, m/s$$



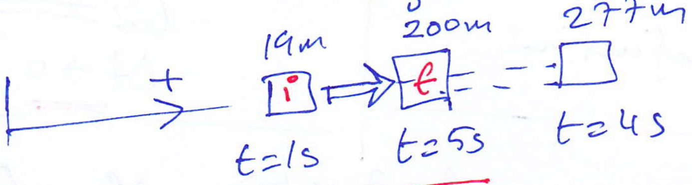
1st $\quad \bar{v} = 86 \, m/s$

2nd $\quad \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

$$= \frac{200 - 277}{5 - 1} \, \frac{m}{s}$$

$$= -77 \, m/s$$

$\bar{v}$ → +, → −



$$\bar{v} = \frac{200 - 19}{5 - 1} = +45.3 \, m/s$$

$$\boxed{v = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \Delta \bar{v}}$$

Derivatie:

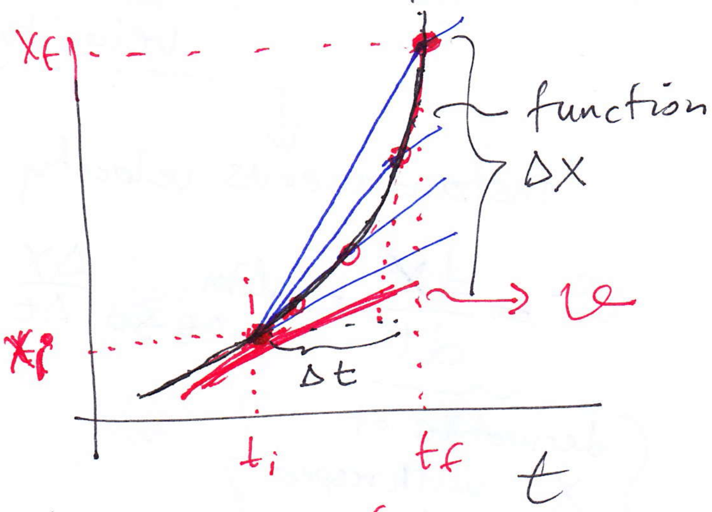
Newton had to invent calculus — derivative, integral ⋮

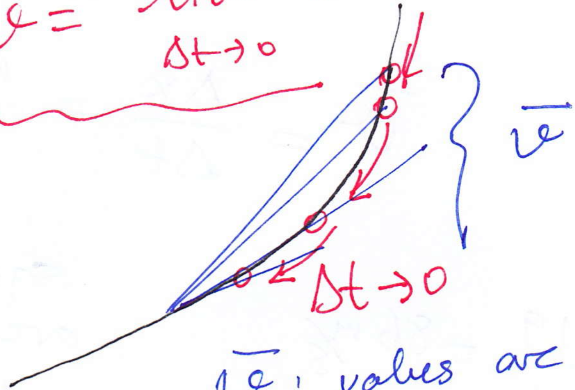
(Leibnitz)

Derivative = türev

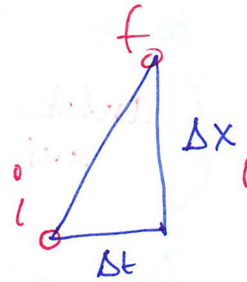
ave. $\bar{v} = \frac{\Delta x}{\Delta t}$

inst. $v = \lim_{\Delta t \to 0} \bar{v}$





$\bar{v}$ values are not equal to each other as t_f, x_f changes.

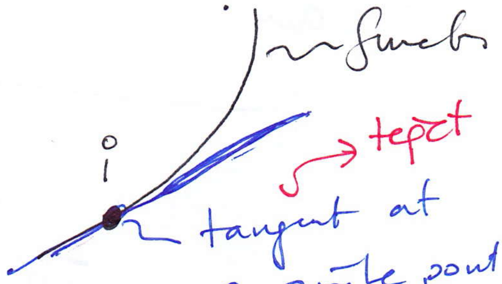


$\bar{v} = \frac{\Delta x}{\Delta t}$ = slope

eğim

Infinitesimally small

sonsuz küçüklük



$\lim_{\Delta t \to 0} \bar{v} = v$

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Derivative definition

Derivative of a polynomial function.

$x = At^n$

$\frac{dx}{dt} = Ant^{n-1}$

$x = 10t^3$

$\frac{dx}{dt} = 10(3)t^2 = 30t^2$

ex = $x = 20\,m + 5\,\frac{m}{s^2}\,t^2$

$[m] = [m] + \left[\frac{m}{\cancel{s^2}}\cancel{s^2}\right]$ ✓ unit balance ✓

(4)

$\Delta X = ?$ $\quad t_i = 1s$ $\quad t_f = 2s$

$X(t=t_i) = X_i = 20 + 5t_i^2 = 25m \qquad X_f(t=2s) = 40m$

$\Delta X = X_f - X_i = 40 - 25 = 15m$

$\bar{v} = ? \qquad \bar{v} = \frac{\Delta X}{\Delta t} = \frac{15}{2-1} = 15 m/s$ ✓

$v(t=1s) = ? \qquad v = \frac{dx}{dt} = \frac{d}{dt}(20 + 5t^2)$

$= 0 + 5(2)t^1$

$v = 10t$

$v(t=1s) = 10(1)$

$= 10 m/s$

make Δt smaller starti from $2s \to 1s$

$\Delta t = 1s$

calculate $\bar{v} = ?$

$\bar{v}$	Δt	t_f	t_i
15	1	2	1
10.5	0.1	1.1	1
10.05	0.01	1.01	1
10.005	0.001	1.001	1
10	$\Delta t \sim 0$	1^+	1

$\bar{v} = \frac{\Delta X}{\Delta t} = \frac{X(t_f) - X(t_i)}{t_f - t_i}$

$X = 20 + 5t^2$

$= \frac{X(1.1) - X(1)}{1.1 - 1} =$

$= \frac{26.05 - 25}{0.1}$

$= 10.5 \, m/s \quad \begin{pmatrix}\Delta t\\ 0.1\end{pmatrix}$

$\bar{v} = \frac{X(1.01) - X(1)}{0.01}$

$= \frac{25.1005 - 25}{0.01}$

$\boxed{\lim_{\Delta t \to 0} \bar{v} = v} \to \frac{dx}{dt} = 10t$

(1)

$= 10 m/s$

✓ 10 = 10 ✓

Physics I Chapter 2 Cont'd 19.10.20 ①

Motion in 1dimension

Last Lecture: Review

$\Delta = \text{final} - \text{initial}$

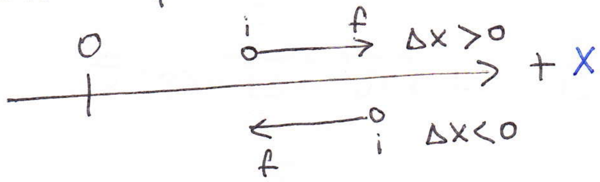
X: location (m)

t: time (s)

$\Delta x = x_f - x_i \equiv$ displacement

$\Delta x \begin{cases} \to + \\ \to - \end{cases}$ vector in 1d

$\vec{\Delta x}$



$v \equiv$ velocity $\Rightarrow$ change of displacement in time

average v

$\bar{v} = \dfrac{\Delta x}{\Delta t}$

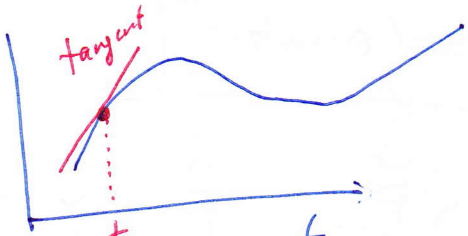
instantaneous v.

$v = \dfrac{dx}{dt} = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \bar{v}$

$$\boxed{\begin{aligned} X(t) &= At^n \\ \frac{dx}{dt} &= Ant^{n-1} \end{aligned}}$$

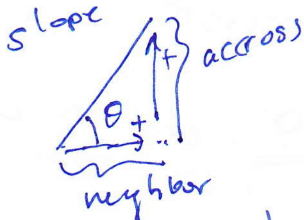
derivative of a polynomial function

meaning of taking a derivative on a graph $\Leftarrow$

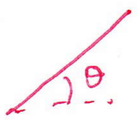


slope (eğim)

tangent (teğet)



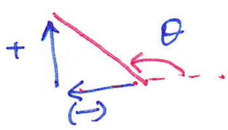
$\tan\theta = \dfrac{\text{across}}{\text{neighbor}}$



slope > 0 (+)

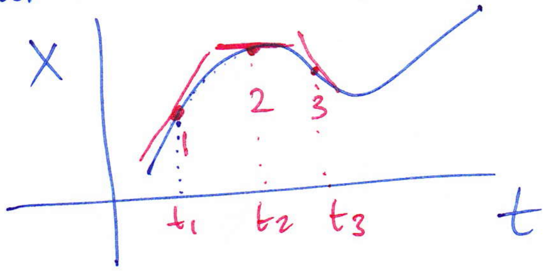
$\tan\theta > 0$

slope $= 0$ $\tan\theta = 0$



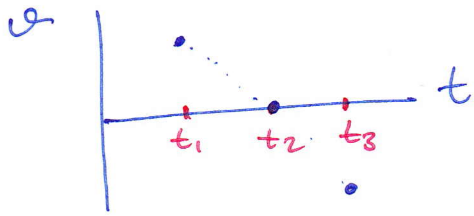
slope < 0 (−)

$\tan\theta < 0$



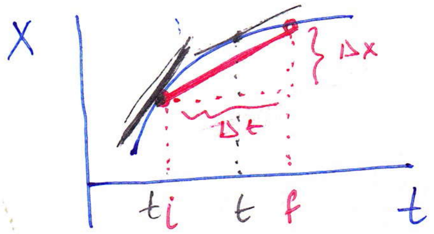
$\dfrac{dx}{dt}$ | x'

$v = dx/dt$	①	②	③
v	+	0	−



$X = At^2 + Bt$ $A \begin{cases} + \\ - \end{cases}$ B

$\dfrac{dx}{dt} = 2At + B$



②

$$\overline{v} = \frac{\Delta x}{\Delta t} \div \frac{x_f - x_i}{t_f - t_i} > 0 \Big\}$$ 2 points value

$$v = \frac{dx}{dt}\Big|$$ at which time value?

$v = \left[\frac{m}{s}\right]$ $\quad \underline{\underline{t=?}} \quad$ $v = \lim_{\Delta t \to 0} \overline{v}$

Acceleration (time)

change of velocity in time

$+$, 0, $-$ $\quad a = \frac{\text{velocity}}{\text{time}} = \frac{m/s}{s} = \left[\frac{m}{s^2}\right]$

$\frac{\Delta v}{\Delta t} = \overline{a}$ (average acc.)

$\frac{dv}{dt} = a$ (inst. acc.)

ex: $v = 60 + 0.5t^2$

a) $t_i = 1s \quad t_f = 3s$ $\quad$ change in velocity $= \Delta v = ?$

$$\Delta v = v_f - v_i = \left[60 + (0.5)3^2\right] - \left[60 + (0.5)1^2\right]$$

$$= 4 \ m/s$$

b) $\overline{a}$ btween 1s ; 3s

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{4}{3-1} = 2 \ m/s^2$$

c) $a = ?$ at $t = 1s$

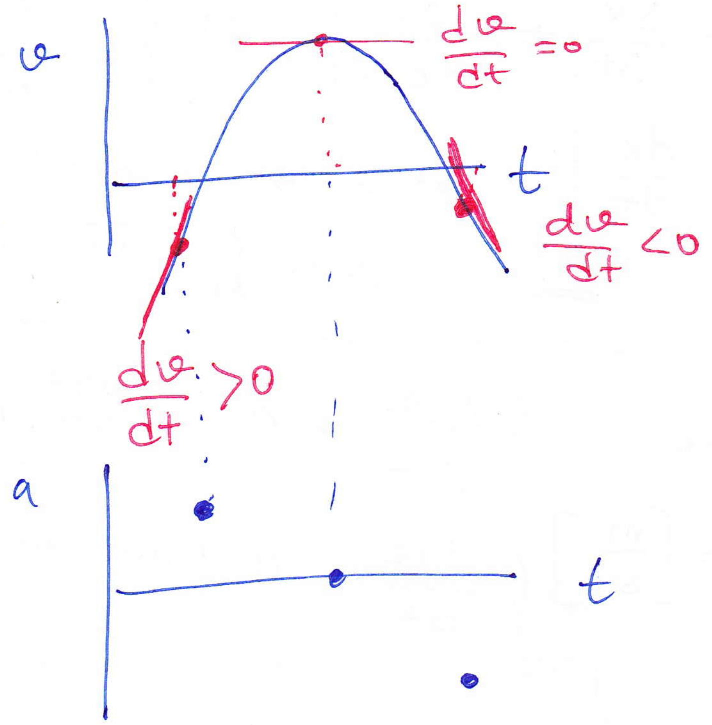
$$a = \frac{dv}{dt} = \frac{d}{dt}\left(60 + 0.5t^2\right) = 0 + (0.5)(2)t^1 = t$$

$$a(t=1s) = 1 \ m/s^2$$

$\overline{a}$	Δt	t_f	t_i
2	2	3	1
1.05	0.1	1.1	1
1.005	0.01	1.01	1
⇓	⇓		
$\left.\frac{dv}{dt}\right\|_{t=1} = a = \underline{1}$	0	1.000000...	1

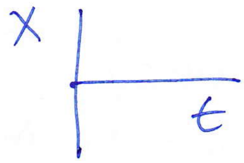
$v = 60 + 0.5t^2$

$$\Rightarrow \frac{\Delta v}{\Delta t} = \frac{v(t=1.1) - v(t=1)}{1.1 - 1}$$

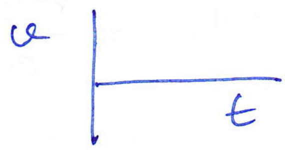


$v = f(t)$

$\frac{dv}{dt} = a$



slope $\Rightarrow v$



slope $\Rightarrow a$

$$v = \frac{dx}{dt} \quad ; \quad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

2nd derivative of X w.r.t time

---

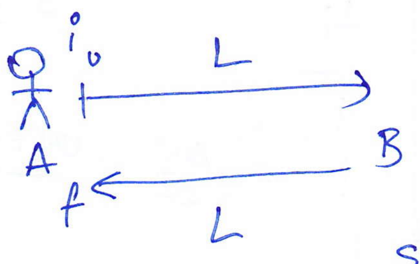
speed (sürat) vs velocity (hız) $\rightarrow \frac{\Delta x}{\Delta t}$; $\frac{dx}{dt}$

odometer (speedometer)

speed > 0.

speed $\neq$ velocity
(+) $\left(\begin{smallmatrix}+\\-\\0\end{smallmatrix}\right)$

$$\text{speed} = \frac{\text{total distance travelled}}{\text{time} = \Delta t}$$



$$\text{velocity} = \frac{X_f - X_i}{\Delta t} = \frac{0-0}{\Delta t}$$

$= 0$

$$\text{speed} = \frac{2L}{\Delta t}$$

$\frac{da}{dt} \neq 0$ acc. may not be constant in a motion

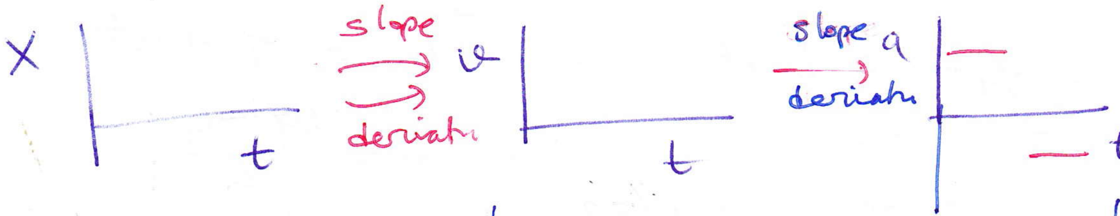
$\frac{dx}{dt}$ = velocity

$\frac{dv}{dt}$ = acc.

