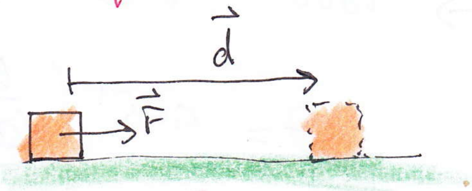
07.12.20

1

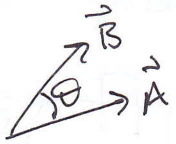
Chapter 6 Work & Kinetic Energy

Chapter 7 Potential Energy.



$\text{Work} = \vec{F} \cdot \vec{d}$ — dot product, scalar.

$\vec{A} \cdot \vec{B} = AB\cos\theta$

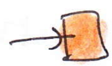


$= F d \cos\theta$

$\text{Work} = F d$

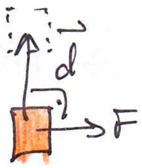
$\text{work} = [N\,m = \text{Joule}]$

if F, or $d \Rightarrow 0$ $\quad W = 0$



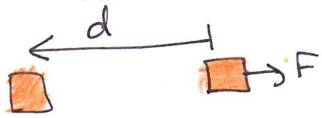
$= \vec{F} \cdot \vec{d} = Fd\cos\theta \to \theta = 0 \quad Fd = W$

$\to 90° \quad 0 = W$



work done by F is ZERO.

$\to \theta = 180°$



$-Fd = W$

$\boxed{W = \vec{F} \cdot \vec{S} = \vec{F} \cdot \Delta\vec{x} = \vec{F} \cdot \Delta\vec{r} = \vec{F} \cdot \vec{d}}$

$S, d, \Delta x, \Delta r$ } displacement

true when $F = \text{const}$

$W = \text{Force} \times \text{displacement}$

$\quad\; N \qquad\quad m$

$= kg \frac{m}{s^2} \quad m$

$\boxed{\text{Joule} = kg \frac{m^2}{s^2}}$

ex: $\vec{F} = (160\hat{i} - 40\hat{j})\,N$

$\vec{S} = (14\hat{i} + 11\hat{j})\,m$

$W = (160\hat{i} - 40\hat{j}) \cdot (14\hat{i} + 11\hat{j})$

$40 \times 160\, \underbrace{\hat{i} \cdot \hat{i}}_{1} + 160 \times 11\, \underbrace{\hat{i} \cdot \hat{j}}_{0} - 40 \times 140\, \underbrace{\hat{j} \cdot \hat{i}}_{0} - 40 \times 11\, \underbrace{\hat{j} \cdot \hat{j}}_{1}$

$W_F = [14 \times 160 - 40 \times 11]\,\text{joule} = 1800\,J$

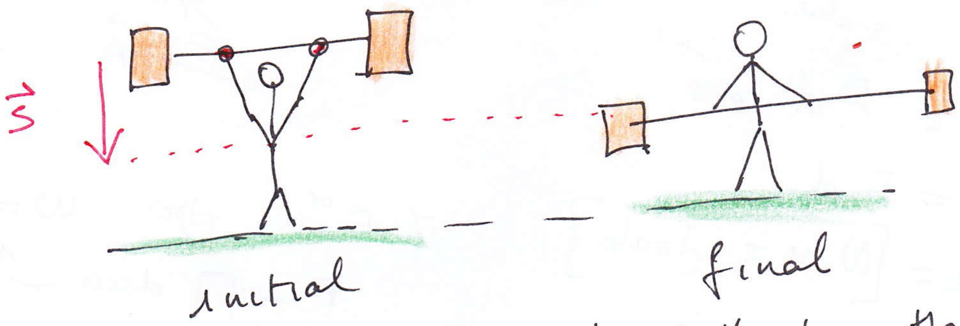
$$W = \vec{F} \cdot \vec{S} = FS \cos\theta$$

$$\hookrightarrow 1800 = \sqrt{160^2 + 40^2}\sqrt{14^2 + 11^2} \cos\theta$$

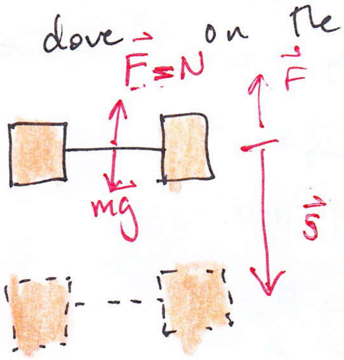
$$\theta = \cos^{-1}\left(\frac{1800}{(210)(18)}\right) = \underline{30^\circ}$$

---

ex:) Lowering of dumbell by the weight lifter.



$\Rightarrow$ the work done on the dumbell by $\underline{\text{the person.}}$



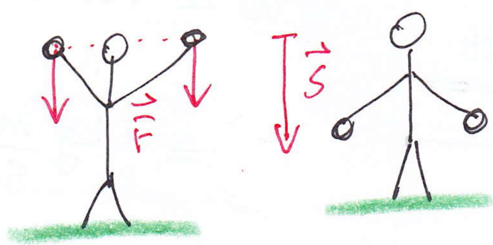
$W_F = ?$

$$\vec{F} \cdot \vec{S} = FS \underbrace{\cos 180}_{(-1)}$$

$$W_F < 0$$

$\Rightarrow$ the work done by the dumbell on the hands of person

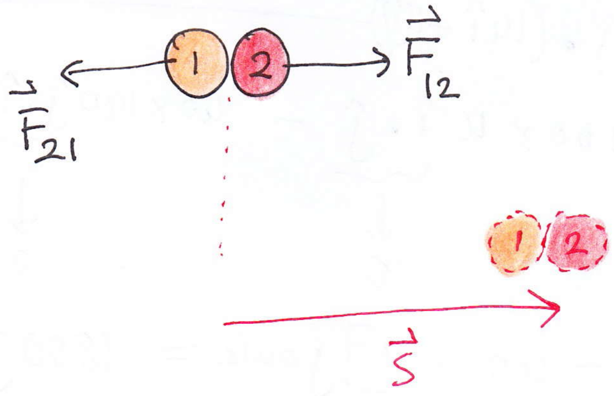
action-reaction pairs



$$W_F = \vec{F} \cdot \vec{S} \doteq FS \underbrace{\cos 0}_{(+1)}$$

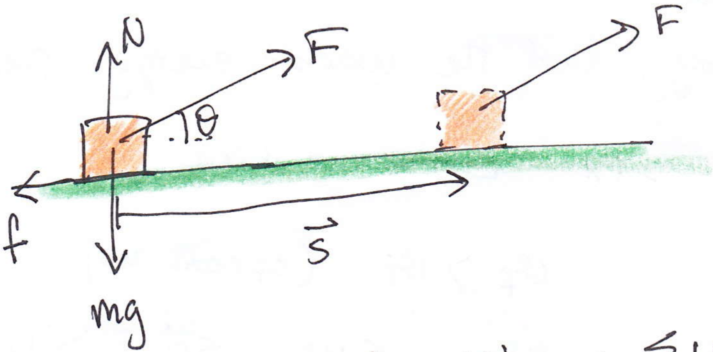
$$\underline{W_F > 0}$$

when there is ~~aaa~~ a contact between objects



$$W_{F12} = - W_{F21}$$

$$FS = -FS$$



$F = 5000N$

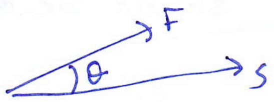
$\theta = 37^\circ$

$f = 3500N$

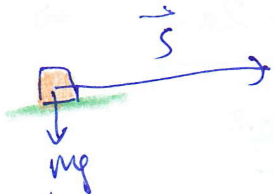
$s = 20\,m$

(2)

W_F , W_{mg} , W_N , W_f ; $\Sigma W = ?$

$W_F \Rightarrow$ 

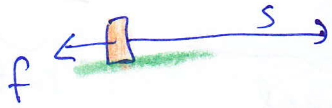
$W_F = (5000)(20)\cos 37 = \underline{80000J}$