$\frac{da}{dt} = ?$

Motion with constant acceleration

19 10 20



$a \rightarrow +$, $\rightarrow -$

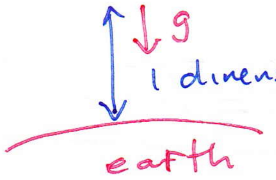
$x = G t^2 + Ht + I$

$v = B t^1 + C$

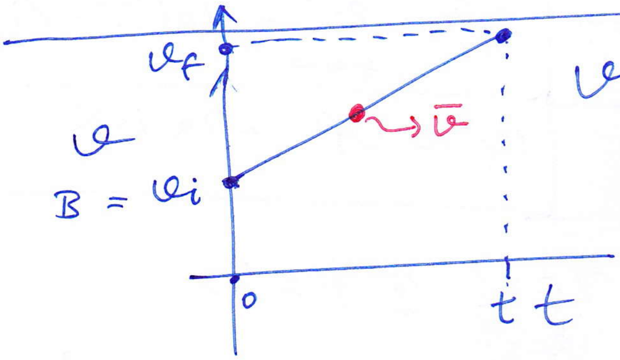
$\boxed{a(t) = A t^0 = \#}$

acc. does not depend (change) on time.

daily life we deal with $g = 9.8\ m/s^2$



1 dimensional motion in y



$v = At + B$ $\qquad a = \text{const}$

$a = \frac{dv}{dt}$

$\int dv = \int a\,dt$

$\boxed{\begin{matrix} t_i = 0 \\ t_f = t \end{matrix}}$

$\Delta v = a\,\Delta t \Big|_{t_i=0}^{t_f=t}$

$\Delta v = a(t-0)$

$v_f - v_i = at$

$\boxed{v_f = v_i + at}$

$v(t) = v_i + at$

when $t = 0$ $\quad v(t) = v_i$

$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{v_f + v_i}{2}$ (from graph)

$\frac{x_f - x_i}{(t-0)} = \frac{v_f + v_i}{2} = \frac{(v_i + at) + v_i}{2}$

$x_f - x_i = \frac{\cancel{2} v_i t}{\cancel{2}} + \frac{at^2}{2}$

✓ $\boxed{x_f = x_i + v_i t + \frac{1}{2} a t^2}$ $\quad a = \text{const}$

✓ $\boxed{v_f = v_i + at}$ $\quad a = \text{const}$

Let

write a new eqn. without time

** $x_f = x_i + v_i t + \frac{1}{2}at^2$; * $v_f = v_i + at$

$t = \frac{v_f - v_i}{a}$

$$x_f = x_i + v_i\left(\frac{v_f - v_i}{a}\right) + \frac{1}{2}\cancel{a}\left(\frac{v_f^2 + v_i^2 - 2v_f v_i}{a^{\cancel{2}}}\right)$$

$$= x_i + \cancel{\frac{v_f v_i}{a}} - \frac{v_i^2}{a} + \frac{v_f^2}{2a} + \frac{v_i^2}{2a} - \cancel{\frac{v_f v_i}{a}}$$

$$x_f = x_i - \frac{v_i^2}{2a} + \frac{v_f^2}{2a} \quad ; \quad v_f = (\dots)$$

$$(x_f - x_i)\,2a = v_f^2 - v_i^2 \Rightarrow \boxed{v_f^2 = v_i^2 + 2a(x_f - x_i)}$$

***

$$\boxed{v_f^2 = v_i^2 + 2a\Delta x}$$

$\boxed{x_f = x_i + v_i t + \frac{1}{2}at^2}$ ✓ $\boxed{v_f = v_i + at}$ ✓

ex:

$a = 4\ m/s^2$

$t = 0s$

$x_i = 5m$

$v_i = 15\ m/s$

a) $x_f = ?$ @ $t = 2s$

$v_f = ?$

$x_f = 5 + 15(2) + \frac{1}{2}(4)\,2^2$

$x_f = 43m$

$v_f = v_i + at = 15 + 4(2)$

$= 23\ m/s$

b) when $v = 25\ m/s$

what is its location?

v_f ✓ t? $x_f = ?$

3rd eqn.

$v_f^2 = v_i^2 + 2a\Delta x$

$25^2 = 15^2 + 2(4)\Delta x$

$\Delta x = x_f - \underset{5}{x_i}$

1st way

$x_f = 55m$

b) 2nd way use 1st 2nd eqn.

time ✓

$v_f = v_i + at$

$25 = 15 + (4)t$

$t = 2.5s$

$x_f = 5 + 15(2.5) + \frac{1}{2}(4)(2.5)^2$

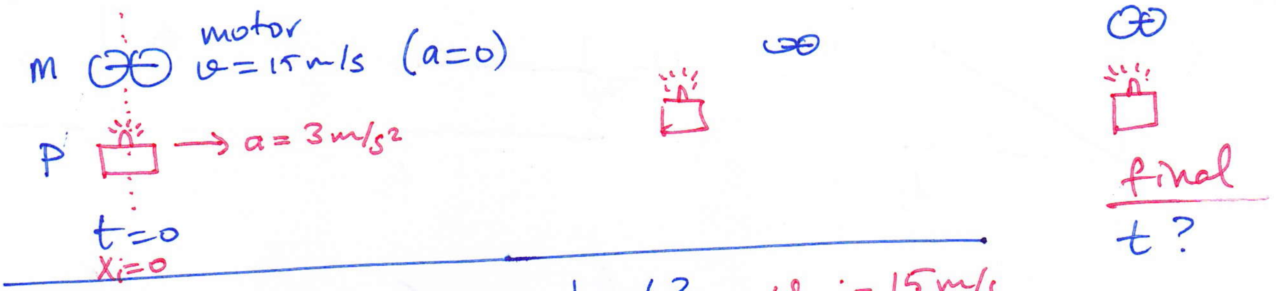
$x_f = 55m$ ✓

ex: a motorist travelling at constant velocity of 15 m/s passes a police car which is stationary (not moving). After passing the police, police car tries to catch the motorist with const. $a = 3\ m/s^2$.

a) when does the police catch the motorist?
$t = ?$

b) at that time (when police ~~cat~~ caught the motorist) what's the police car's velocity?

c) how far does each vehicle travelled?



$$X_{mF} = X_{mi} + v_{mi} t + \frac{1}{2} a_m t^2 \qquad v_{mi} = 15\ m/s, \quad a_m = 0, \quad X_{mi} = 0$$

a) $X_{mF} = 0 + 15t + 0 = 15t$

$$X_{pF} = X_{pi} + v_{pi} t + \frac{1}{2} a_p t^2 \qquad v_{pi} = 0, \quad X_{pi} = 0, \quad a_p = 3\ m/s^2$$

$$X_{pF} = \frac{1}{2}(3) t^2$$

when police catches the motorist $X_{mF} = X_{pF}$

$$15\cancel{t} = \frac{3}{2} t^{\cancel{2}} \quad ; \quad \underline{t = 10\ sec} \ \checkmark$$

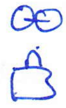
b) whats $v_{pF} = ?$ at $t = 10s$

$$v_{pF} = \cancel{v_{pi}}^{0} + at = 3(10) = 30\ m/s$$

$$v_{pF} > v_{mF}$$
$$30 > 15 = \text{const.}$$

c)

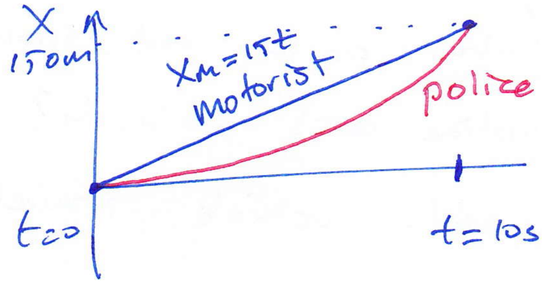
How far does each moved/travelled? $(t = 10s)$



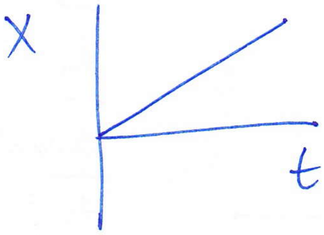
$X_{MF} = 15t$

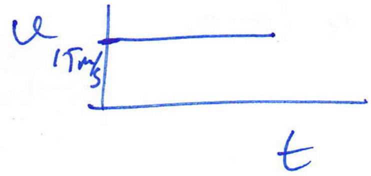
$X_{PF} = \frac{3}{2}t^2$

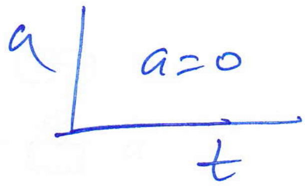
$X_{MF} = 150m$ $\quad X_{PF} = \frac{3}{2}(10)^2 = 150m$



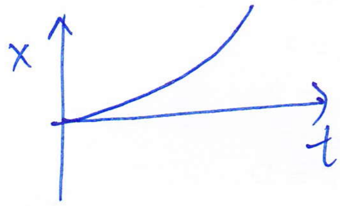
motorist

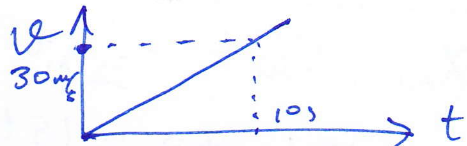


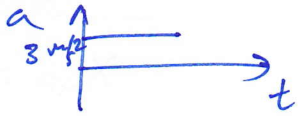




police







$v_{PF} = \cancel{v_i}^{\,0} + at$

$= 3t$

---

imagine police waited 5 seconds after the motorist passes it.

→ previous question time started synchronously / at the same time,

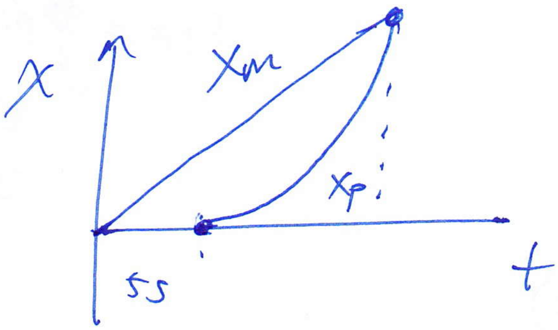
$t_p = t_m$

→ $t_p \neq t_m$

$t_p = t_m - 5$

$t_m = t$

$t_p = t - 5$



$X_m = 15t \quad X_p = \frac{3}{2}(t-5)^2$

FREELY FALLING OBJECTS

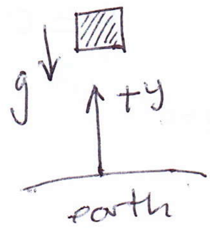
21.10.20

(1)

$x_f = x_i + v_i t + \frac{1}{2} a t^2$

$v_f = v_i + at$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$



$a = -g$

$x \rightarrow y$

$v_x \rightarrow v_y$ (in y direction)

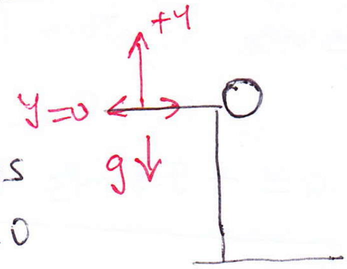
$y_f = y_i + v_i t - \frac{1}{2} g t^2$

$v_f = v_i - gt$

$v_f^2 = v_i^2 - 2g(y_f - y_i)$

$g = 9.8\ m/s^2$

Ex.



$t = 0s$

$v_i = 0$

t	y_F (m)	v_F (m/s)
1	-4.9	-9.8
2	-19.6	-19.6
3	-45.1	-29.4

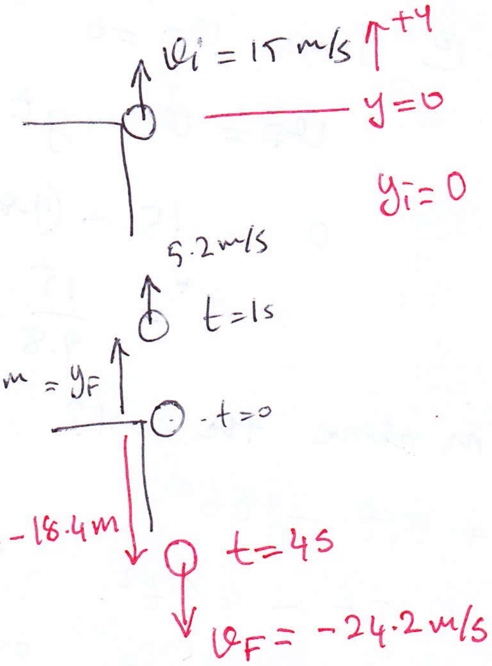
- Decide on where $y = 0$

$y_F = 0 + 0 - \frac{1}{2}(9.8)1^2$

$= -4.9m$

$v_F = 0 - (9.8)1$

Ex.



a) $y_F, v_F = ?$ @ $t = 1s$

$v_F = v_i - gt = 15 - 9.8(1) = 5.2\ m/s$

$y_F = 0 + 15(1) - \frac{9.8}{2}(1^2) = 10.1m$

b) y_F, v_F ? @ $t = 4s$

$v_F = 15 - 9.8(4) = -24.2\ m/s$

$y_F = 0 + 15(4) - \frac{9.8}{2}(4^2) = -18.4m$

c) what's the ball's velocity when its height is 5m?

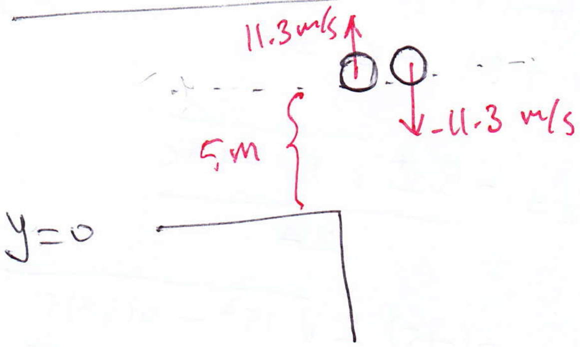
$v_F = ?$ @ $y_F = 5m$

$v_F^2 = v_i^2 - 2g(y_F - y_i)$

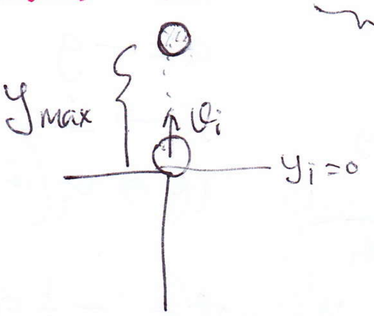
$v_F = \sqrt{15^2 - 2(9.8)[5-0]}$

$v_F = \sqrt{127} \rightarrow -11.3\ m/s$

$\rightarrow +11.3\ m/s$



d) whats ball's max height?



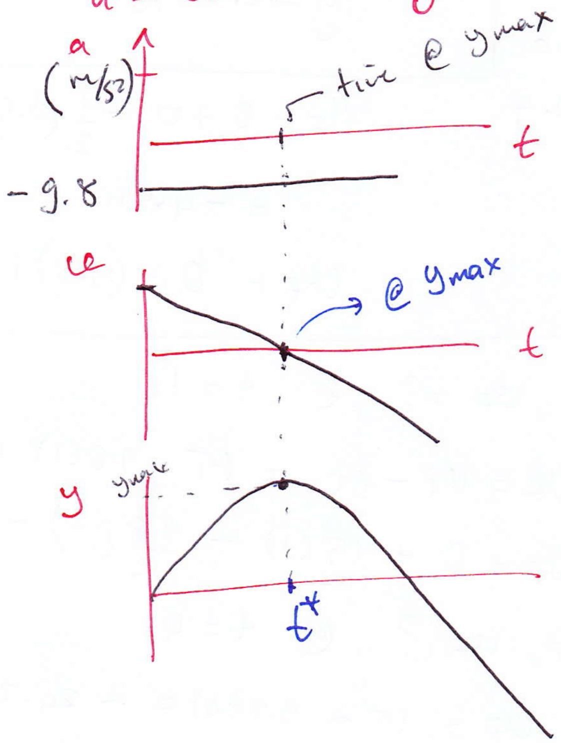
$v_f = 0$

$$v_f^2 = v_i^2 - 2g(y_F - y_i)$$
$$0^2 = 15^2 - 2(9.8)[y_{max} - 0]$$
$$y_{max} = 11.5\ m$$

e) what's the acceleration of ball @ max. height?

tricky question
since it stops @ y_{max}
$a \overset{?}{=} 0$ (wrong)
$a = -9.8\ m/s$ @ y_{max} ✓

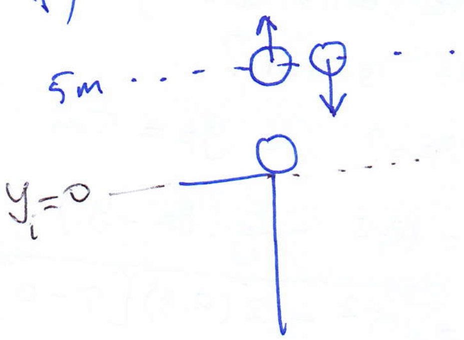
$a = \text{const} = -g$



when does it reach to max height?

@ y_{max} $v_F = 0$
$$v_F = v_i - gt$$
$$0 = 15 - (9.8)t^*$$
$$t^* = \frac{15}{9.8} = 1.53\,s$$

f) At what times the ball is 5m above the roof?

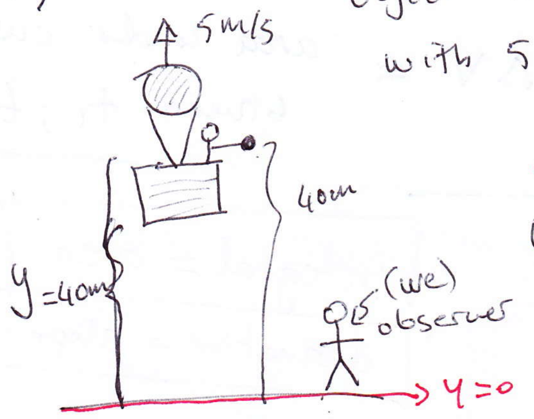


$$y_F = y_i + v_i t - \frac{1}{2} g t^2$$
$$5 = 0 + 15t - 4.9t^2$$
$$4.9t^2 - 15t + 5 = 0$$
2nd degree of poly..
$$At^2 + Bt + C = 0$$
$$t_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$t_{1,2} = \frac{-(-15) \pm \sqrt{15^2 - 4(4.9)5}}{2(4.9)}$$

$$\frac{15 \pm 11.3}{9.8}$$

0.37s ✓ 2.7s ✓

Ex) 2.44 object is released from a hot-air balloon going up with 5 m/s. → bırakılıyor



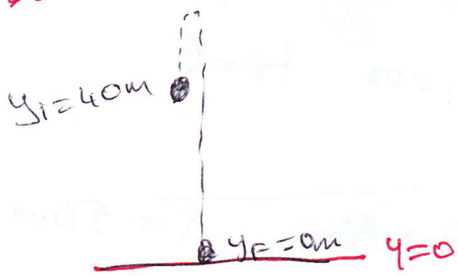
a) when does the object hit the ground?

balloon, object, the guy ↑ 5 m/s

$v_i = 5\ m/s\ (+)\uparrow$ $\quad t = ?$

$v_F = v_i - gt \qquad y_F = y_i + v_i t - \frac{1}{2} g t^2$

~~set~~ decide on where $y=0$



$$0 = 40 + 5t - 4.9t^2$$

$$At^2 + Bt + C = 0$$

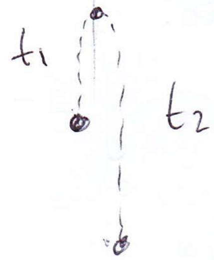
$$t = \frac{5 \pm \sqrt{809}}{9.8} = \frac{5 \pm 28.4}{9.8}$$

$t < 0$ cannot be ✓

$t \to -\cancel{2.39}\ s$

$t \to 3.41\ s$ ✓

$t = 3.41s$ total time travelled.

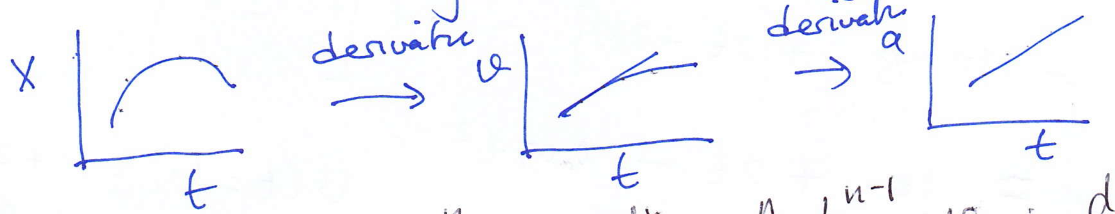


$t_1 + t_2 = 3.41s$

t_1: how long it takes reach y_{max}

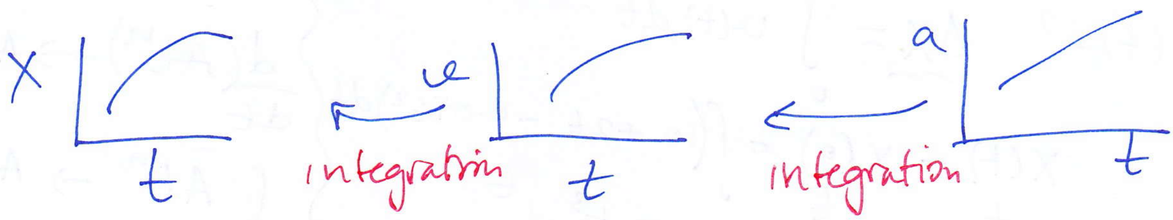
t_2: how long it takes to reach to ground from y_{max}

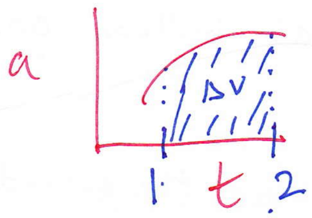
Velocity & Position by Integration



derivative = slope

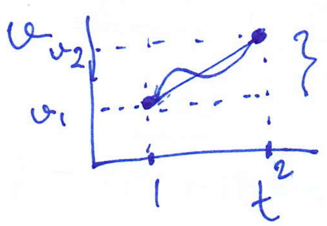
$$x = At^n \to \frac{dx}{dt} = Ant^{n-1} = v \ ; \ \frac{dv}{dt} = An(n-1)t^{n-2} = a$$





$$a(t) = f(t)$$

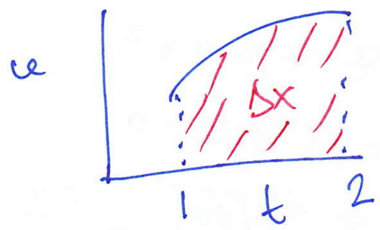
$$\int_{t_1}^{t_2} a(t)\,dt = \Delta V = \text{area under curve between } t_1 ; t_2$$



$$\} \Delta v = v_2 - v_1 = \int a(t)\,dt$$

$$\boxed{\text{integral} = \text{area}}$$

$$\boxed{\text{derivative} = \text{slope}}$$



$$\int_{t_1}^{t_2} v(t)\,dt = \Delta X = X_2 - X_1$$

$$t_1 = 0s \qquad t_2 = t$$

ex: at $t = 0s$ $\quad v = 10\,m/s$ $+x$ direction. $X = 50m$

$$a_x = 2 - 0.1t$$

a) Find v_x ; X as a function of time.

$$\Delta v = \int_{t_1}^{t_2} a(t)\,dt = \underbrace{V(t)}_{?} - V(0) = \int_0^t (2 - 0.1t)\,dt$$

$$v(t) = v(0) + \int_0^t 2\,dt - 0.1 \int_0^t t\,dt$$

$$= v(0) + 2t \Big|_0^t - 0.1 \frac{t^2}{2} \Big|_0^t$$

$$= 10\,m/s + \left[2t - 2\cancel{(0)}\right] - 0.1 \left[\frac{t^2}{2} - \cancel{\frac{0^2}{2}}\right]$$

$$\boxed{v(t) = 10 + 2t - 0.05t^2}$$

$$v(t = 1s) = 10 + 2 - 0.05(1) = 11.95\,m/s$$

$$X(t) = ? \quad \Delta X = \int_0^t v(t)\,dt$$

$$X(t) - X(0) = \int (10 + 2t - 0.05t^2)\,dt$$

$$X(t) = 50 + 10t \Big|_0^t + \frac{2t^2}{2} \Big|_0^t - 0.05 \frac{t^3}{3} \Big|_0^t$$

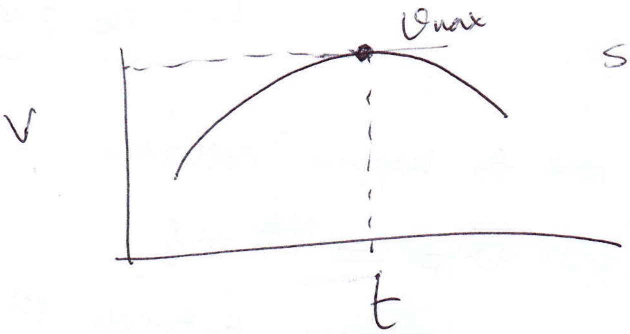
$$\left(X(t) = 50 + 10t + t^2 - \frac{0.05}{3} t^3\right)$$

$$\frac{d(At^n)}{dt} \rightarrow Ant^{n-1}$$

$$\int At^n \rightarrow A \frac{t^{n+1}}{n+1}$$

③

b) what's the value of max velocity? $v_x = \max$



slope @ v_{max} $0 = \text{slope} = \frac{dv}{dt}$

$$a = \frac{dv}{dt} = 2 - 0.1t = 0$$

$$t = \frac{2}{0.1} = 20s$$

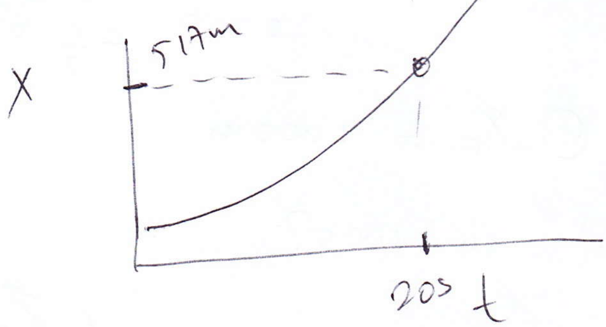
$$v_{max}(t = 20s) = 10 + 2t - 0.05t^2$$

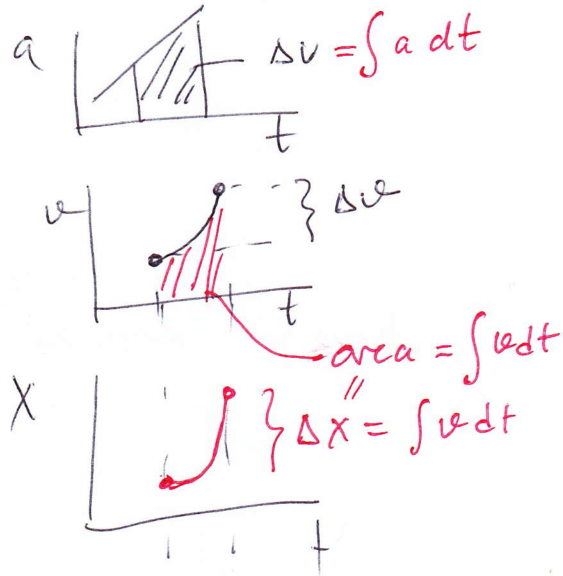
$$= 30 \text{ m/s}$$

c) where is the car when it reaches max velocity?

$$x(t = 20s) = 50 + 10(20) + 20^2 - \frac{0.05}{3}(20^3)$$

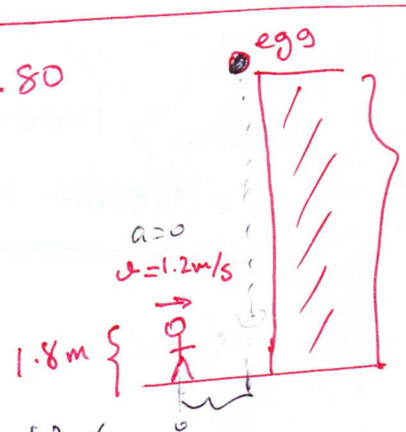
$$= \underline{517 \text{ m}}$$

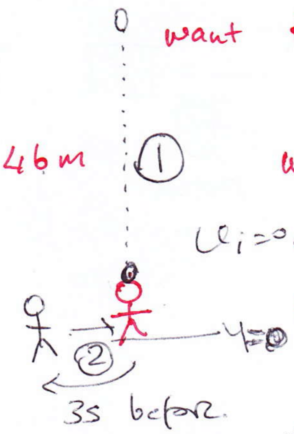




$$v(t_2) - v(t_1) = \Delta V = \int_{t_1}^{t_2} a(t)\, dt$$

2.80





want to release the egg so it hits to physics prof.

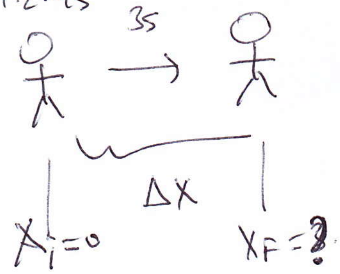
① where would prof. be when you ($v_i = 0$) release the egg?

①: how long will it take for egg to reach $y = 1.8m$? $t = ?$

$$y_F = y_i + v_i t - \frac{1}{2} g t^2$$

$$1.8 = 46 + 0 - 4.9t^2$$

$$t = \sqrt{\frac{46 - 1.8}{4.9}} = \underline{3 \text{ s}}$$



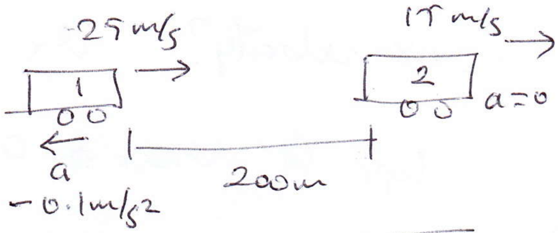
$$x_F = \cancel{x_i}^{0} + v_i t + \cancel{\frac{1}{2} a t^2}^{0}$$

$$x_F = 1.2(3)$$

$$= \underline{3.6 \text{ m}}$$

2.70 ••

•• } medium
••• } more calculation
• } easy calc.



will the 1st car hit the 2nd car?

collision?

not to happen collision

$v_{1F} = 15 \text{ m/s}$

distance between 1st – 2nd

$\Delta L \geq 0$

1st way

how long it takes 1st to slow down to 15 m/s?

$v_F = v_i + at$

$15 = 25 - (0.1)t$

$t = \frac{10}{0.1} = 100 s$

total distance travelled by 1st in 100 s?

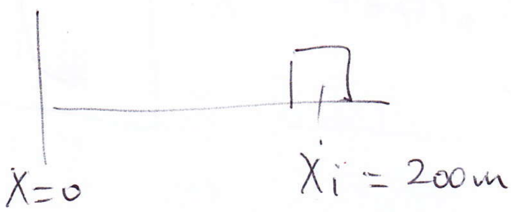
$X_F = X_i + v_i t + \frac{1}{2} a t^2$

$0 + 25(100) + \underbrace{\frac{1}{2}(-0.1)(100)^2}$

$2500 \quad -\frac{1}{2} 10^3$

(1) $X_F = 2000 m$

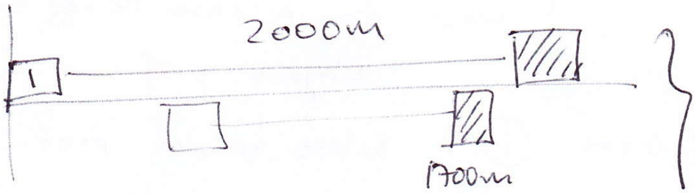
how far does the 2nd car travel in 100 s?