W_{mg} 

$W_{mg} = mg\,(20)\cos 90^\circ = 0$

W_N 

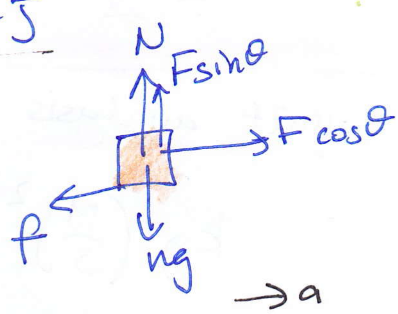
$W_N = N\,(20)\cos 90 = 0$

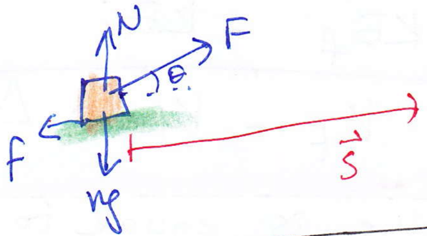
W_f 

$W_f = (3500)(20)\cos 180$

$= -70000\,J$

$\Sigma W = 80kJ + 0 + 0 - 70kJ = 10kJ$





$\Sigma W = \underbrace{\Sigma \vec{F}}_{\text{net force}} \cdot \vec{s}$

$a_x = a$

$a_y = 0$

$\Sigma F_y = m\cancel{a_y}^{0}$

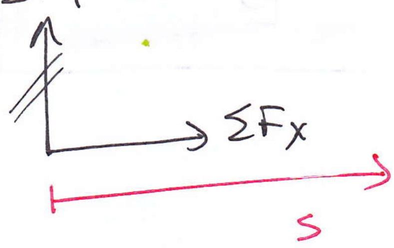
$\boxed{N + F\sin\theta - mg = 0}$

$\Sigma F_y = 0$

$\Sigma F_x = ma$

$\underbrace{(F\cos\theta - f)}_{\text{net force}}$

$\Sigma F_y = 0$



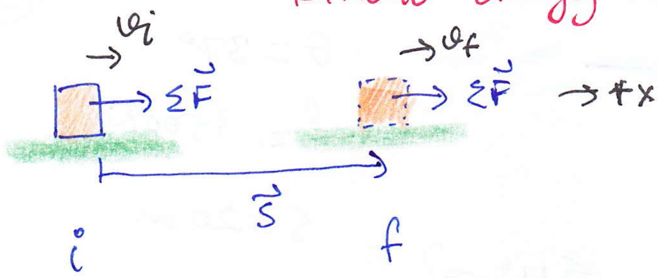
✓ check

$\Sigma F = (5000\cos 37 - 3500) = \underline{500N} = \Sigma F$

$\Sigma W = \Sigma \vec{F} \cdot \vec{s}$

$= (500)(20)\cos 0 = 10000J = \underline{+10kJ}$

Kinetic Energy and the work-energy theorem



Linear motion

$v_f > v_i$ (speeds up)

$W_{tot} = \Sigma W = \Sigma\vec{F}\cdot\vec{s} > 0$

$\Sigma\vec{F} = m\vec{a}$; $\Sigma\vec{F} = \Sigma F_x = \Sigma F$

$\Sigma F = ma$ $\qquad a = \dfrac{\Sigma F}{m}$

should be equal

$$v_f^2 = v_i^2 + 2a\Delta x = v_i^2 + 2aS \Rightarrow a = \frac{v_f^2 - v_i^2}{2S}$$

$$\frac{\Sigma F}{m} = \frac{v_f^2 - v_i^2}{2S}$$

$$\underbrace{\Sigma F\, S}_{\text{Total work}} = \underbrace{\underbrace{\frac{1}{2} m v_f^2}_{\text{final kinetic E.}} - \underbrace{\frac{1}{2} m v_i^2}_{\text{initial kinetic E.}}}_{\text{change in kinetic energy}}$$

Describe kinetic energy

$$\boxed{KE \equiv \frac{1}{2} m v^2}$$

unit analysis

$$kg\left(\frac{m}{s}\right)^2 = kg\,\frac{m^2}{s^2}$$

$KE = [\text{Joule}]$

$$\Sigma F\, S = KE_f - KE_i$$

$$\boxed{W_{tot} = K_f - K_i = \Delta K}$$

total work done is equal to change in kinetic energy

$W_{tot} > 0 \qquad \Delta K > 0$

$K_f > K_i$

$v_f > v_i$ speeds up

$W_{tot} < 0 \qquad \Delta K < 0$

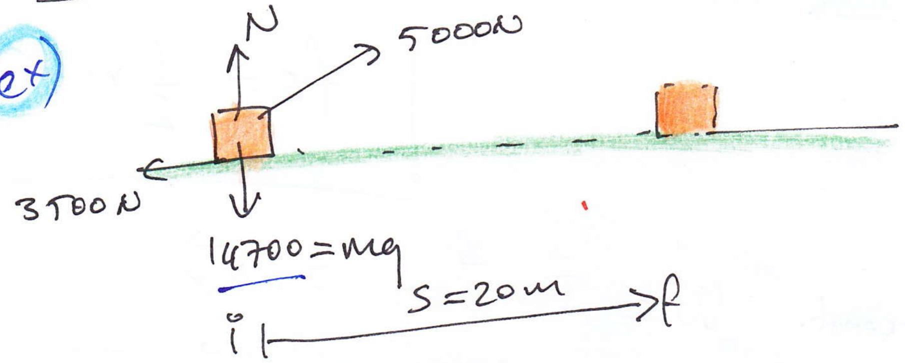
$K_f < K_i$

$v_f < v_i$ slows down

$KE = \frac{1}{2} m v^2 =$ always positive ≥ 0 ☺

---

ex



$v_i = 2\, m/s$

$v_f = ?$

$W_{tot} = 10000\, J$

$$W_{tot} = \Delta K = K_f - K_i = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$m = \frac{14700}{9.8} = 1500\, kg$$

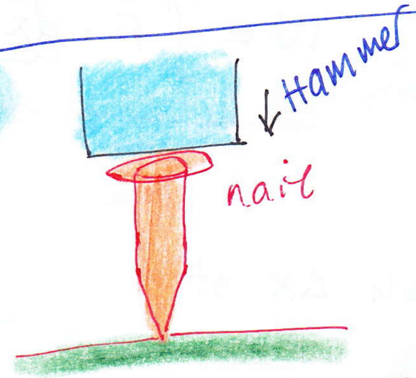
$$1000\cancel{0} = \frac{1}{2}\, 150\cancel{0} (v_f^2 - 2^2)$$

$$100 = \frac{1}{2} 15 v_f^2 - \frac{1}{\cancel{2}} 15 \cancel{4}^{2}$$

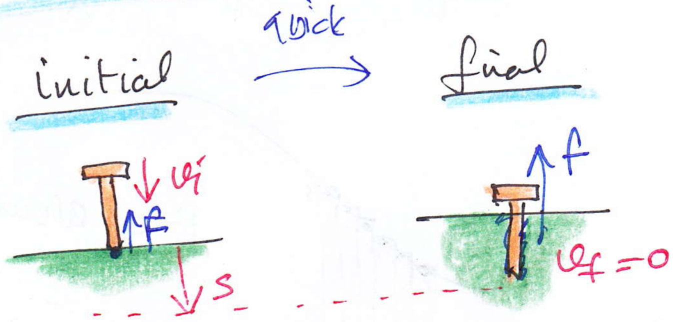
$$\frac{2(130)}{15} = v_f^2 \Rightarrow v_f = 4.2\, m/s$$

---

ex



~~pu~~ hit the nail with hammer.



- after the hamer hits the nail, nail has v_i velocity;

nail goes down till its velocity is ZERO $v_f = 0$.

f is a frictional force stopping the nail.

assume f is const.

$$W_{tot} = \Delta K$$

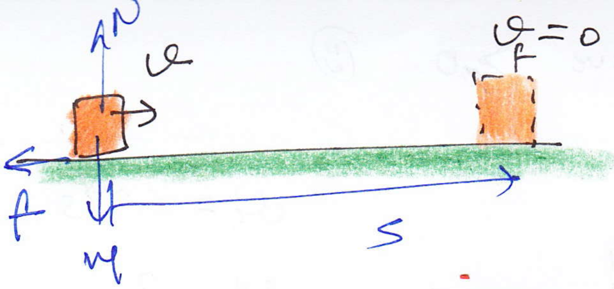
$$\vec{f} \cdot \vec{s} = K_f - K_i$$

$\uparrow f$ $\downarrow s$

$$-fs = 0 - \frac{1}{2} m v_i^2$$

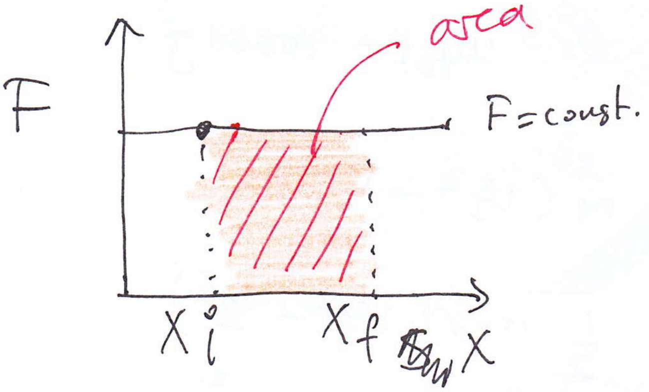
if $s = 1\, cm$
$m = 0.01\, kg$
$v_i = 50\, m/s$
✓