$X_F = X_i + v_i t + \cancel{\frac{1}{2} a t^2}^{\,0}$

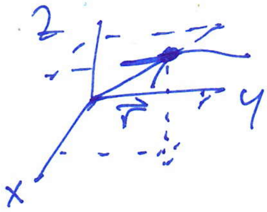
$= 200 + 15(100)$

$= 1700 m$



2000m > 1700m
collision ✓

Chapter 3 — Motion in 2d & 3d



Chapter 2 — motion in 1d

$$\begin{matrix} x & \to & \vec{r} \\ v & \to & \vec{v} \\ a & \to & \vec{a} \\ & & \text{3d} \end{matrix}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$x \Rightarrow \Delta x \Rightarrow \frac{\Delta x}{\Delta t} = \bar{v} \text{ (ave. vel.)} \qquad \vec{\bar{v}} = \frac{\Delta \vec{r}}{\Delta t} \text{ ave. vel. 3d}$$

$$\hookrightarrow \frac{dx}{dt} = v \text{ (ins. vel.)} \qquad \frac{d\vec{r}}{dt} = \vec{v} \text{ inst. vel. 3d}$$

$$\vec{v} \Rightarrow \frac{\Delta \vec{v}}{\Delta t} = \vec{\bar{a}} \text{ (ave. acc. 3d)}$$

$$\hookrightarrow \frac{d\vec{v}}{dt} = \vec{a} \text{ (inst. acc.) 3d}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

inst. vel. $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

inst. acc. $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$$\boxed{\frac{dx}{dt} = \text{derivative (polynomial func)}}$$

$$\boxed{\Delta x = x_F - x_i = x(t_F) - x(t_i)}$$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}}$$

$$\frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$\boxed{\vec{\bar{v}} = \bar{v}_x\hat{i} + \bar{v}_y\hat{j} + \bar{v}_z\hat{k}}$$

$$\vec{\bar{a}} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \frac{\Delta v_z}{\Delta t}\hat{k}$$

ave. acc. $\boxed{\vec{\bar{a}} = \bar{a}_x\hat{i} + \bar{a}_y\hat{k} + \bar{a}_z\hat{k}}$

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

inst. acc. $\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$

Phys I Ch3 ① 26.10.20

3d

position: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

velocity $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

acceleration $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

inst. vel. $\vec{v} = \frac{d\vec{r}}{dt}$

inst acc. $\vec{a} = \frac{d\vec{v}}{dt}$

Δ = last − initial

Δ = final − initial

; ave vel. $\frac{\Delta \vec{r}}{\Delta t} = \vec{v}$

ave acc. $\frac{\Delta \vec{v}}{\Delta t} = \vec{a}$

ex: coordinates of an object

$x = 2 - 0.25t^2$

$y = t + 0.025t^3$

$z = 0$

2d motn.

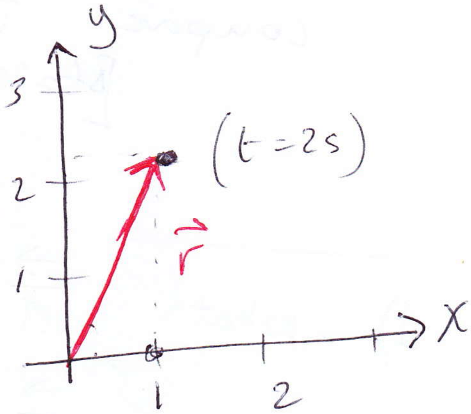
a) what's the location of the object @ $t = 2s$?

$\vec{r}(t=2s) = ?$

$\vec{r} = (2 - 0.25t^2)\hat{i} + (t + 0.025t^3)\hat{j}$

$t = 2s$ $\vec{r} = (1\hat{i} + 2.2\hat{j})$ @ $t = 2s$

$|\vec{r}| = \sqrt{1^2 + 2.2^2} = 2.4m$



b) what's $\vec{v}$ between 0s & 2s?

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$

$\Delta x = x_f - x_i = 2 - 0.25(2^2) - [2 - 0.25(0^2)]$

$= -1$

$\Delta y = 2 + \underbrace{0.025(2^3)}_{0.2} - [2 + 0.025\, 0^3]$

$= 0.2$; $\Delta t = 2 - 0 = 2$

$\vec{v} = \frac{-1}{2}\hat{i} + \frac{0.2}{2}\hat{j} = -0.5\hat{i} + 0.1\hat{j}$ m/s

$|\vec{v}| = \sqrt{0.5^2 + 0.1^2}$

$= 0.51$ m/s

direction

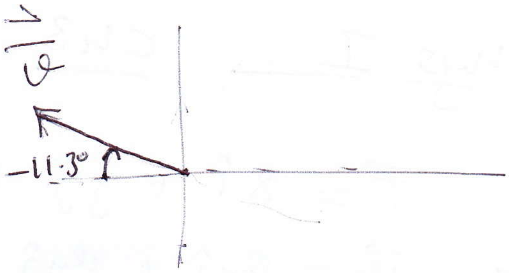
$\theta = \tan^{-1}\left(\frac{+0.1}{-0.5}\right)$

$\theta = -11.3°$

$|\vec{v}| = 0.51$ m/s

$\theta = -11.3°$

$= \frac{+0.1}{-0.5}$



---

c) $\vec{v} = ?$ $\quad \frac{d\vec{r}}{dt} = \frac{d}{dt}\left([2 - 0.25t^2]\hat{i} + [t + 0.025t^3]\hat{j}\right)$

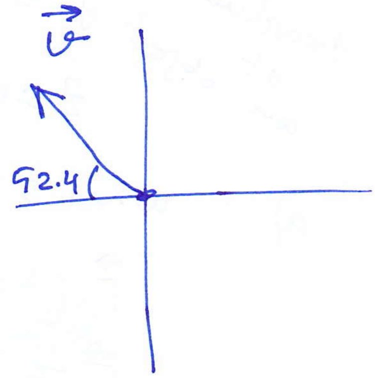
$$\vec{v} = -0.5t\,\hat{i} + [1 + 0.075t^2]\hat{j}$$

what's $\vec{v} = ?$ @ $t = 2s$

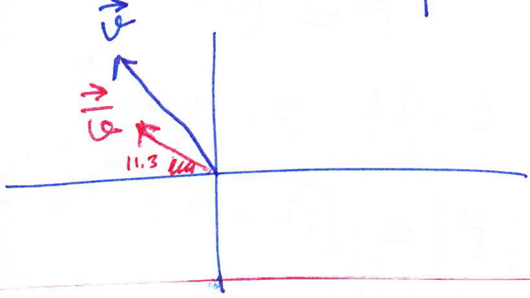
$(t = 2s)\ \vec{v} = [-1\,\hat{i} + 1.3\,\hat{j}]$ m/s

$|\vec{v}| = \sqrt{1^2 + 1.3^2} = 1.64$ m/s

$\theta = \tan^{-1}\left(\frac{+1.3}{-1}\right) = -52.4°$



compare $\vec{v}_{[\Delta t = 2s - 0s]}$ and $\vec{v}_{t=2s}$



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d) what's $\vec{a}\,[t = 2s, 0s] = ?$ $\quad \vec{a}\ ;\ \bar{a}$

ave acc.

$$\bar{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t=2s) - \vec{v}(t=0s)}{2 - 0}$$

$\vec{v} = -0.5t\,\hat{i} + [1 + 0.075t^2]\hat{j}$

$\vec{v}(t=2) = -1\,\hat{i} + [1.3]\hat{j}$

$\vec{v}(t=0) = 0\,\hat{i} + 1\,\hat{j}$

$\frac{\Delta\vec{v}}{\Delta t} = \frac{-1\hat{i} + 1.3\hat{j} - [1\hat{j}]}{2} = [-0.5\,\hat{i} + 0.15\,\hat{j}]$ m/s²

$|\vec{a}| = 0.52$ m/s²

$\theta = -16.7°$

$$\theta = \tan^{-1}\left(\frac{+0.15}{-0.5}\right) = \underline{-16.7^\circ} \quad ; |\vec{a}| = 0.52 \text{ m/s}^2$$ (2)

---

e) $\vec{a}(t=2s) = ?$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[-0.5t\,\hat{i} + (1+0.075t^2)\hat{j}\right]$$

$$\vec{a} = \left[-0.5\,\hat{i} + 0.15t\,\hat{j}\right]$$

$$\vec{a}(t=2s) = \left[-0.5\,\hat{i} + 0.3\,\hat{j}\right] \text{ m/s}^2$$

$$|\vec{a}| = \sqrt{(-0.5)^2 + 0.3^2} = 0.58 \text{ m/s}^2$$

$$\theta = \arctan\left(\frac{+0.3}{-0.5}\right) = \underline{-31^\circ}$$

$\vec{v}$; $\vec{a}$; Δt ; $t_f ; t_i$

$\vec{v}$, $\vec{a}$, $\underline{t = 2s}$

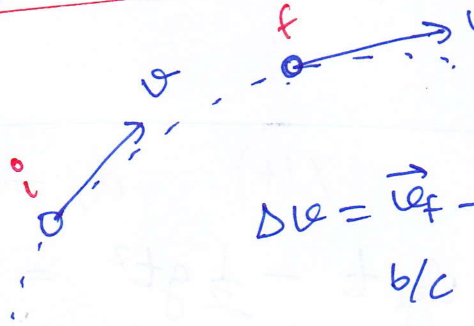
$\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$

$\vec{r} \xrightarrow{\frac{d}{dt}} \vec{v} \xrightarrow{\frac{d}{dt}} \vec{a}$; $\dfrac{\Delta \vec{r}}{\Delta t} = \vec{v}$

If an object is moving on a curved path; its acceleration is always NON ZERO!! $a \neq 0$

---

even if $v = \text{const.}$



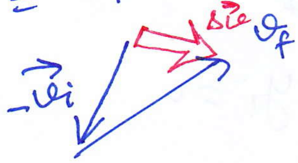
$$\frac{\Delta \vec{v}}{\Delta t} = \vec{a}$$

$$\Delta v = \vec{v}_f - \vec{v}_i \neq 0$$

b/c direction of v_f ; v_i are different

$$v_f = v_i = v$$

$$\vec{\Delta v} = \vec{v}_f - \vec{v}_i$$



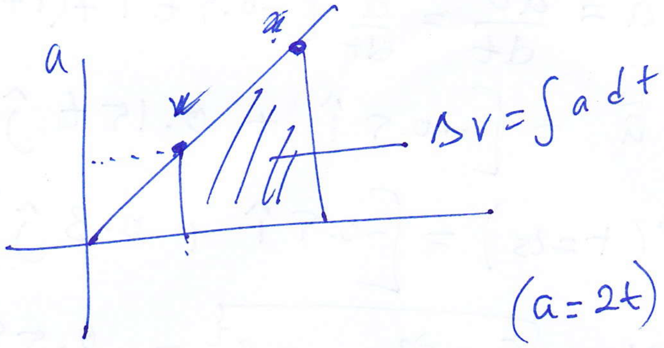
$$\bar{a} = \frac{\Delta v}{\Delta t} \neq 0$$

uniform circular motion
↳ ($v = \text{const}$)
$a \neq 0$

/ nonuniform circular motion
$v \neq \text{const}$ $a \neq 0$

$r \rightarrow v \rightarrow a$ turev:

int



$(a = 2t)$

$$a = \frac{dv}{dt}$$

$$\int a\,dt = \int_i^f dv = \Delta v \Big|_i^f$$

$2t\,dt$

$$2\frac{t^2}{2}\Big|_{t_i}^{t_f}$$

$$\int 2t\,dt$$

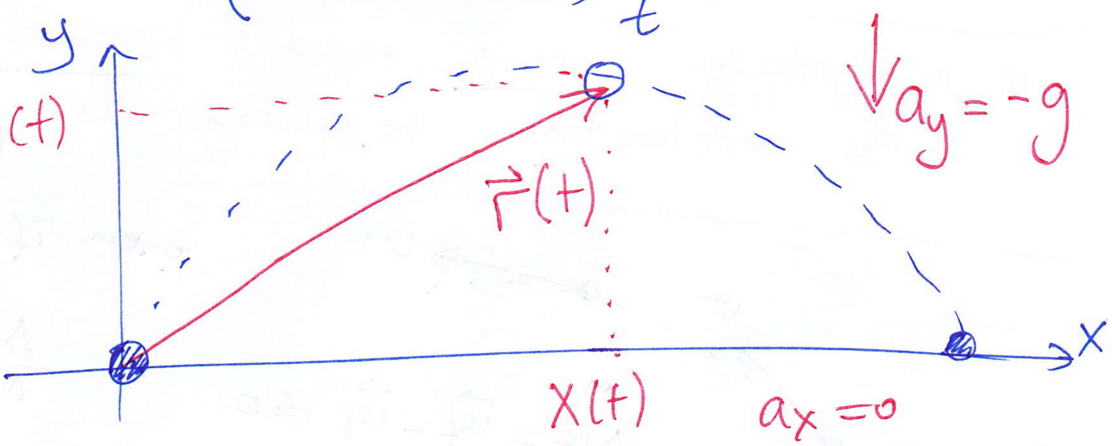
$$2\frac{t^2}{2} + c = \Delta v = v_f - v_i$$

$$\left[t_f^2 - t_i^2\right] = v_f - v_i$$

sometimes constants can be given in v_i ; v_f values.

PROJECTILE MOTION (EGIK ATIS)

(2d)



$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = \cancel{a_x}\hat{i} + a_y\hat{j}$$

0

$$\vec{a} = -g\hat{j}$$

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2 = v_{yi}t - \frac{1}{2}gt^2$$

$$x_f = x_i + v_{xi}t + \cancel{\frac{1}{2}a_x t^2} = v_{xi}t$$

$$x_i = 0 \;;\; y_i = 0$$

$$v_{yf} = v_{yi} - gt = \frac{d}{dt}(y_f)$$

inst. velocity in y direction.

$$= \frac{d}{dt}\left(v_{yi}t - \frac{1}{2}gt^2\right)$$

$$\underline{\underline{v_{xf}}} = \frac{dx}{dt} = \frac{d}{dt}(v_{xi}\, t) = \underline{\underline{v_{xi}}}$$

velocity ; (speed) is const. along X direction

$$\boxed{v_{xf} = v_{xi} = v_x} \quad a_x = 0 \quad t_i = 0,\ t_f = t$$

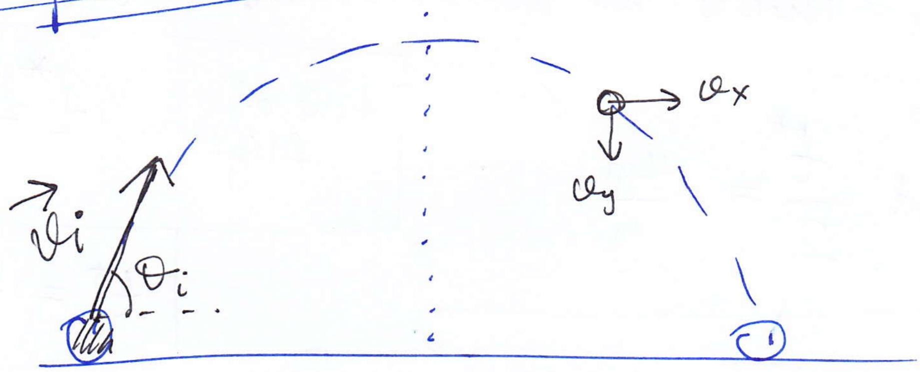
(4) $$\boxed{x = v_x t} \quad x_i = 0,\ x_f = x$$

y ; $\hat{j}$ direction

$$v_{yf} = v_{yi} - gt \quad (1)$$

$$y_f = v_{yi} t - \frac{1}{2} g t^2 + y_i \quad (2)$$

$$v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i) \quad (3)$$



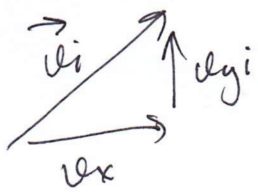
- How high does it go? $y_{max} = ?$

- How long does it stay in air? $t_{max} = ?$

- How far does it go in x direction
Range (menzil) =?

$\boxed{\vec{v}_i \,;\, \theta \text{ must be given.}}$

$$\vec{v}_i = v_x \hat{i} + v_{yi} \hat{j}$$



$v_x = v_i \cos\theta$

$v_{yi} = v_i \sin\theta$

How high does it go?

@ y max $\quad v_{yf} = 0$

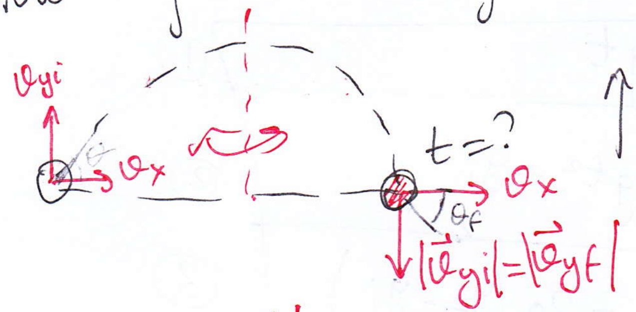
$$y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$$

$$v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i)$$

$$0 = v_{yi}^2 - 2g(y_{max} - 0)$$

$$\boxed{y_{max} = \frac{v_{yi}^2}{2g} = \frac{(v_i \sin\theta)^2}{2g}}$$

- How long does it stay in air?



symetric motion
b/c there is no air friction!

$$v_{yf} = v_{yi} - g t$$

$$-v_{yi} = v_{yi} - g t$$

time to max height $\quad \boxed{\frac{t^*}{2} = \frac{v_{yi}}{g}}$

time of flight $\quad \boxed{t^* = \frac{2 v_{yi}}{g}}$

$$t^* = 2 \frac{v_i \sin\theta}{g}$$

Range of motion ; Xmax = ?

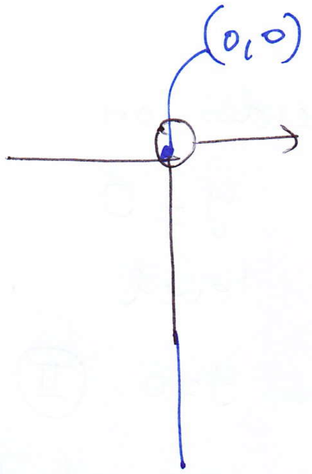
$$X = v_x t^* = \text{Range}$$

$$R = v_x \frac{2 v_{yi}}{g}$$

$$= v_i \cos\theta \frac{2 v_i \sin\theta}{g}$$

$$\boxed{R = 2 v_i^2 \frac{\cos\theta \sin\theta}{g}}$$

ex:



$v_{yi} = 0$

location ? @ t = 2s

$x_f = ?$ $x_i = 0$

$y_f = ?$ $y_i = 0$

$$y_f = \cancel{y_i}^{0} + \cancel{v_{yi} t}^{0} - \frac{1}{2} g t^2 = -\frac{9.8(4)}{2}$$

$$x_f = v_x t = 9(2) = 18\,m$$

$$y_f = -19.6\,m$$

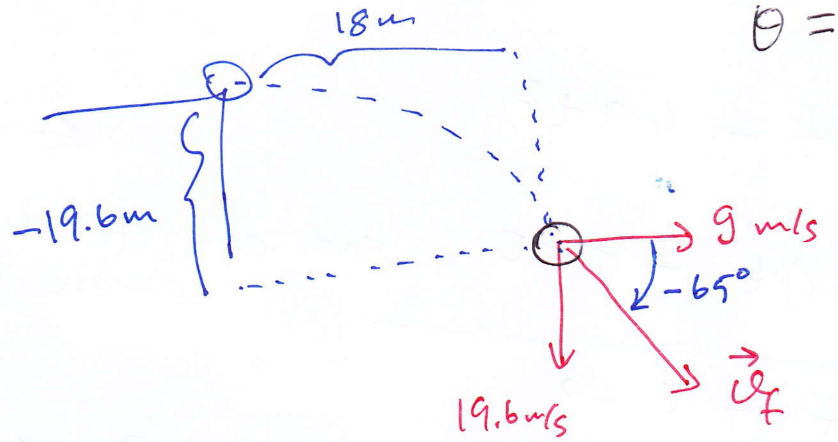
~~v_{yf}=?~~ b) $\vec{v}(t = 2s) = ?$

$$v_{yf} = \cancel{v_{yi}}^{0} - g t = -9.8(2) = -19.6\,m/s$$

$$\vec{v} = v_x \hat{i} + v_{yf} \hat{j} = [9\hat{i} - 19.6\hat{j}]\ m/s$$

$$|\vec{v}| = \sqrt{9^2 + 19.6^2} = \underline{21.6\,m/s}$$

$$\theta = \tan^{-1}\left(\frac{-19.6}{9}\right) = -65°$$



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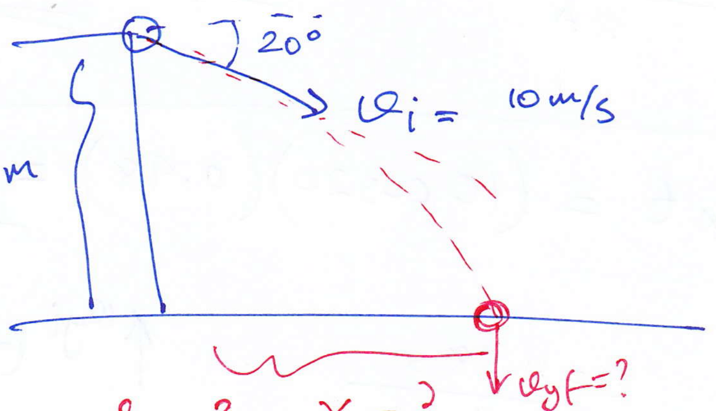
ex:



How far horizontally will the ball hit the ground

$$X = v_x t = (v_i \cos\theta)\, t$$

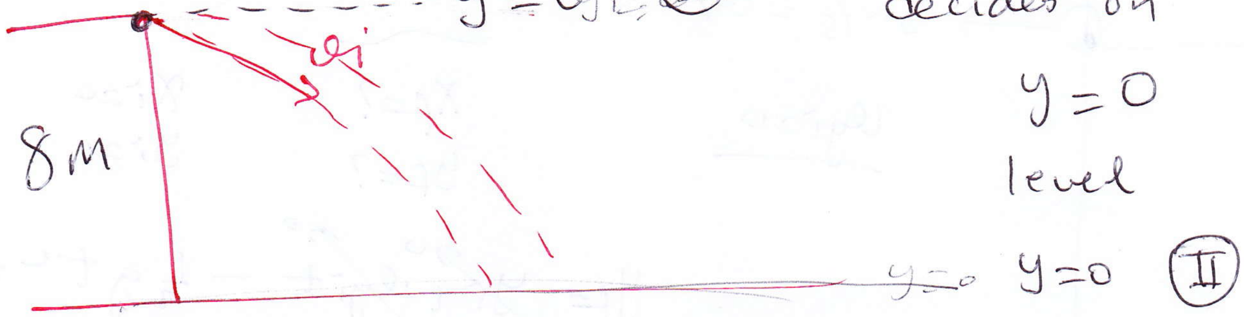
?

t = how long will it stay in air?

~~$v_{yi} - g t = v_{yf}$~~

$$y_i + v_{yi} t - \frac{1}{2} g t^2 = y_f$$

$v_{yi} = v_i \sin\theta$



decides on $y = 0$ level

(I) $y_i = 0$ $\quad y_f = -8m \Rightarrow y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$ $\quad$ +y ↑ ↓g

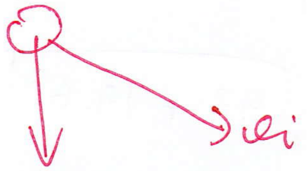
$-8 = 0 + (-10\sin 20)t - \frac{9.8}{2}t^2$

or

(II) $y_i = 8m$ $\quad y_f = 0$

$v_{yi} \begin{cases} \to (+) \\ \to (-) \end{cases}$?

+y



$v_{yi} = v_i \sin\theta = -10\sin 20 = -3.42$ m/s

$-8 = (-3.42)t - 4.9t^2$

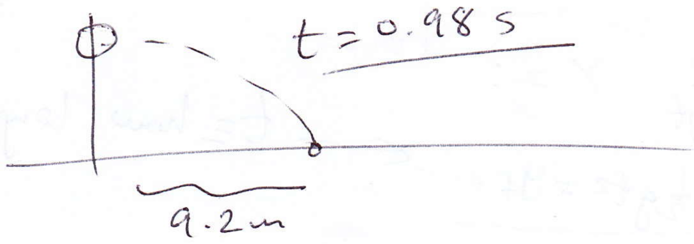
$\boxed{4.9t^2 + (3.42)t - 8 = 0}$ same equ if you choose (II)

$At^2 + Bt + C = 0$

$t_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ $\begin{cases} \to \cancel{-1.7}\ s \\ \to +0.98\ s \ \checkmark \end{cases}$

$X = v_x t = (10\cos 20)(0.98) = 9.2m$

+y ↑ ↓g



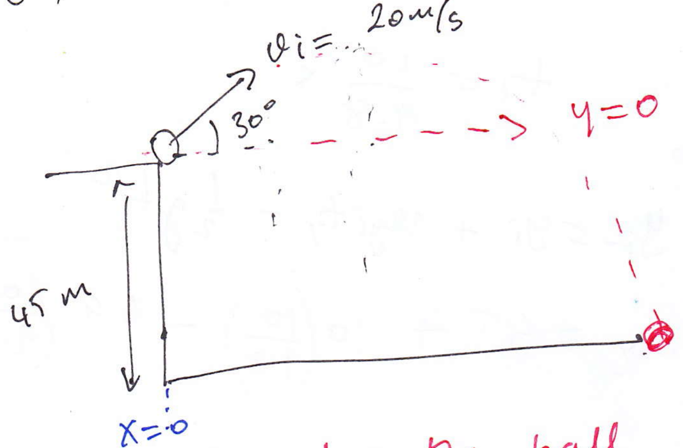
↑v_{yi} (+)

↓v_{yi} (−)

projectile motion

① 02. 11. 2020

ex:)



a) whats the time of flight? (havada kalma süresi)

$v_{iy} = v_{yi} = v_i \sin\theta$

$= 20 \frac{1}{2} = 10 m/s$

$y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$

$-45 = 0 + 10t - 4.9t^2$

$4.9t^2 - 10t - 45 = 0$

$t_{1,2} = \frac{+10 \pm \sqrt{10^2 + 4(45)(4.9)}}{2(4.9)}$

$t_1 = t_+ = \underline{4.22 s}$ ✓

$t_- = \underline{\underline{t_2 < 0}}$ X

b) How far does the ball go in x direction?

Range = ?

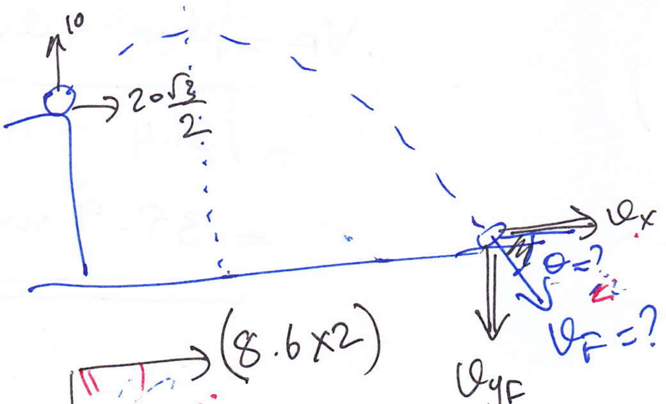
$x_f = x_i + v_x t$

$R = x_f - 0 = (v_i \cos\theta) t$

$= 20 \frac{\sqrt{3}}{2} (4.22)$

$= \underline{73.1 m}$ //

c) whats the final velocity just before it hits the ground?



$v_F = ?$ magnitude

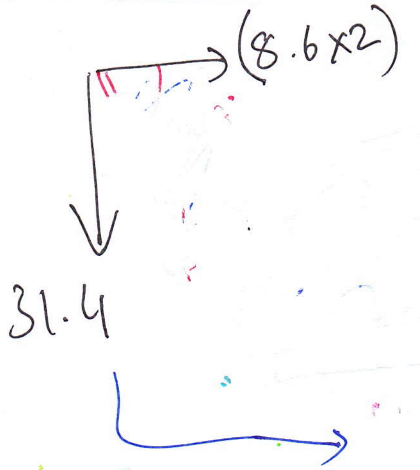
$\theta = ?$ direction

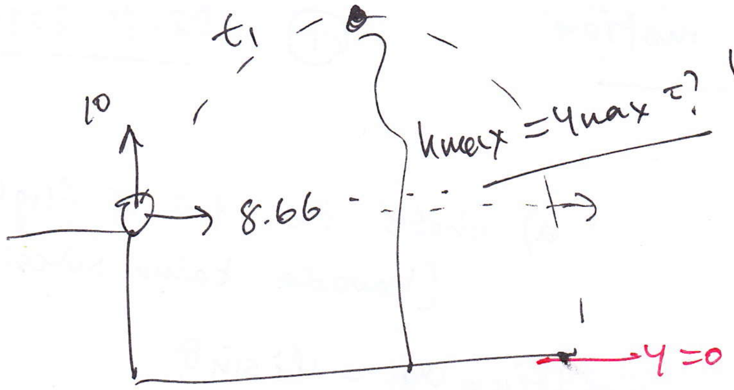
$\vec{v}_F = \vec{v}_{yF} + \vec{v}_x$

$v_{yF} = v_{yi} - gt$

$= 10 - (9.8)(4.22)$

$v_{yF} = -31.4 m/s$





$$v_{yF} = v_{yi} - gt$$

$$0 = 10 - 9.8(t_1)$$

$$t_1 = \frac{10}{9.8} \text{ s}$$

$$y_F = y_i + v_{yi} t_1 - \frac{1}{2} g t_1^2$$

$$y_{max} = 45 + 10\left(\frac{10}{9.8}\right) - 4.9\left(\frac{10}{9.8}\right)^2$$

$$=$$

II nd way

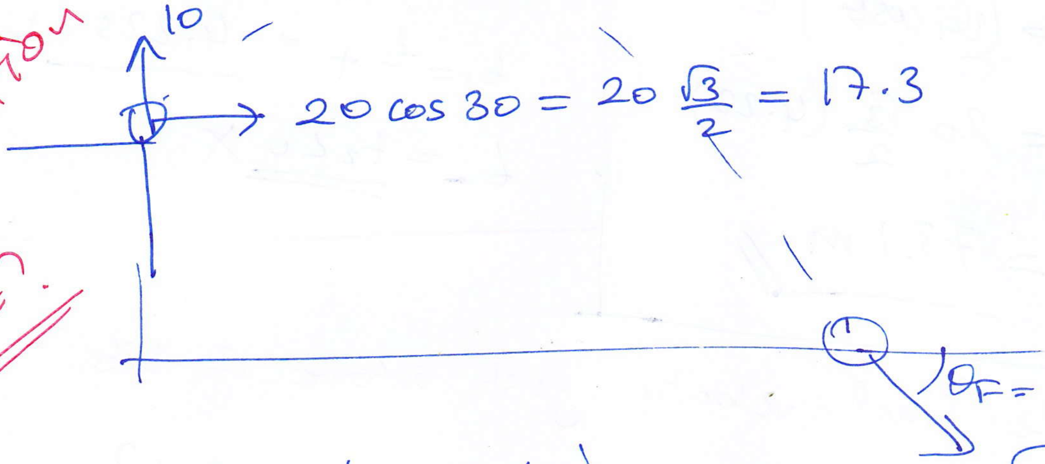
$$v_{yF}^2 = v_{yi}^2 - 2g(y_F - y_i)$$

$$0 = 10^2 - 2(9.8)(y_{max} - 45)$$

$$45 + \frac{100}{19.6} = y_{max}$$

correction on

$\theta_F = ?$



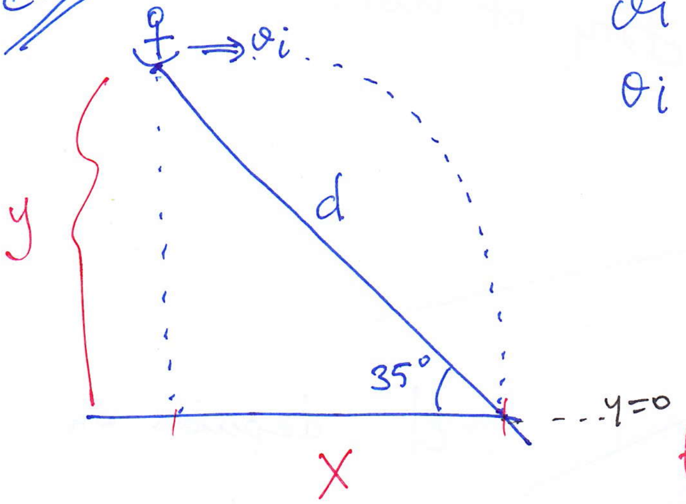
$$20 \cos 30 = 20 \frac{\sqrt{3}}{2} = 17.3$$

$$V_F = \sqrt{v_{yF}^2 + v_x^2} = \sqrt{31.4^2 + 17.3^2} = 35.9 \text{ m/s}$$

$$\theta_F = \tan^{-1}\left(\frac{-31.4}{17.3}\right)$$

$$\theta_F = -61.1^\circ$$

ex:



$v_i = 25$ m/s } horizontal velocity.
$\theta_i = 0$

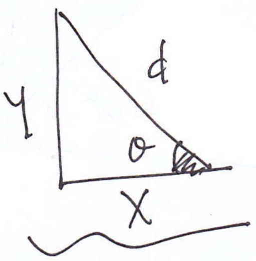
where does the sportsmen / skier land? d = ?