$f = ?$

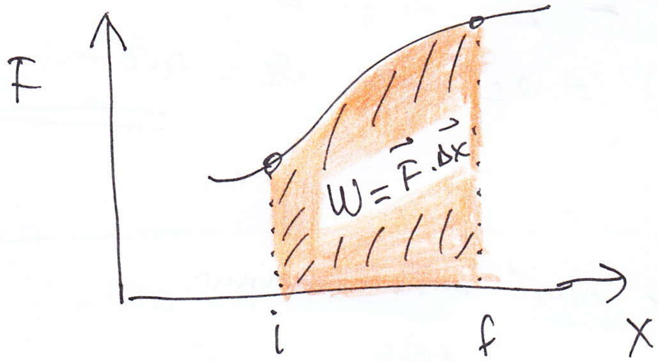


$$W_{tot} = -fs = \frac{1}{2} m \cancel{v_f}^2 - \frac{1}{2} m v_i^2$$

$$\boxed{fs = \frac{1}{2} m v_i^2}$$



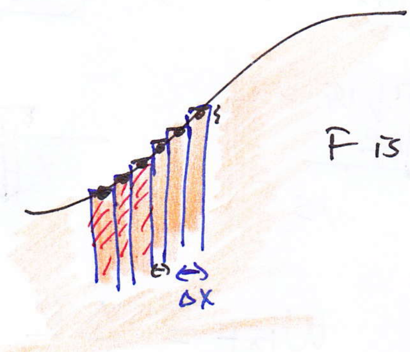
$$W = \vec{F} \cdot \vec{s} = \vec{F} \cdot \Delta \vec{x}$$
$$= \vec{F} \cdot (\vec{x}_f - \vec{x}_i)$$



when F is not const.

$$W = F \cdot s \Rightarrow W = \int \vec{F} \cdot d\vec{s}$$

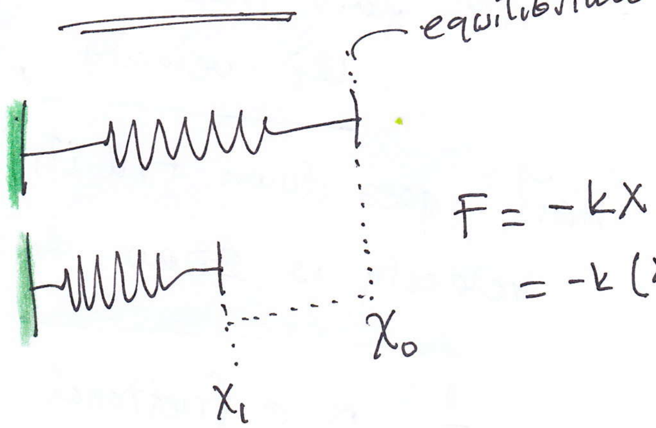
$$W = \int \vec{F} \cdot d\vec{x}$$



F is changing at each Δx step

$$\underline{F(x)} \; -$$

ex: spring force



$$F = -kx$$
$$= -k(x_1 - x_0)$$

$$\int \vec{F} \cdot d\vec{x}$$

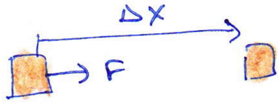
$$W = \int kx \, dx \cos 0$$

$$k \int x \, dx = \frac{k x^2}{2} \equiv \text{work done by the spring force}$$

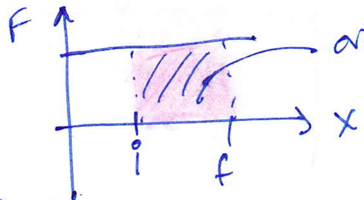
14.12.20

1

Work

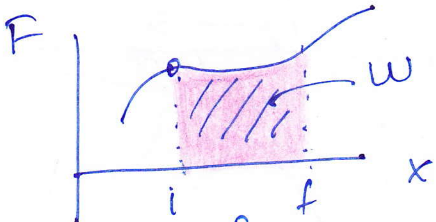


$W = \vec{F} \cdot \vec{\Delta x}$

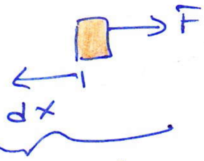


area under curve.

if $F \neq$ const

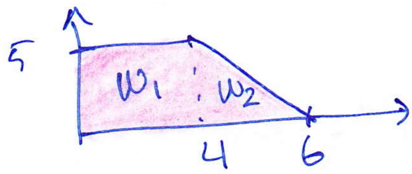


$W = \int_i^f \vec{F} \cdot \vec{dx}$



$W < 0$

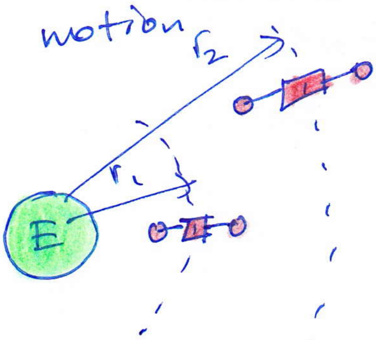
ex) F (N)



$\sum W = W_1 + W_2 = 5 \times 4 + 5 \times \frac{2}{2} = 25$ Joule

---

ex). Satellite motion



a satellite goes from $r_1 \to r_2$ distance

where $F = -\frac{1.3 \times 10^{22}}{x^2}$

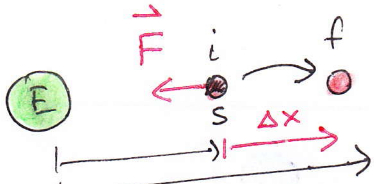
$\vec{F} = -\frac{1.3 \times 10^{22}}{r^2} \hat{r}$

whats the work done by $F = ?$

$r_1 = 1.5 \times 10^{11}$ m $= x_1 = i$

$r_2 = 2.3 \times 10^{11}$ m $= x_2 = f$

$W = \int_i^f \vec{F} \cdot \vec{dx}$



~~$F(\Delta x) =$~~

$= -1.3 \times 10^{22} \int_i^f \frac{dx}{x^2}$

$\int \frac{dx}{x^2} = \int dx\, x^{-2} \Rightarrow \frac{x^{(-2+1)}}{(-2+1)} = \frac{x^{-1}}{-1}$

$= -1.3 \times 10^{22} \left[-\frac{1}{x} \right]_{x_i}^{x_f}$

$= 1.3 \times 10^{22} \left[\frac{1}{2.3 \times 10^{11}} - \frac{1}{1.5 \times 10^{11}} \right] = 1.3 \times 10^{11} \left[\frac{1.5 - 2.3}{(1.5)(2.3)} \right]$

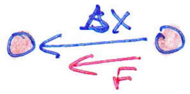
F ← → Δx

$W = -3 \times 10^{11}$ Joule

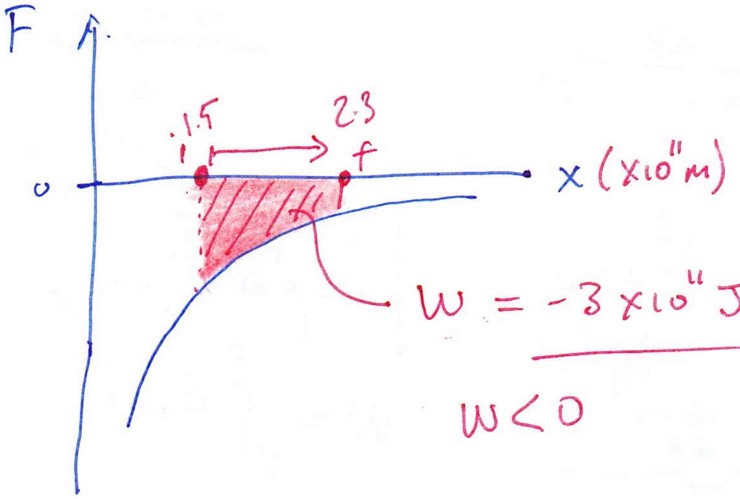
$W < 0$

if satellite from 2.3×10^{11} to 1.5×10^{11}

$W = -1.3 \times 10^{22} \left[-\frac{1}{x} \right]_{2.3 \times 10^{11}}^{1.5 \times 10^{11}} = +3 \times 10^{11}$ J



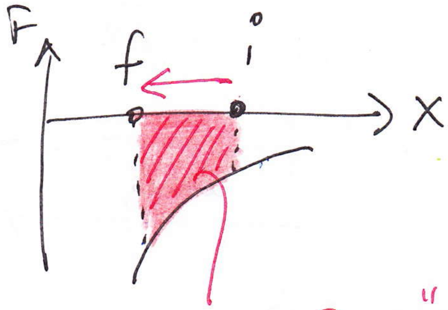
$W > 0$



$W = -3\times10^{11}$ J

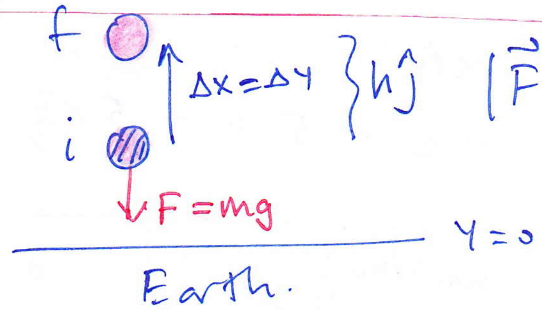
$W < 0$

$F = -\dfrac{const}{x^2} < 0$



$W = +3\times10^{11}$ J

$\vec{F} = mg(-\hat{j})$



$|\vec{F}| = const$

$W = \int \vec{F}\cdot d\vec{x}$

$= \vec{F}\cdot\underbrace{\int d\vec{x}}_{\Delta\vec{x}}$

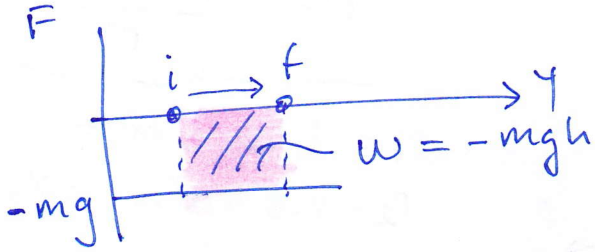
$W = \vec{F}\cdot\Delta\vec{x}$

$W = \vec{F}\cdot\Delta\vec{y}$

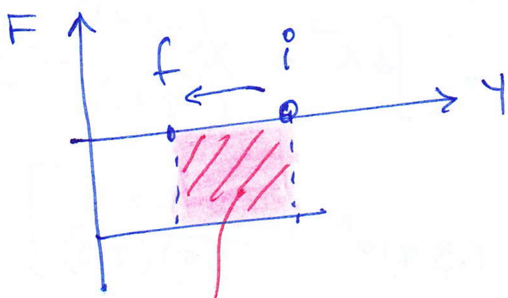
$W = F\cdot\Delta y$

$= mg(-\hat{j})\cdot h\hat{j}$

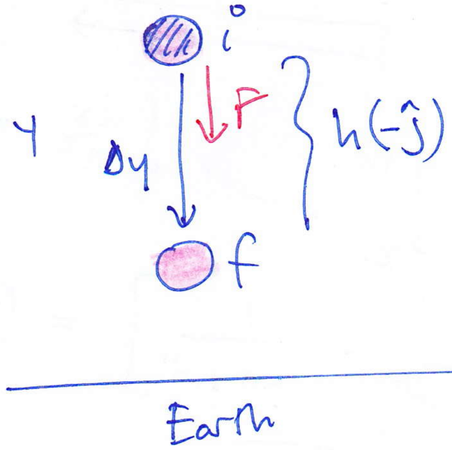
$W = -mgh < 0$



$W = -mgh$



$W = mgh > 0$



$W = \vec{F}\cdot\Delta\vec{y}$

$= mg(-\hat{j})\cdot h(-\hat{j})$

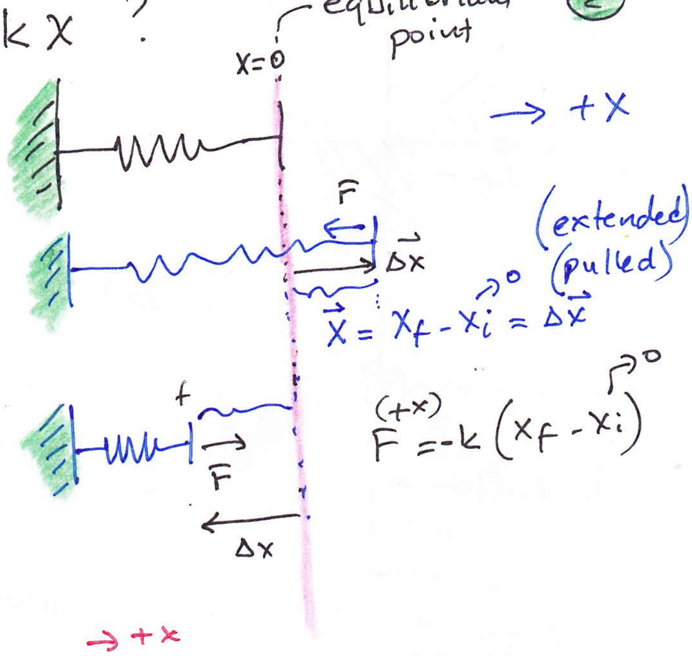
$= mgh > 0$

Spring force
(yay kuvveti)

$\vec{F}_s = -k\Delta\vec{x}$

$F_s = -kx$?

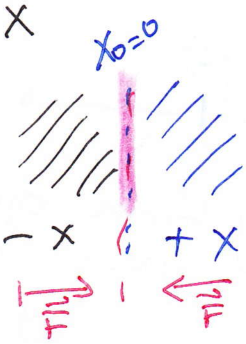
F is always towards equilibrium point

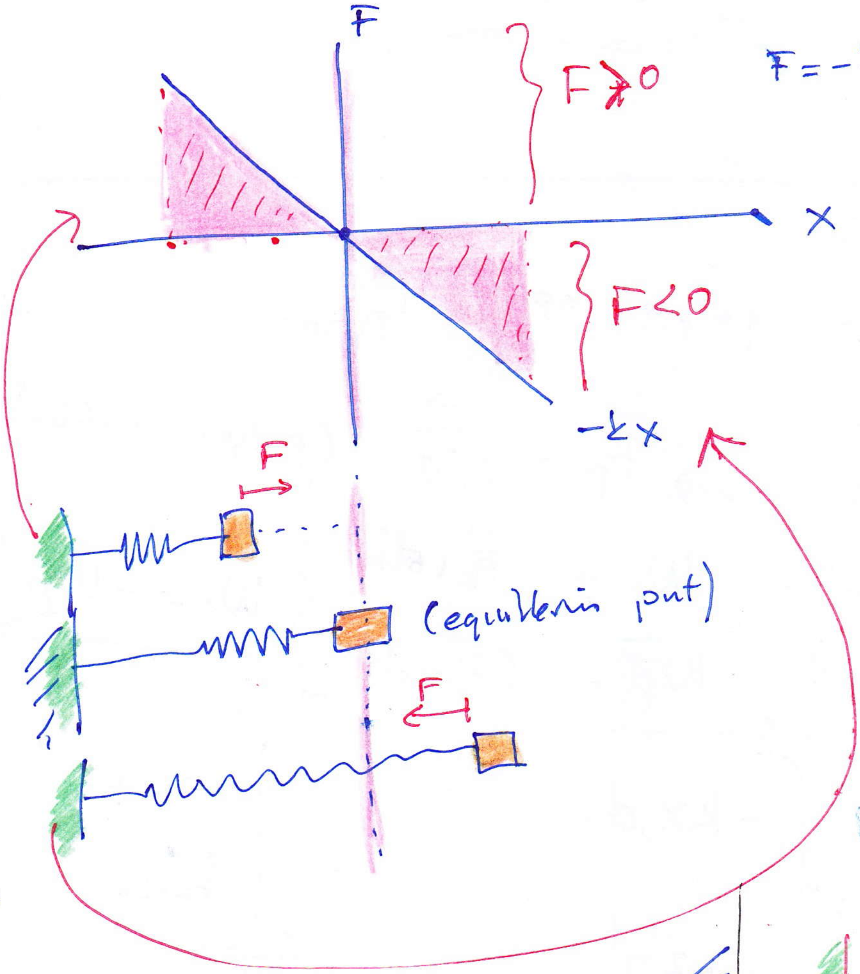


$\Delta x = x_f - x_i \Rightarrow x$

$x_i = 0 \qquad x_f = x$

$F_s = -kx$

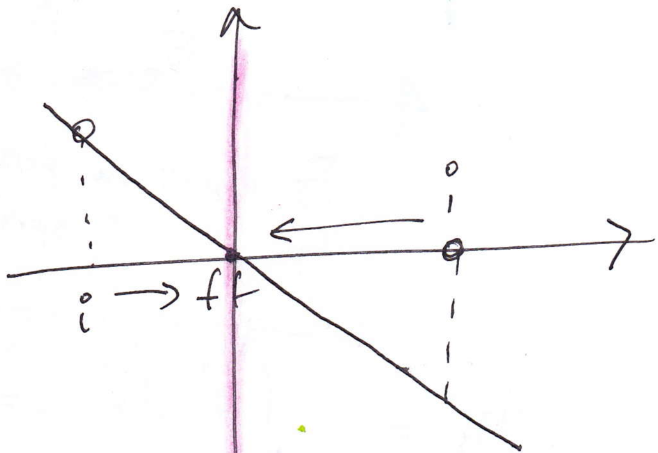


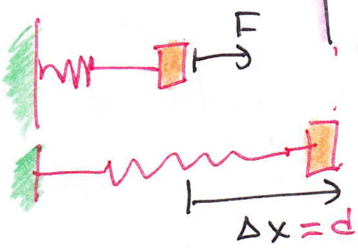


work done by spring force ?