time of fligh is $\underline{t}$

$v_x = v_i$

$$\boxed{X = v_x t = 25t}$$

$$y_F = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$
$$0 = y + 0(t) - 4.9t^2$$
$$\boxed{y = 4.9t^2}$$



$$d = \sqrt{x^2 + y^2} = \frac{X}{\cos\theta}$$

$$\tan 35 = \tan\theta = \frac{y}{x} = \frac{4.9t^{\cancel{2}}}{25\cancel{t}} = 0.7$$

$$t = \frac{0.7(25)}{4.9}$$

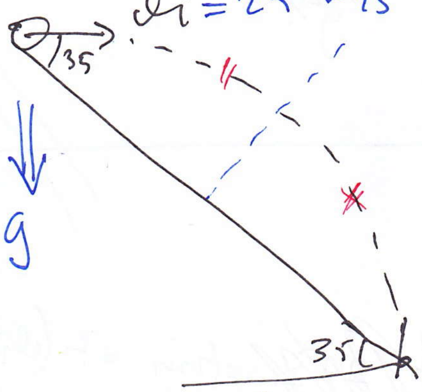
$$\boxed{t = 3.57\,s}$$

$$X = 25(3.57) = 89.3\ m$$

$$y = 4.9(3.57)^2$$
$$y = 62.5\ m$$

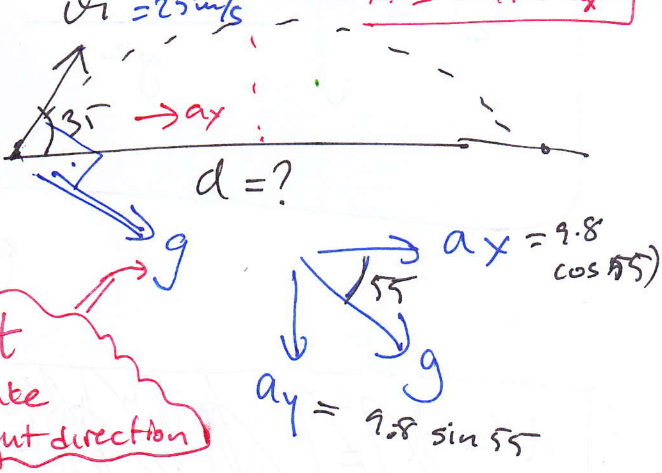
$$d = \sqrt{89.3^2 + 62.5^2}$$

$$d = 109.0\ m$$



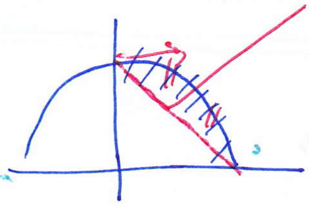
Rotate

$\neq$



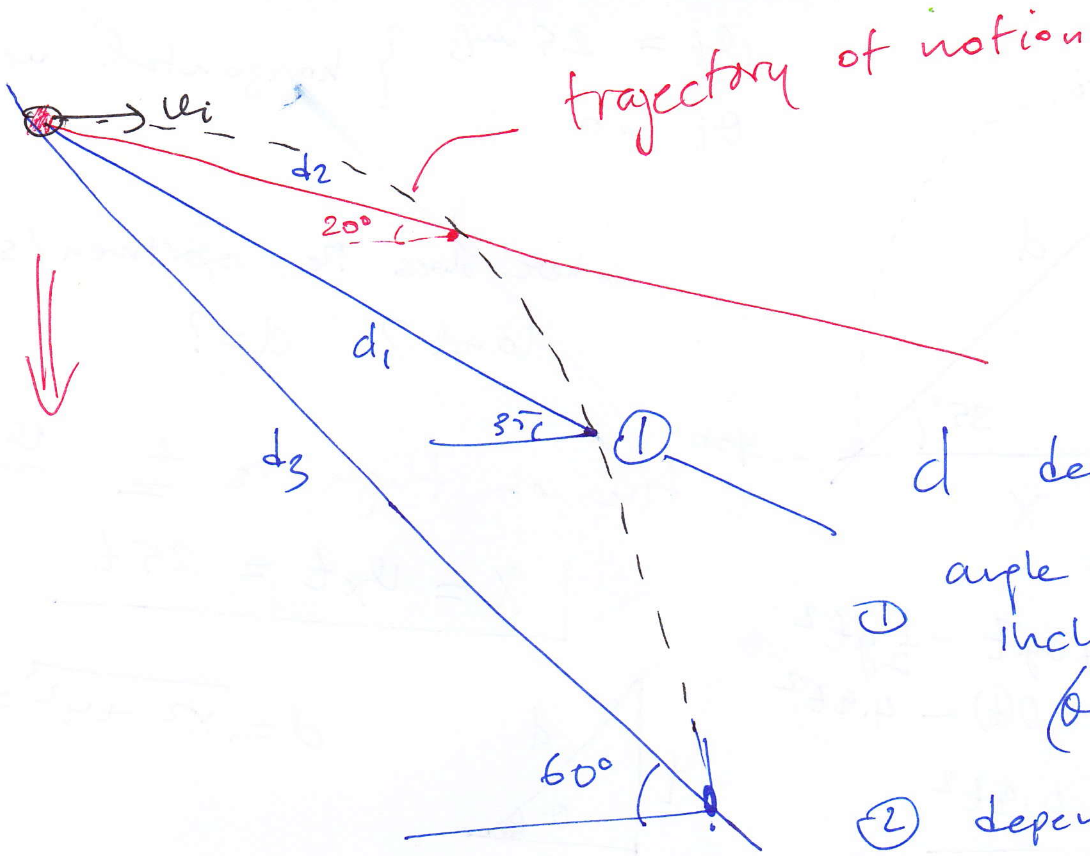
$\boxed{v_{xF} = v_{xi} + a_x t}$

Not equivalent unless you take g in the right direction



$$\text{Range} = \frac{2\,v_i^2 \sin 35 \cos 35}{(9.8)} = 59.8\ m$$

Range formula won't work here!!



d depends on

1. angle of inclined plane ($\theta = 35°$)
2. depends on v_i

$$d = \sqrt{x^2 + y^2}$$
$$= \sqrt{(v_i t)^2 + \left(\tfrac{1}{2} g t^2\right)^2}$$

$$\tan\theta = \frac{y}{x} = \frac{4.9t^2}{} = \frac{\frac{1}{2}gt^2}{v_i t}$$

$$t = \frac{2v_i \tan\theta}{g}$$

$$d = \sqrt{\frac{4 v_i^4 \tan^2\theta}{g^2} + \frac{1}{\cancel{4}} \cancel{g^2} \frac{\cancel{16}4 v_i^4 \tan^4\theta}{g^{2\cancel{4}}}}$$

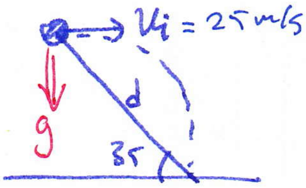
θ, v_i //

$$\boxed{d = \frac{2v_i^2 \tan\theta}{g}\sqrt{1+\tan^2\theta}}$$

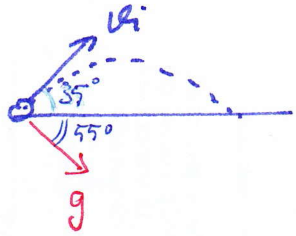
d is a function of (v_i, θ)!

$$d = \frac{2(25)^2 \tan 35}{9.8}\sqrt{1+[\tan(35)]^2} = 109\ \text{m}$$

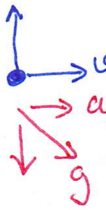
Mahir Cem Aşkın'ın çözümü



rotate



$v_i \sin 35 = 14.34\,m/s = v_{yi}$



$v_i \cos 35 = 20.48\,m/s = v_{xi}$

$a_x = g\cos 55 = 5.62\,m/s^2$

$a_y = g \sin 55 = 8.03\,m/s^2$

time of flight $= t = 2\dfrac{v_{yi}}{a_y} = 3.57s = \dfrac{2(14.34)}{8.03}$

Range $\quad x_F = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

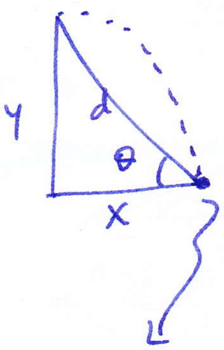
$= 0 + (20.48)(3.57) + \frac{1}{2}(5.62)(3.57)^2 = 108.93 \simeq 109\ m$

* **Not:** Bu çözümde x ve y yönünde ivme var onun için x yönündeki a_x ivmesini uygun şekilde kullanmak gerekiyor.

---

genel bir soru

Bu sorudaki çözümümüzde



$\tan\theta = \dfrac{y}{x}$ kullanmıştık

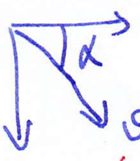
$\tan\theta \neq \dfrac{v_y}{v_x}$ eşit değildir; böyle yazamayız.

çünkü $v_{yF} = v_{yi} - gt = (-9.8)t$

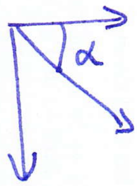
$v_x = v_i = 25\ m/s$

topun yüzeye değdiği noktadaki

$\alpha \neq 35°$

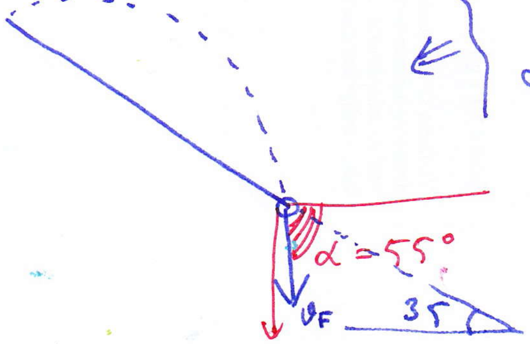


$v_{yF} = (9.8)(3.57) = 34.99 \simeq 35$



$\alpha = \tan^{-1}\left(\dfrac{-35}{25}\right)$

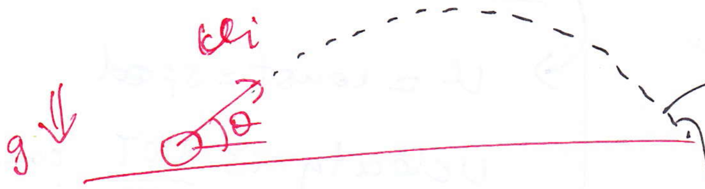
$\alpha = -54.5 \simeq -55°$



$$\boxed{\alpha = 55° \neq 35° = \theta}$$

projectile motion

3



trajectory (yörünge)
(kütlenin izi)

parabola

$y(t) = y_i + v_{yi}t - \frac{1}{2}gt^2$

$y(x) = ?$ Function of trajectory in terms of X.

$$y(t) = y_i + v_{yi}\frac{x}{v_x} - \frac{1}{2}g\frac{x^2}{v_x^2} \qquad x = v_x t$$

$$t = \frac{x}{v_x}$$

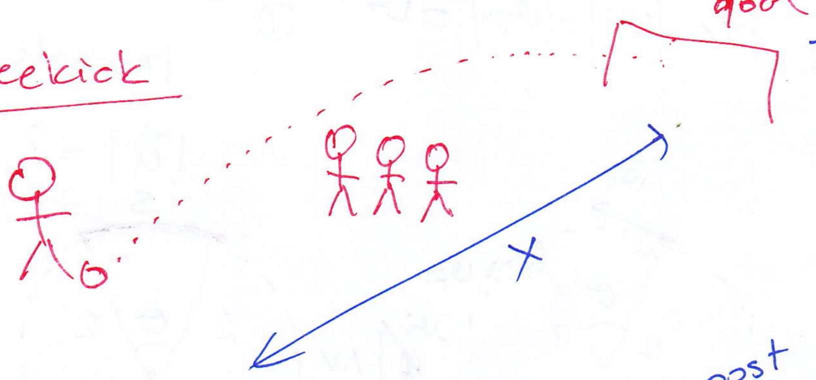
$\Downarrow$
x

$$y(x) = y_i + \cancel{v_i}\sin\theta\frac{x}{\cancel{v_i}\cos\theta} - \frac{g\,x^2}{2\,v_i^2\cos^2\theta}$$

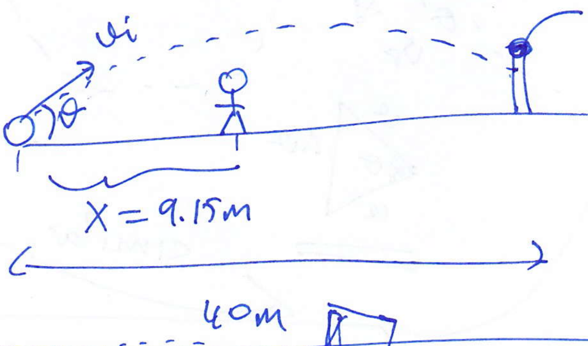
$$\boxed{y(x) = y_i + \tan\theta\; x - \frac{g}{2v_i^2\cos^2\theta}\,x^2}$$

$$y(x) = A + Bx + Cx^2$$

Freekick

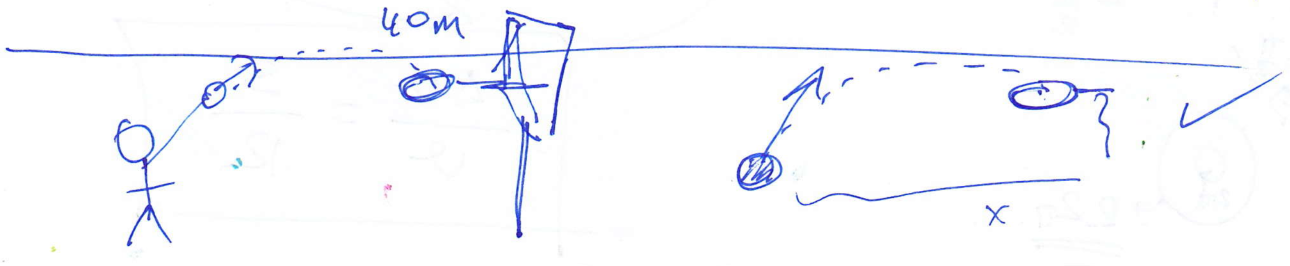


$x = v_x t$
$y_F = y_i + v_{yi}t - \frac{1}{2}gt^2$
$v_{yF} = v_{yi} - gt$
$v_{yF}^2 = v_{yi}^2 - 2g(\Delta y)$



$y(x = 9.15m) >$ opponents height

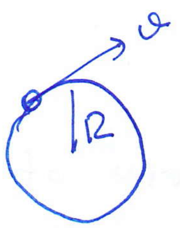
$y(x = 40m) <$ height of top post



motion in 2d & 3d

↳ projectile motion ✓

↳ circular motion ✓



Uniform circular motion

→ $v = \text{const} = \text{speed}$

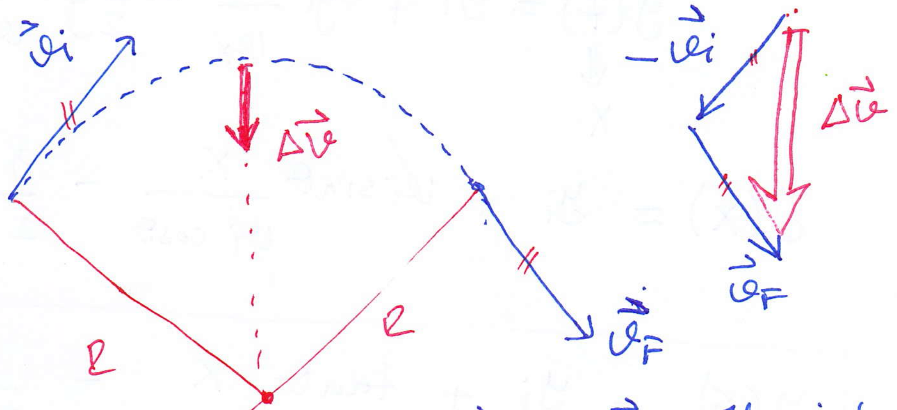
velocity is **NOT** const.

→ radius const.

$$\boxed{@ \text{ UCM} \quad v = \text{const} \quad R = \text{const}}$$

acceleration of UCM?