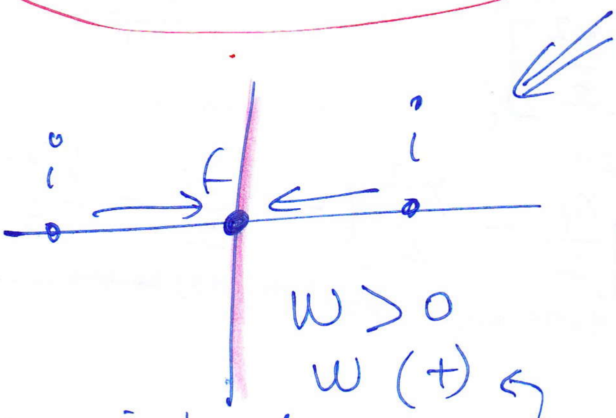
$F = -kx$ (not const)

$W_s = \int \vec{F} \cdot d\vec{x}$



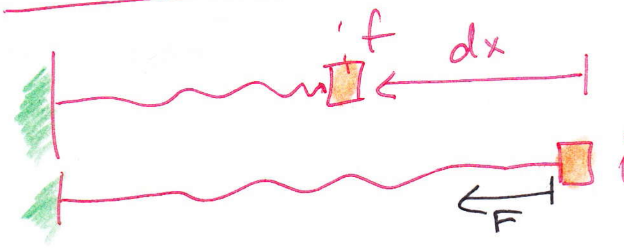


$W = \int \underbrace{\vec{F} \cdot d\vec{x}}_{(+)}$

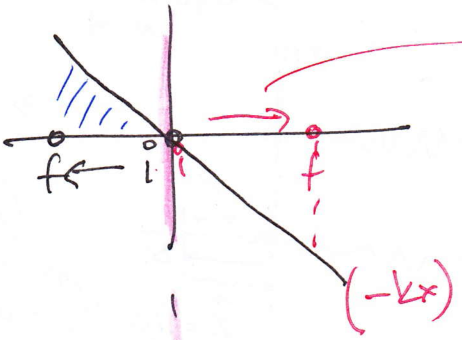


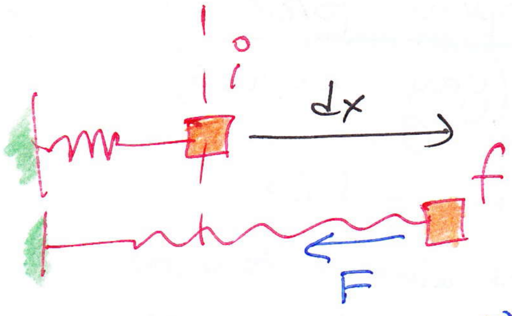
W>0
W (+)

out going towards equilibrium



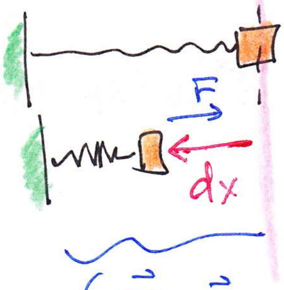
W>0
W(+)





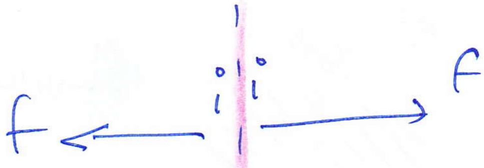
$$\underbrace{\int \vec{F}\cdot\vec{dx}}_{(-)}$$

$$\underline{\underline{W<0}}$$



$$\underbrace{\int \vec{F}\cdot\vec{dx}}_{(-)}$$

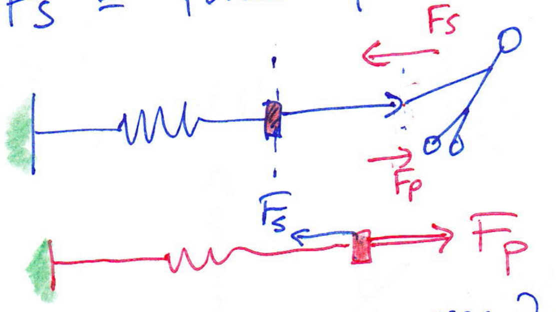
$$\underline{\underline{W<0}}$$



away from equilibri point to out

$$\underline{\underline{W<0}}$$

F_s = force by spring



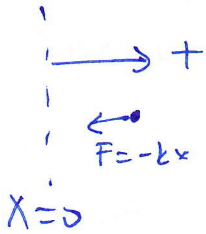
you pull spring F_{Person}

$\vec{F_p} = -\vec{F_s}$ (action-reaction)

F_s applies on person
F_p " " spring

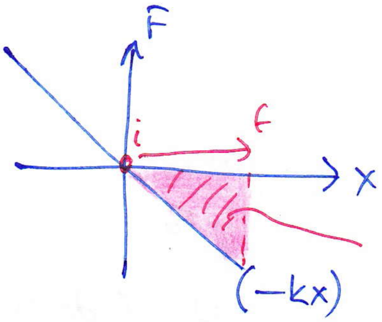
$$\left.\begin{aligned} W_s &= \int \vec{F_s}\cdot\vec{dx} \\ W_p &= \int \vec{F_p}\cdot\vec{dx} \end{aligned}\right\} \quad \underline{W_s = -W_p}$$

$$W_s = \int_i^f \vec{F}\cdot\vec{dx} = \int_i^f -kx\,dx$$

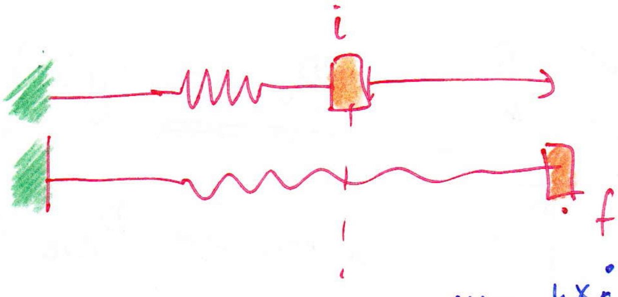


$$= -k\int_i^f x\,dx = -k\left[\frac{x^2}{2}\right]_i^f$$

$$W_{spring} = -k\left[\frac{x_f^2}{2} - \frac{x_i^2}{2}\right] = \frac{kx_i^2}{2} - \frac{kx_f^2}{2}$$

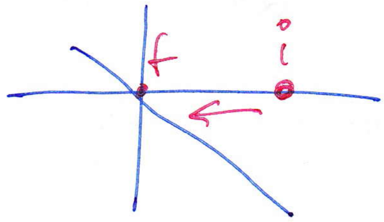


$\frac{kx^2}{2}$



$x_0 = 0$

$$W_s = \frac{kx_i^2}{2} - \frac{kx_f^2}{2}$$



$$\frac{kx_i^2}{2} - \frac{kx_f^2}{2}$$

$x_f = 0$

$$W_s = \frac{kx_i^2}{2} > 0$$

if $x_i \Rightarrow 0$

$$W_s = -\frac{kx_f^2}{2} < 0$$

$$F = -kx$$

k: spring const.

$$k = \frac{F}{x} = \left[\frac{N}{m}\right]$$

ex

$x_i = 0.2\,m$ $\quad k = 500\,N/m$

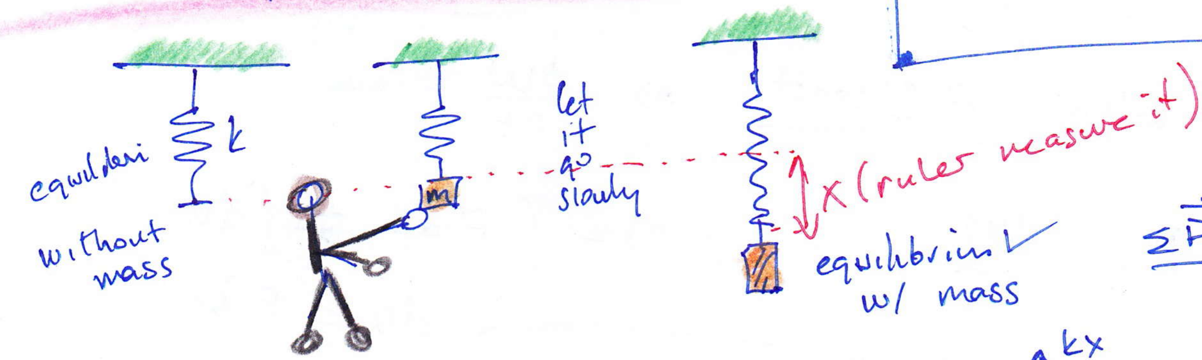
$x_f = 0.5\,m$

$$W_s = ? = k\frac{x_i^2}{2} - k\frac{x_f^2}{2}$$

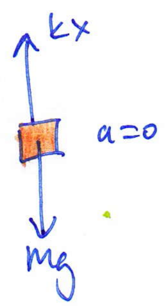
$$= \frac{500}{2}\left[0.2^2 - 0.5^2\right]$$

$$= -52.5 \text{ Joule}$$

How do you measure k?