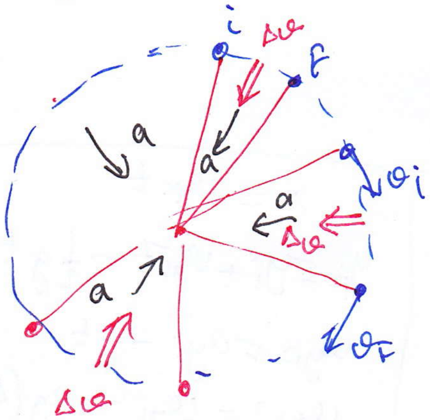
$a \overset{?}{=} \frac{v^2}{R}$



average acceleration

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_F - \vec{v}_i}{\Delta t}$$

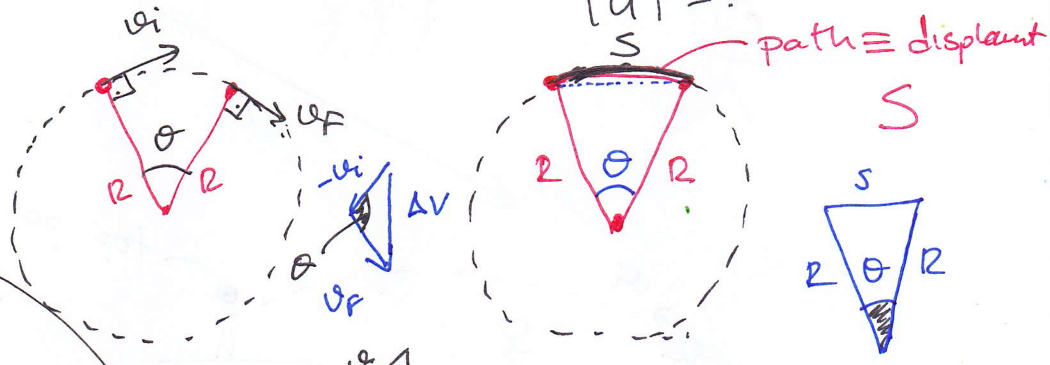
$\Delta\vec{v}$ or $\vec{a}$ will point towards center.



b/c $|\vec{v}_i| = |\vec{v}_F| = v$

① direction of a towards center ✓

$|\vec{a}| = ?$

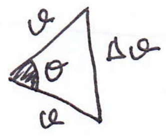


S = arclength

= yayuzunluğu

$$\boxed{S = R\theta}$$

↑ radian



similar

$$\boxed{\frac{\Delta v}{v} = \frac{S}{R}}$$

→ $R2\pi$

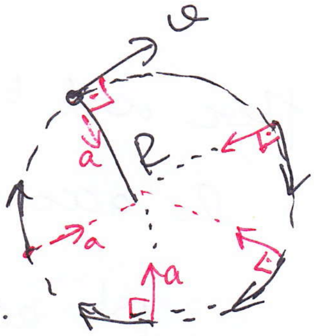
$$\frac{\Delta v}{v} = \frac{s}{R} \; ; \quad a = \frac{\Delta v}{\Delta t} = \frac{v \frac{s}{R}}{\Delta t} = \frac{v}{R} \frac{s}{\Delta t} \; v$$

$$\Delta v = v \frac{s}{R}$$

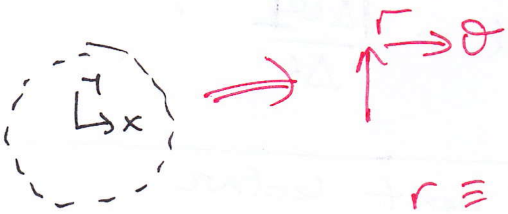
$$\boxed{a = \frac{v^2}{R}}$$

$$\text{velocity} = \frac{s}{\Delta t} = \frac{\text{displacement}}{\text{time}}$$

$$\boxed{a \text{ towards center} \; ; \quad a = \frac{v^2}{R} \quad \text{UCM}}$$



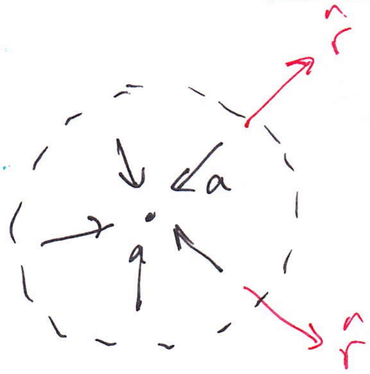
we dont want to write in x, y coordinates.



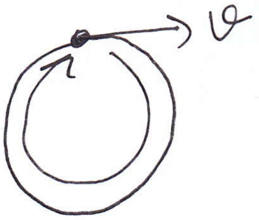
changes where you are
↳ on your location

r ≡ direction from center to out

θ ≡ " of rotation



$$\boxed{\vec{a} = a(-\hat{r}) = \frac{v^2}{R} \, -\hat{r}} \quad \text{toward the center}$$



1 revolution ≡ 1 turn

$$X = v t$$

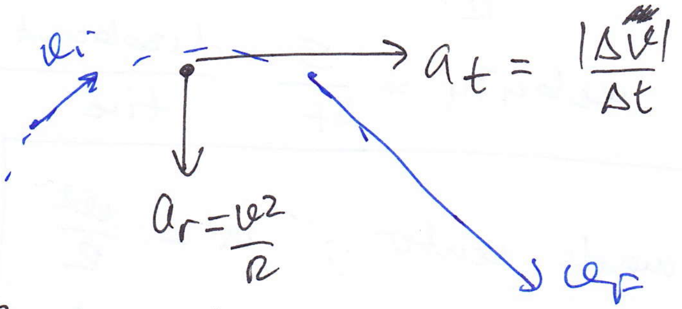
$$2\pi R = v T$$

T: period
time it takes for one revolution.

$$a = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2} \quad (\text{UCM})$$

non uniform circular motion.

$v \neq$ const



there will be
2 acceleration

1st acc = $a_{radial} = \frac{v^2}{R}$ (like uniform circular motion)

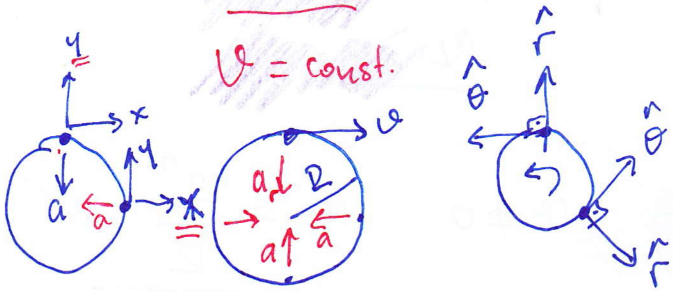
$a_{tangential} = \frac{|\Delta v|}{\Delta t}$

more on next lecture

① 04.11.20

Uniform circular motion.

$v = \text{const.}$



non-uniform circular motion.

$v \neq \text{const.}$ → speed up / → slow down

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \to a_{radial}, \quad \to a_{tan} \equiv \text{tangential}$$

$$a_r = a_{rad} = \frac{v^2}{R} \equiv \text{merkezcil ivme}$$

$$\boxed{\vec{a}_r = \frac{v^2}{R}(-\hat{r})}$$

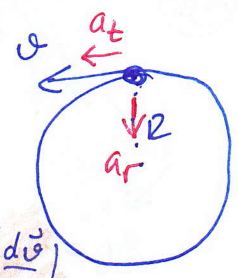
$x, y \to r, \theta$

because its more convenient!

$v \neq \text{const}$

$$a_{rad} = \frac{v^2}{R}$$

$$a_{tan} = \frac{d|\vec{v}|}{dt} = \left|\frac{d\vec{v}}{dt}\right|$$

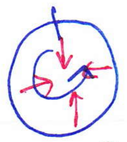


$\vec{a}_r \perp \vec{v}$; $\vec{a}_t \parallel \vec{v}$

T = period ; time it take for 1 rotation

UCM

$$a_r = \frac{v^2}{R}$$



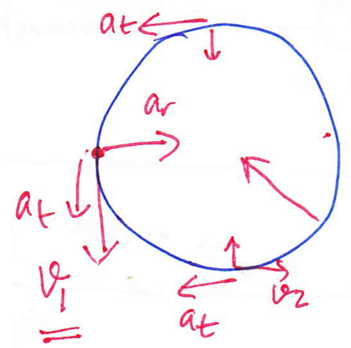
$$a_r = 4\pi^2 \frac{R}{T^2} \qquad \frac{2\pi R}{T} = v$$

If $R = 5$ m ; 1 rotation takes 4 sec.

$$v = \frac{2\pi R}{T} = \frac{2(3.14)5}{4} \text{ m/s}$$

$$a_r = \frac{v^2}{R} \checkmark$$

NUCM



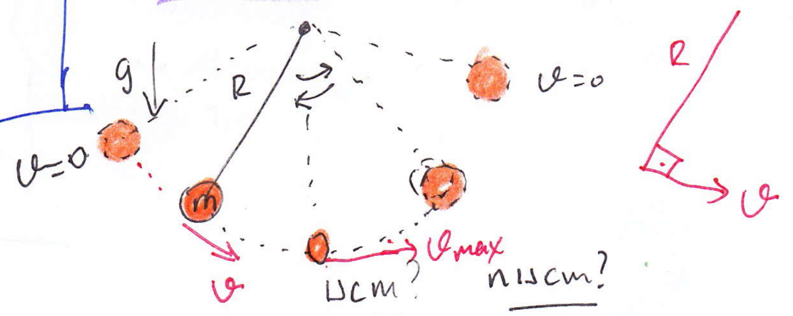
$$a_r = \frac{v^2}{R}$$

$$a_t = \left|\frac{d\vec{v}}{dt}\right|$$

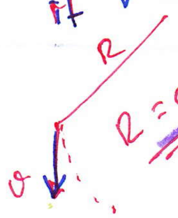
$$\vec{a} = a_r(-\hat{r}) + a_t\,\hat{\theta}$$

$$\vec{a} = \frac{v^2}{R}(-\hat{r}) + \left|\frac{d\vec{v}}{dt}\right|\hat{\theta}$$

Pendulum = Sarkaç

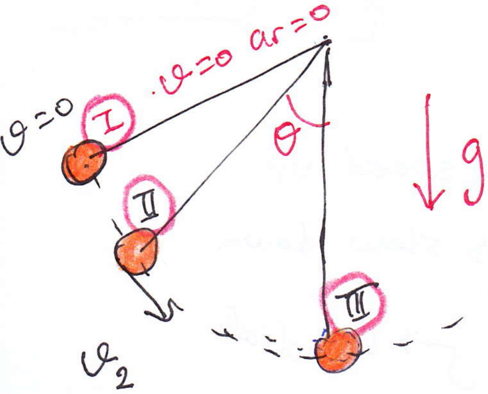


If v is not perpendicular to R



R = changes!

this is asked rarely!



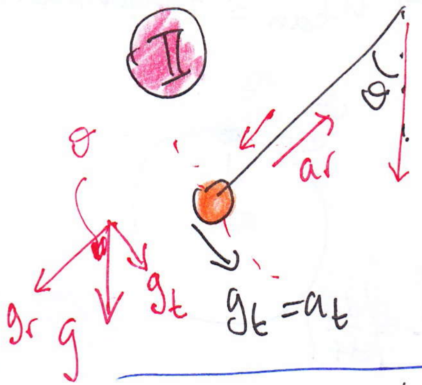
I $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ b/c it's speeding up.

$\underline{a_t > 0}$

II $v \neq 0$ $\underline{a_r = \frac{v_2^2}{R}}$ ✓

$a_t = ?$

II



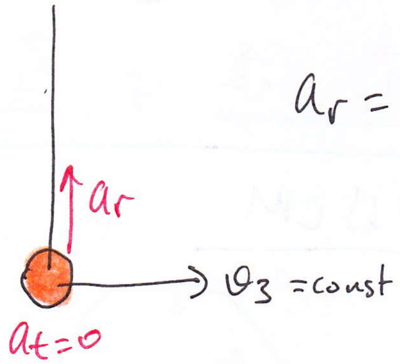
$g_r = g\cos\theta$; $R = \text{const.}$

string is attached.

$g_t = g\sin\theta$

$\underline{\underline{a_t}} = g_t = g\sin\theta = \left|\frac{d\vec{v}}{dt}\right|$

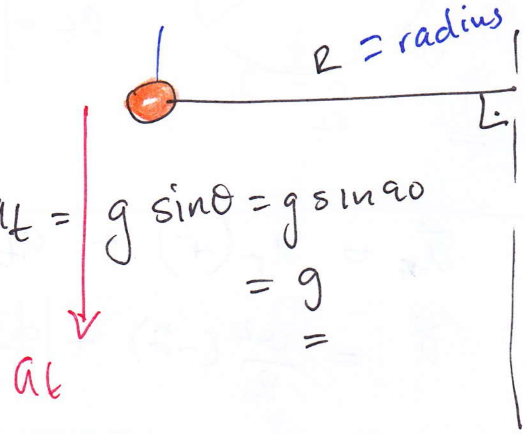
III



$a_r = \frac{v_3^2}{R}$ ✓

$a_t = g\sin\theta = g\sin 0 = \underline{\underline{0}}$

IV

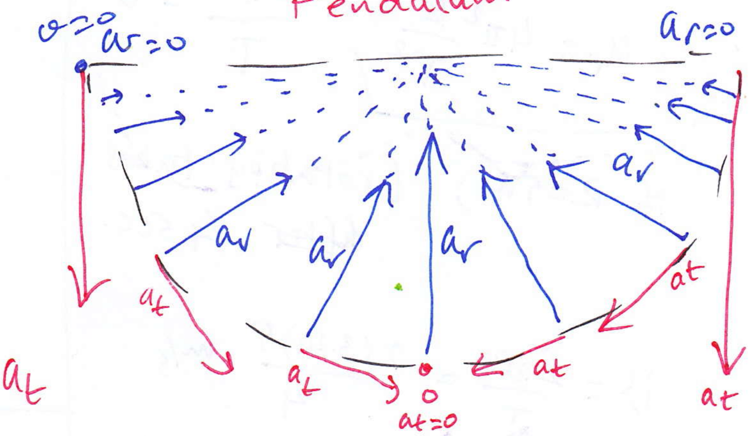


$a_t = g\sin\theta = g\sin 90$

$= g$

$=$

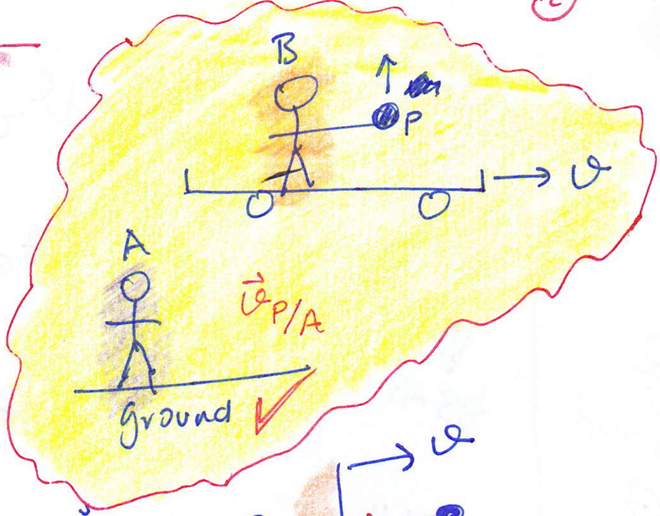
Pendulum



a

pendulum with no air resistance
$$\begin{cases} a_t = g\sin\theta \\ a_r = \frac{v^2}{R} \end{cases}$$

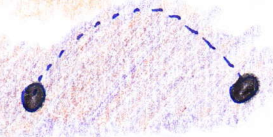
Relative motion
(Bağıl hareket)
göreceli

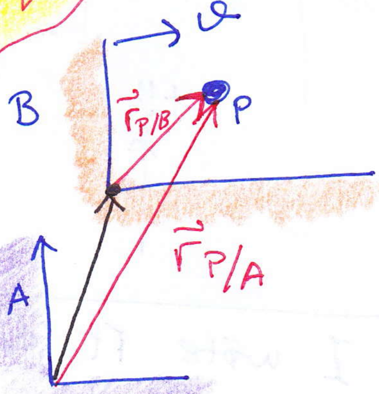


observe / measure motion

what B sees.

what A sees

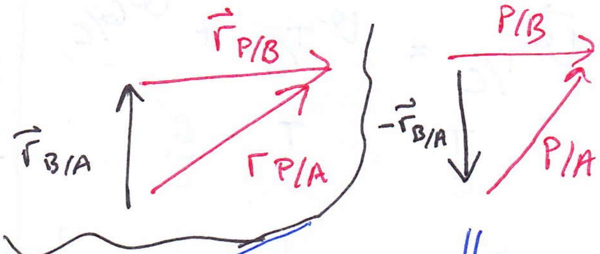




$\vec{r}_{P/B}$: point P with respect to B

$\vec{r}_{P/A}$: P w.r.t. A

$\vec{r}_{B/A}$: B w.r.t. A



$$\vec{r}_{B/A} + \vec{r}_{P/B} = \vec{r}_{P/A}$$

$$-\vec{r}_{B/A} + \vec{r}_{P/A} = \vec{r}_{P/B}$$

position ***
$$\boxed{\vec{r}_{P/B} + \vec{r}_{B/A} = \vec{r}_{P/A}}$$

same!