$$\Sigma\vec{F} = 0$$



$$\Sigma\vec{F} = m\vec{a}$$

$$kx - mg = 0$$

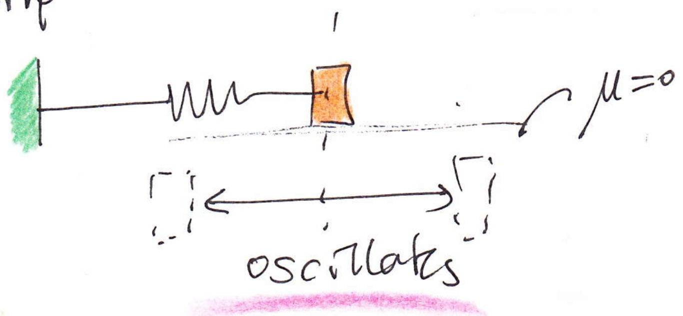
$$k = \frac{mg}{x}$$

$$W_{tot} = \Delta K \quad \text{(work - KE theorem)}$$

$$\int \Sigma\vec{F}\cdot d\vec{x}$$

$$\Sigma\vec{F}\cdot\Delta\vec{x} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

spring - mass - on a frictionless surface



only force F_s

$$W_{tot} = \Delta K = W_s$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

ch 7 → potential energy

Work ✓
KE ✓
spring

Power (giq)

averge powr $P_{ave} = \dfrac{\Delta W}{\Delta t} = \dfrac{\text{work}}{\text{time}}$

instantaneous power $P = \lim_{\Delta t \to 0} \dfrac{\Delta W}{\Delta t} = \dfrac{dW}{dt}$ (derivative of work by time)

$$P = \frac{\text{work} = \text{Joule}}{\text{time} = \text{sec.}} \Rightarrow \left[\frac{J}{s} = \text{Watt}\right] = P$$

$1\,hp$ = horse powr $= 746\,W$ 100 hp car $\Rightarrow$ $7.46 \times 10^4\,W$ 74.6 kW

$\Rightarrow$ Electric bills : (Energy bills) 500 kW.hr (kW.sa)

power $\times$ time $\equiv$ work $\equiv$ energy

$500\,kW.hr = 500 \times 10^3 \times 3600\,(Ws)$ Joule

$P_{ave} \Rightarrow$ when $F \cong \text{const} \Rightarrow \dfrac{\Delta W}{\Delta t} = P_{ave}$

$$P = \frac{dW}{dt} \quad ; \quad W = \int F\,dx \Rightarrow \int dW$$

$$dW = F\,dx$$

$$= \frac{d}{dt}(F\,dx) = \frac{dF}{dt}\cdot dx + F \cdot \frac{dx}{dt}$$

$$P = 0 \qquad\qquad \vec{F}\cdot\vec{v}$$

$F \simeq$ const.

$$\boxed{P = \vec{F}\cdot\vec{v}} \qquad \boxed{P_{ave} = \frac{\Delta W}{\Delta t}}$$

(ex) Airbus A380 plane develops $F = 322\,000\,N$ flying at $\vartheta = 250\ m/s$ ($900\ km/hr$) (4)

$P = ?$

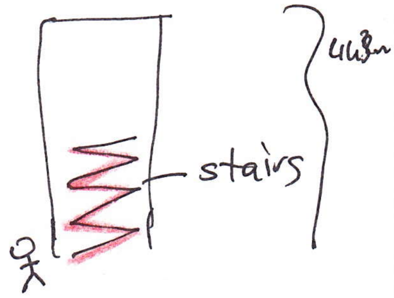
$$P = \vec{F}\cdot\vec{\vartheta} = 322 \times 250 \times 10^3\,W = 8.05 \times 10^7\,W$$

$$= \underline{\underline{108\,000\ hp}}$$

~~1W = 76~~
$1\,hp = 746\,W$

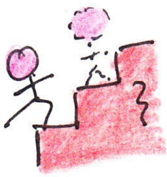
(ex) A 50 kg mass of runner, runs up 443 m tall tower in 15 minutes.

What's her average power output?



$$P_{ave} = \frac{\Delta W}{\Delta t} \Rightarrow 15\,min = \underline{\underline{15 \times 60\,s}}$$

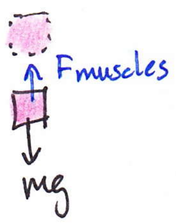
$\Delta W = ?$



lifting up yourself by step length.

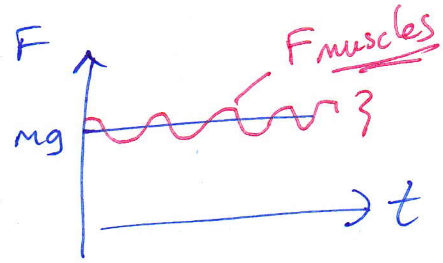
↳ muscles are carrying you up.

⇓



$$\Sigma\vec{F} = 0 \Rightarrow a = 0$$

$$F_m = mg$$

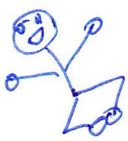


$\Delta W =$ F_m ... $y = 443\,m$; F_m ... $y = 0$

$$W_{muscles} = F_m\,\Delta y$$
$$= mg\,\Delta y$$
$$= 50(9.8)(443)$$
$$= 2.17 \times 10^5\,J$$

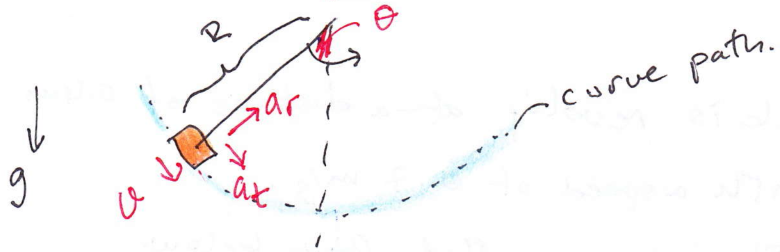
$$P_{ave} = \frac{2.17 \times 10^5\,J}{900\,s} = 241\,W = 0.241\,kW$$
$$= 0.323\,hp$$

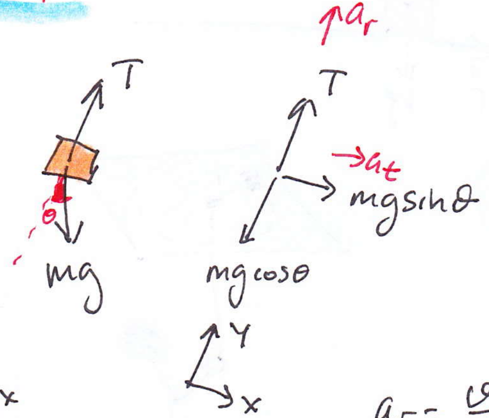
$$P = \vec{F}\cdot\vec{\vartheta} \qquad \vartheta = \frac{\Delta y}{\Delta t}$$
$$= mg\,\frac{443\,m}{900\,s}$$
$$= 50 \times 9.8 \times \frac{443}{900}\ W$$
$$\underline{P = 241\,W}$$



End of Ch 6

Work-Energy Theorem along a curve path





$\Sigma F_y = m a_y$

$T - mg\cos\theta = m a_r$

$T = m\frac{v^2}{R} + mg\cos\theta$

$\Sigma F_x = m a_x$

$mg\sin\theta = m a_t$

$g\sin\theta = a_t = \frac{d|\vec{v}|}{dt}$

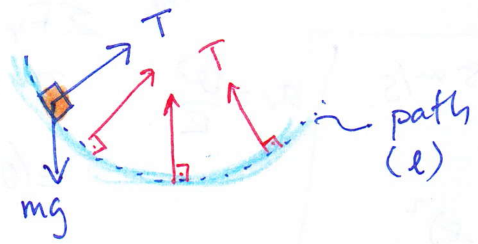
$a_r = \frac{v^2}{R}$

$\theta\downarrow \quad a_t\downarrow \quad a_r\uparrow$

work-energy work done by each force.

$W = \vec{F}\cdot\vec{l}$; (f const)

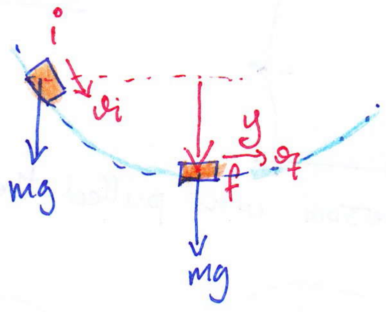
$T \neq$ const $T(\theta)$ $\theta\uparrow$ $T\uparrow$



$W_T = \int \vec{T}\cdot d\vec{l} = \int T\,dl\cos 90$

$W_T = 0$

always $\theta = 90°$



$y \equiv$ displacement

$W_{mg} = m\vec{g}\cdot\vec{y} = mgy > 0$

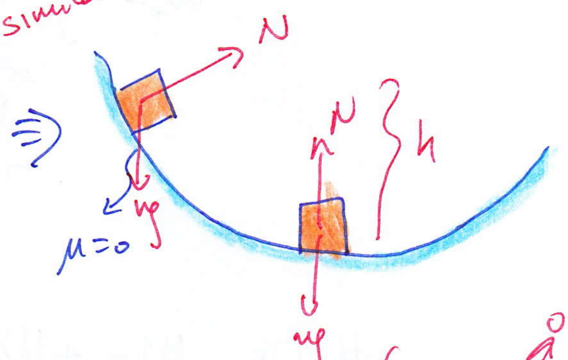
$W_{tot} = \Delta K$ ⇒ work-KE theorem.

$W_T + W_{mg} = \frac{1}{2}m v_f^2 - \frac{1}{2}m v_i^2$

$\boxed{mgy = \frac{1}{2}m v_f^2 - \frac{1}{2}m v_i^2}$

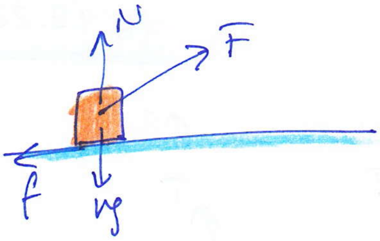
End of ch 6

similar problem



$W_N = \int N\cdot dl = \int N\,dl\cos 90$

$W_N = 0$ $W_{mg} = mgh$



$W_{tot} = W_F + W_{mg} + W_N + W_f = \Delta K$

work - KE theorem.

6.75

Block is revolving at a distance of 0.4 m

with a speed of 0.7 m/s.

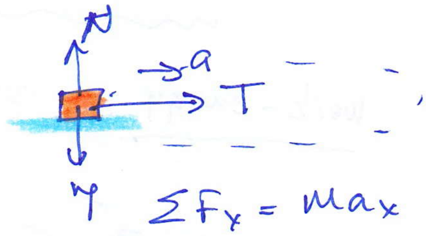
The rope is pulled from below.

$m = 0.09\,kg$ $\quad \underline{\underline{\mu = 0}}$

after pull now the block is revolving at a radius of 0.1 m.

$v_{new} = 2.8\ m/s$

a) what's the tension on the rope originally when $v = 0.7\ m/s$?



$\Sigma F_x = m a_x$

$T = m a_r$

$T = (0.09)\dfrac{(0.7)^2}{(0.4)}$

$\underline{\underline{T_i = 0.11\ N}}$

b) what's the tension when $v = 2.8\ m/s$.

$a_r = \dfrac{v^2}{R}$

$\Sigma F = ma_x \Rightarrow \quad T = m\dfrac{v^2}{R}$

$= (0.09)\dfrac{(2.8)^2}{(0.1)}$

$= 64 \times (0.11)$

$T_f = 7.04\ N$

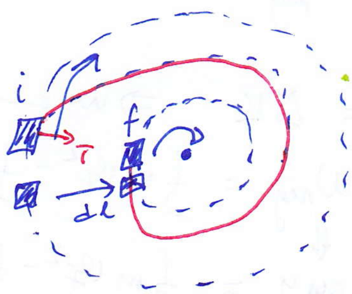
c) How much work done by the person who pulled the rope?

$W_T = \int \vec{T} \cdot d\vec{l}$

hard

$T \neq const$

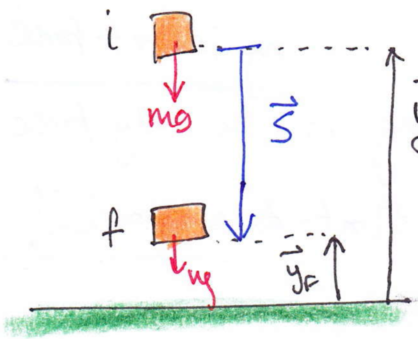
dl ?



$W_{tot} = W_T + \underset{0}{\cancel{W_N}} + \underset{0}{\cancel{W_{mg}}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

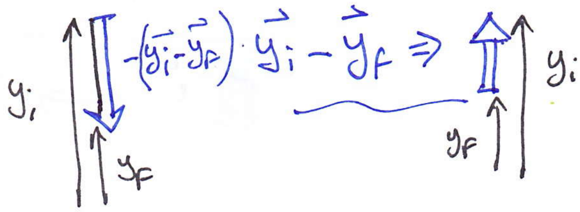
$= \frac{1}{2}\,0.09\left[(2.8)^2 - (0.7)^2\right]$

$= 0.33$ Joules



$W_{mg} = m\vec{g}\cdot\vec{s}$

$y=0 \longrightarrow y=0$ ↑ $+y$



$-(\vec{y}_i-\vec{y}_F) = \vec{s}$

$(\vec{y}_F-\vec{y}_i) = \vec{s}$

$W_{mg} = m\vec{g}\cdot(\vec{y}_F-\vec{y}_i)$

$= m\vec{g}\cdot\vec{y}_F - m\vec{g}\cdot\vec{y}_i$

$= mg\,y_F\cos 180 - mg\,y_i\cos 180$

$W_{mg} = mg\,y_i - mg\,y_F = mgS$

$W_{mg} = U_i - U_F$

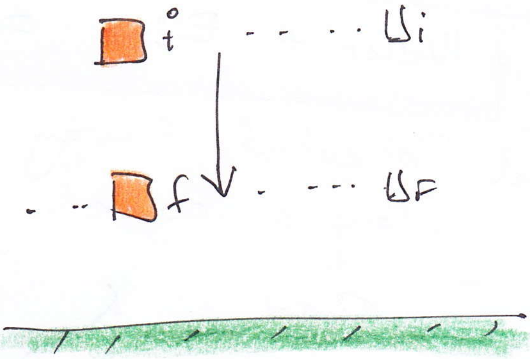
$W_{mg} = -(U_F - U_i) = -\Delta U$

$U = PE \equiv$ potential energy

$U \equiv mgy$

$\Delta \equiv$ final − initial

$$\boxed{W_{mg} = -\Delta U} \quad ***$$



$\Delta U < 0$

$-\Delta U > 0$

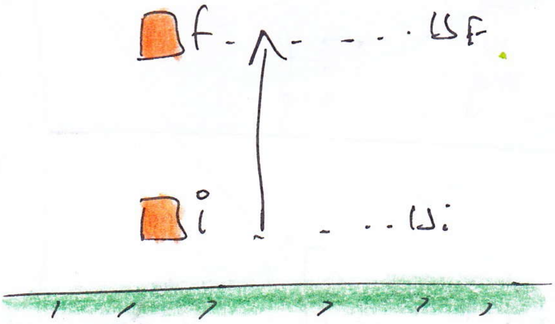
$W_{mg} > 0$

Work − KE theorem

If only mg is the force.

$W_{tot} = W_{mg} = \Delta K$

$W_{mg} = -\Delta U$



$\Delta U > 0$

$-\Delta U < 0$

$W_{mg} < 0$

$\Delta K = -\Delta U$

$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -(mg\,y_F - mg\,y_i)$

$\frac{1}{2}mv_i^2 + mg\,y_i = \frac{1}{2}mv_F^2 + mg\,y_F$

$\underbrace{K_i + U_i = K_F + U_F}_{\text{conservation of mechanical energy}}$

$E = U + K$

U → pot E

K → kin E

conservation of mechanical energy:

$$E_i = E_f$$

$$\boxed{K_i + U_i = K_f + U_f}$$

when $\Sigma F = mg$

mg is net force!

$+ \; mg$ is the only force that does work!!

$i \rightarrow K_i + U_i$

equal

$f \rightarrow K_f + U_f$

when there are other forces than mg.

$D_i \xrightarrow{\Delta \vec{s}} U_f$

$$W_{tot} = W_{mg} + W_{other} = \Delta K$$

$$\boxed{W_{mg} = -\Delta U} \; ***$$

$$-\Delta U + W_{other} = \Delta K$$

$$W_{other} = \Delta K + \Delta U = K_f - K_i + U_f - U_i$$

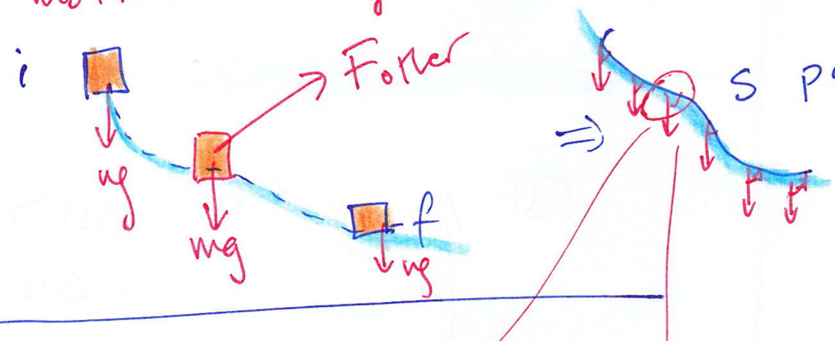
$$\boxed{W_{other} + K_i + U_i = K_f + U_f} \; ***$$

$$\boxed{W_{other} + E_i = E_f} \; ***$$

other forces = frictional, spring force.

$$\boxed{\begin{aligned} K &= \tfrac{1}{2} m v^2 \\ U &= mgy \end{aligned}}$$

gravitational Potential Energy for motion along a curved path.



$$W_{mg} = \vec{mg} \cdot \vec{s}$$

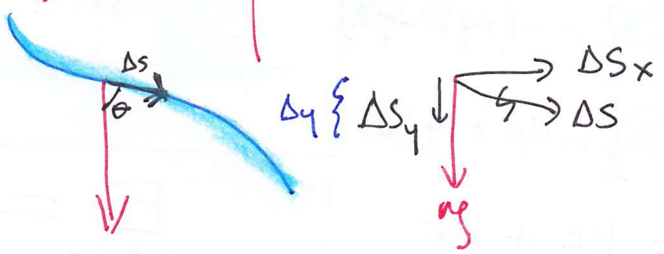
$mgs \cos \theta$

θ = not const.