$$\boxed{\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A}}$$

$$\frac{P}{\cancel{B}} \quad \frac{\cancel{B}}{A} = \frac{P}{A}$$

$\frac{d\vec{r}}{dt} = \vec{v}$

*

B

A

$$-\vec{r}_{B/A} = +\vec{r}_{A/B}$$

– B w.r.t A equals to + B w.r.t. A

$\frac{d}{dt}$

***
$$\boxed{\vec{v}_{P/B} + \vec{v}_{B/A} = \vec{v}_{P/A}}$$

most of the time

$v_{B/A} = \text{const.}$

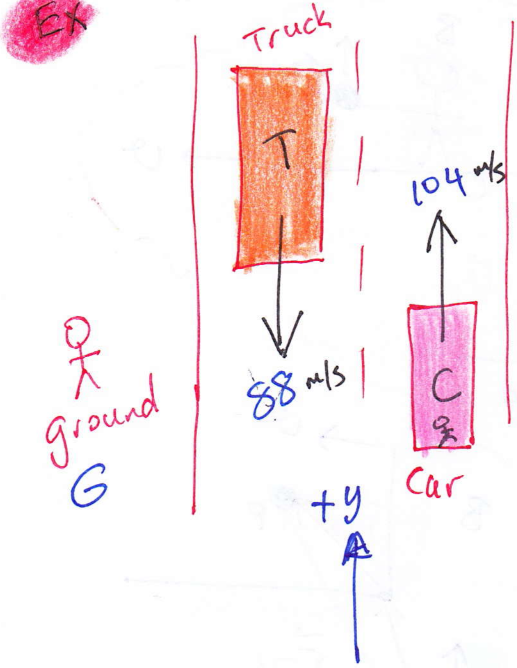
$\frac{d}{dt}$

$$\boxed{\vec{a}_{P/B} + \vec{a}_{B/A} = \vec{a}_{P/A}}$$

$a_{B/A} = 0$

$$\boxed{\vec{a}_{P/B} = \vec{a}_{P/A}}$$ ***

Ex



$v_{Truck/G} = 88$ km/hr (truck driver sees on speedometer)

$v_{car/G} = 104$ km/hr

velocity of truck w.r.t. car?

$$\vec{r}_{T/G} = \vec{r}_{T/C} + \vec{r}_{C/G}$$

$$\vec{v}_{T/G} = \underset{?}{\vec{v}_{T/C}} + \vec{v}_{C/G}$$

$$88(-\hat{j}) = \vec{v}_{T/C} + 104(+\hat{j})$$

$$192(-\hat{j})\ m/s = \vec{v}_{T/C}$$

$\downarrow \vec{v}_{T/C}$

2nd way

if I wrote it like this

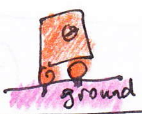
$$\vec{v}_{T/C} = \vec{v}_{T/G} + \vec{v}_{G/C}$$

$$\frac{T}{C} = \frac{T}{\cancel{G}} \ \frac{\cancel{G}}{C}$$

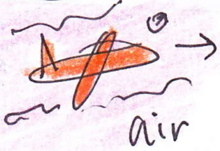
$$\vec{v}_{T/C} = \underset{88}{\downarrow} + \underset{104}{\downarrow} = \underset{192\ m/s}{\downarrow}$$

✓ same answer!!

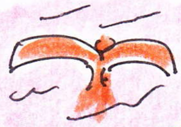
Ex.) A plane's compass (pusula) shows North direction.
Its speed is 240 km/hr.
If there is a wind from West to East with a speed of 100 km/hr.
what is the velocity of plane w.r.t. earth?

car  $v_{car/ground}$

ship  $v_{ship/water}$

 $v_{plane/air}$

$v_{plane/wind}$

 $v_{bird/air}$

air ≃ wind

$$\boxed{\text{If wind} = 0 \Rightarrow \text{air} \quad \text{air} = \text{ground}}$$

- speed of car is w.r.t. ground
- speed of ship is w.r.t. water
- speed of plane is w.r.t. air

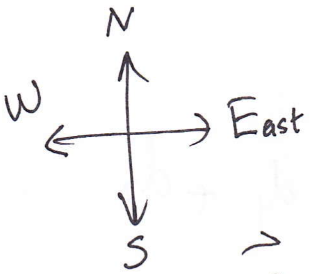
what their speedometer reads!

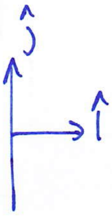
$v_{P/W} = 240$ km/hr direction of plane

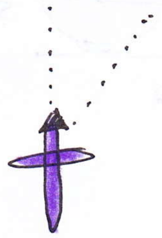
↑ North compass w.r.t earth

3

$v_{W/Earth = ground} = 100$ km/hr direction wind West → East

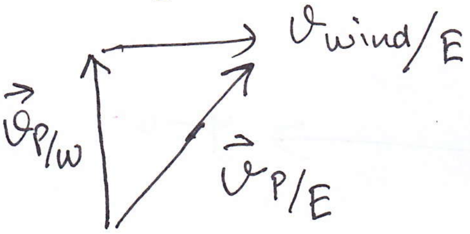


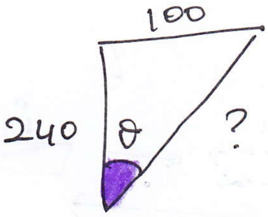




$$\vec{v}_{P/E} = \vec{v}_{P/W} + \vec{v}_{W/E}$$

?



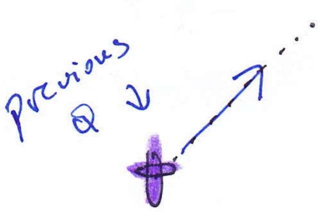


$$|\vec{v}_{P/E}| = \sqrt{100^2 + 240^2}$$
$$= 260 \text{ km/hr} \equiv \text{speed}$$

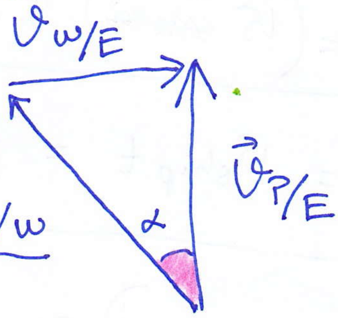
direction of $\vec{v}_{P/E}$ =? $\tan\theta = \frac{100}{240}$; $\theta = \tan^{-1}\left(\frac{100}{240}\right) = 22.6°$ $\cong 23°$

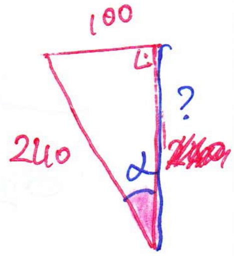
⇒ same question: $v_{P/W} = 240$ km/hr ; $v_{W/E} = 100$

what would be the direction of plane so that it flies towards North?



Burnunu ne kadar ne nereye doğru çevirmeli ki North (kuzeye) doğru uçsun?





$$v_{P/E} = \sqrt{240^2 - 100^2}$$
$$= 218 \text{ km/hr}$$

$240? = \vec{v}_{P/W}$

$\alpha \neq \theta$

24.6° 22.6°

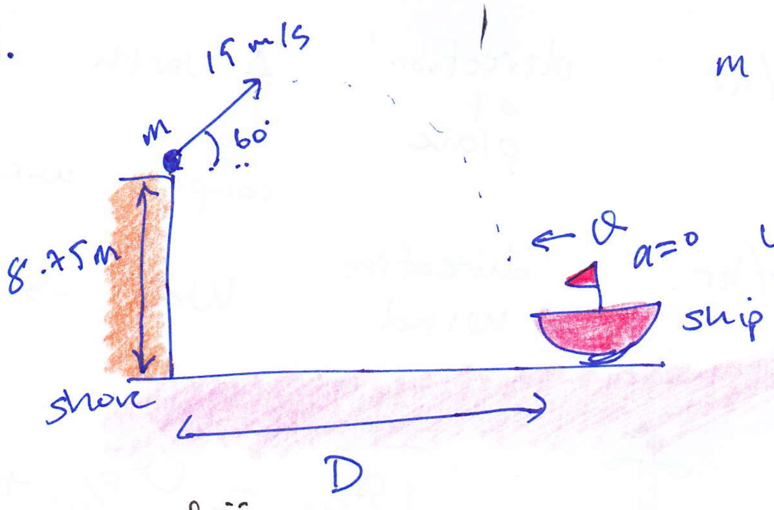
$\sin\alpha = \frac{100}{240}$

$\alpha = \sin^{-1}\left(\frac{100}{240}\right) = 24.6°$ $\cong 25°$

End of chapter 3

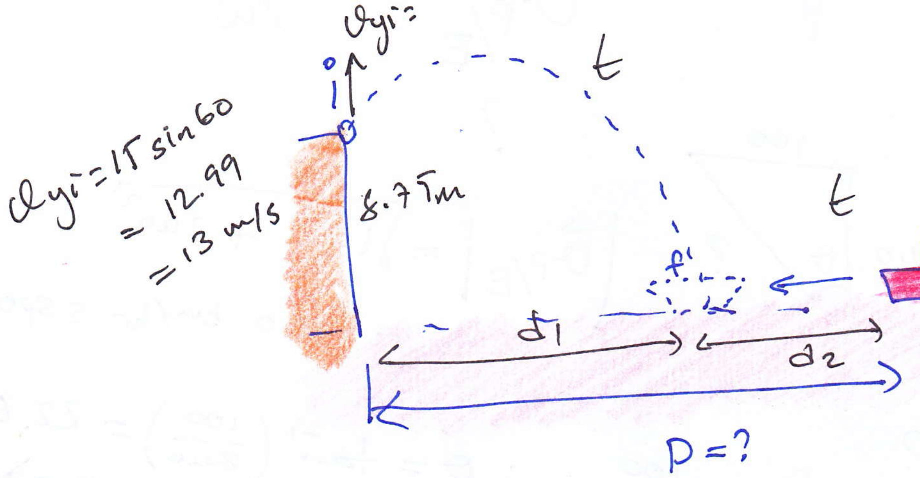
3.56) ...

m to be land on the ship deck.



$v_{ship} = 45\ cm/s$

$D = ?$



$D = d_1 + d_2$

$\longrightarrow y = 0$

$y_F = y_i + v_{yi} t - \frac{1}{2} g t^2$

$0 = 8.75 + 13t - 4.9t^2$; $\dfrac{-13 \pm \sqrt{13^2 - 4(8.75)(-4.9)}}{-2(4.9)}$

$\underline{\underline{t = 3.21 s}} \leftarrow \dfrac{-13 \pm 18.5}{-9.8}$

$d_1 = v_x t$

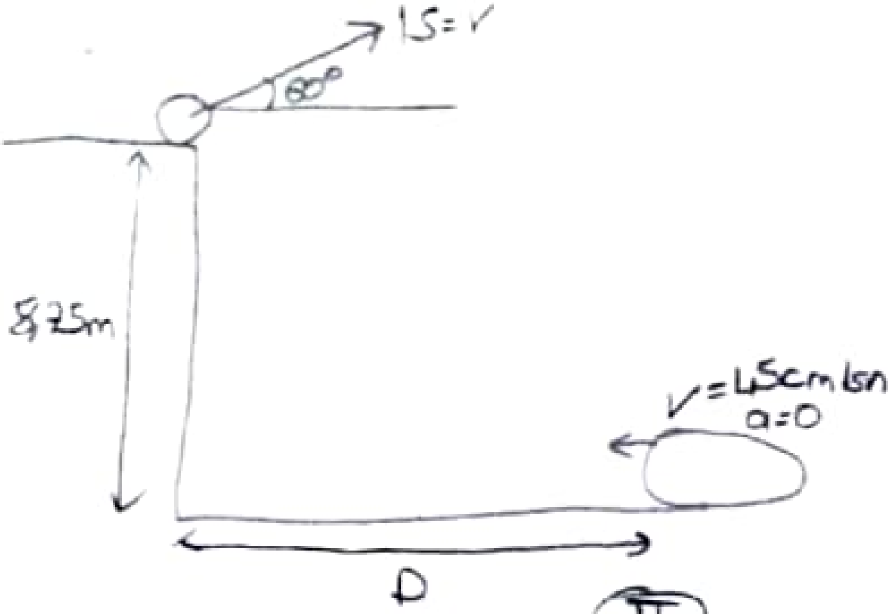
$= (15 \cos 60)\ 3.21 = 24.08 m$

$d_2 = v_{ship} t = (45\ cm/s)(3.21 s) = 1.44 m$

$D = d_1 + d_2 = \underline{25.52 m}$

Ersin Er

Aydoğmuş'un çözümü grafik ile ↓



$V_x = 15 \cdot \cos 60 = 7{,}5 \text{ m/s}$ (I)

$V_y = 15 \cdot \sin 60 = 13 \text{ m/s}$

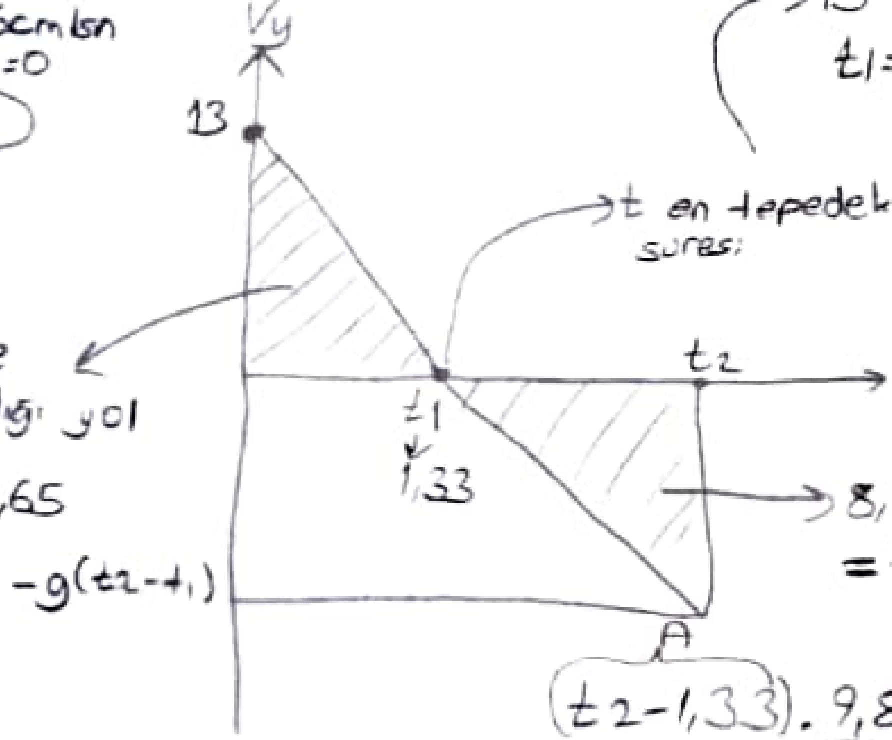
$13 - 9{,}8\, t_1 = 0$

$t_1 = 1{,}33$ sn

(II)

Alan = en tepeye çıkarken aldığı yol

$\frac{13 \cdot 1{,}33}{2} = 8{,}65$



t en tepedeki süresi

$8{,}75 + 8{,}65$

$= 17{,}4$

(III)

$\underbrace{(t_2 - 1{,}33)}_{A} \cdot \frac{9{,}8}{2} \cdot \underbrace{(t_2 - 1{,}33)}_{A} = 17{,}4$

$A^2 \cdot 4{,}9 = 17{,}4$

$A^2 = 3{,}56$

$A = 1{,}89$

$t_2 - 1{,}33 = 1{,}89$

$t_2 = 3{,}22$

(IV)

Kayığın yatayda aldığı yol $\Leftarrow 45 \cdot 3{,}22 = 144{,}9$ cm

$\Downarrow$

$1{,}45 \text{ m} = d_1$

Cismin yatayda aldığı yol. $\leftarrow V_x \cdot t_2 = 7{,}5 \cdot 3{,}22 = 24{,}15 \text{ m} = d_2$

$d_1 + d_2 = 24{,}15 + 1{,}45$

25,6 m