$\vec{mg} \cdot \vec{\Delta s}$

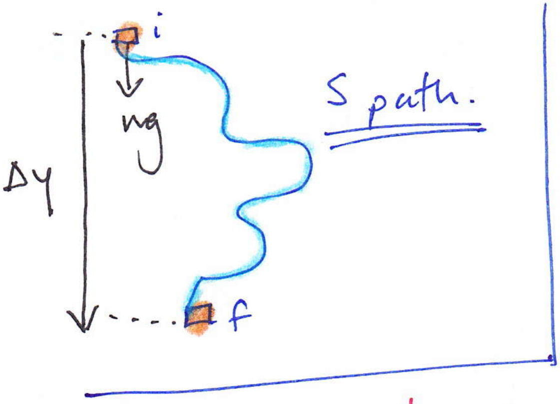
$mg(-\hat{j}) \cdot [\Delta s_x \hat{i} + \Delta s_y \hat{j}]$

$0 \; -\hat{j} \cdot \hat{i} = 0$



$$mg \, \Delta s_y (-1) = \vec{mg} \cdot \vec{\Delta s}$$

$$\boxed{W_{mg} = -mg \Delta y} \equiv \text{only the vertical displacement matters!!}$$



Two situations

projectile motion

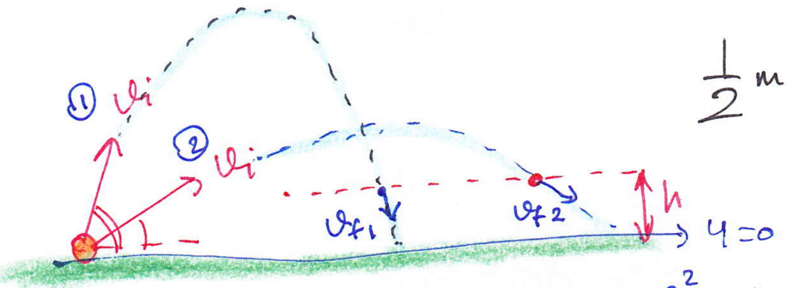
mg is the only force.

at any point on this path.

$$K_i + U_i = K_F + U_F$$

$$\frac{1}{2}m v_i^2 + mg y_i = \frac{1}{2}m v_f^2 + mg y_F$$

if $y_i = 0$



① $\frac{1}{2}m v_i^2 + 0 = \frac{1}{2}m v_{f1}^2 + mgh$

② $\frac{1}{2}m v_i^2 + 0 = \frac{1}{2}m v_{f2}^2 + mgh$

$\Big\}$ $v_{f1} = v_{f2}$ ✓

no matter what the θ_i is ; if at same height they will have same final velocity.

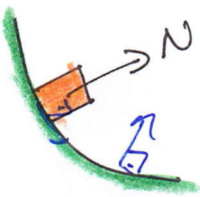
$$K_i + \overset{0}{U_f} = K_f + U_f$$

$$0 + mg y_i = \frac{1}{2}m v_f^2 + \cancel{mg\,0}$$

$$\boxed{v_f = \sqrt{2 g y_i}}$$

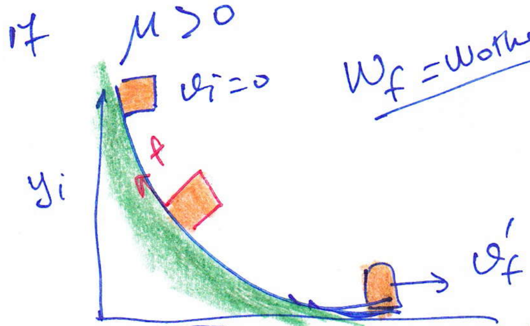
is mg the only for here? ⇒ NO !

$$W_N = \vec{N}\cdot\vec{S} = 0 \quad \vec{N} \perp \vec{S}$$



---

if $\mu > 0$



$W_f = W_{other}$

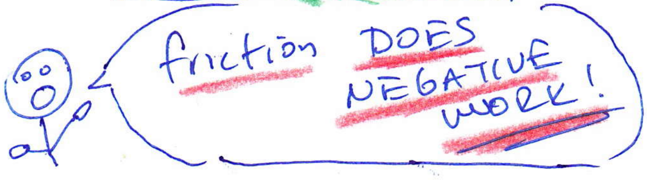
$$W_{other} + K_i + U_i = K_f + \overset{0}{U_f}$$

$$\boxed{W_f + mg y_i = \frac{1}{2}m v_f'^2}$$

$v_f' < v_f$ (no friction)

$\vec{f}$ $\quad$ $\vec{\Delta s}$

W_f $\quad$ $W_f < 0$



Ch7 Potential Energy

21.12.20

21.12.20

1

Energy conservation.

$$W_{other} + K_i + U_i = K_f + U_f$$

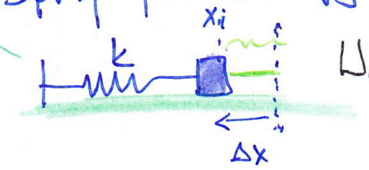
$K = \frac{1}{2} m v^2$

$U \Rightarrow$ grav. pot. energy

$U_g = mgy$

$\Downarrow$

spring pot. energy



U_e

$W_{spring} = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$

work done by spring ; x_i = distance from eq. point

x_f = " " " "

any compression / extension $\Rightarrow$ stored energy in the spring

$U_{spring} = U_{elastic}$ (spring elastic potential energy) $= \frac{1}{2} k x^2$

$\rightarrow \; W_{spring} = U_{el,i} - U_{el,f} = -\Delta U_{el}$

remember $W_{grav} = -\Delta U_{grav}$; $U_{grav} = mgy$

if spring ; grav. pot simultaneously present

$$\boxed{W_{other} + K_i + U_{g,i} + U_{e,i} = K_f + U_{g,f} + U_{e,f}}$$

Energy conservation

$K = \frac{1}{2} m v^2$; $U_g = mgy$; $U_e = \frac{1}{2} k x^2$

W_{other} = other forces than grav. spring i.e. frictional force

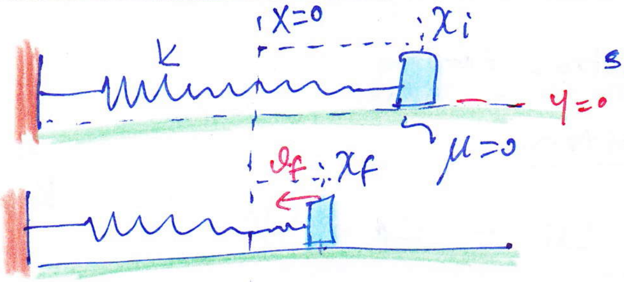
$\Big\} \frac{1}{2} k y^2$

$$\boxed{W_{tot} = \Delta K}$$

$W_{other} + W_{spring\,elastic} + W_{grav} = \Delta K$

$W_{other} + (-\Delta U_e) + (-\Delta U_g) = \Delta K$

ex).



spring is streched 0.1 m and the mass released

$v_i = 0$

$k = 5\ N/m$; $m = 0.2\ kg$

$x_i = 0.1\ m$; $x_f = 0.08\ m$

$v_f = ?$

$$\cancel{U_{g,i}}^{0} + \cancel{W_{other}} + K_i + U_{e,i} = K_f + U_{e,f} + \cancel{U_{g,f}}^{0}$$

since there no change in y direction

0 ; μ=0

$U_{g,i} = U_{g,f}$ ✓

$$\frac{1}{2} m \cancel{v_i}^2 + \frac{1}{2} k x_i^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} k x_f^2$$

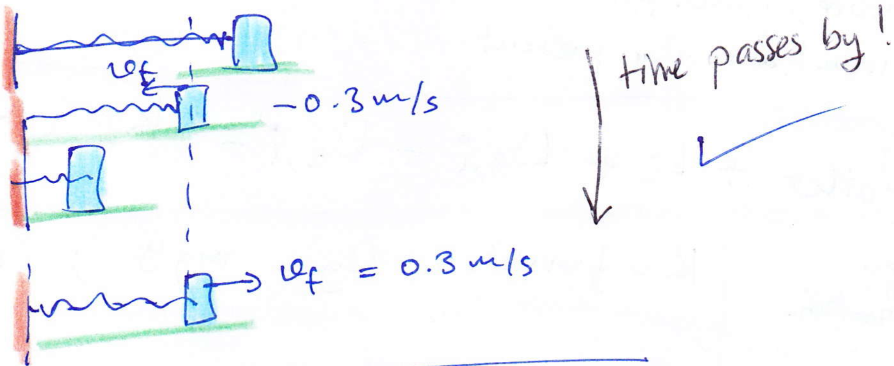
0

$$\frac{1}{2} 5 (0.1)^2 = \frac{1}{2} (0.2) v_f^2 + \frac{1}{2} 5 (0.08)^2$$

$$0.025\ J = 0.1\ v_f^2 + 0.016\ J$$

$$\frac{0.009\ J}{0.1} = v_f^2$$

both are correct { $+0.3\ m/s$; $-0.3\ m/s$

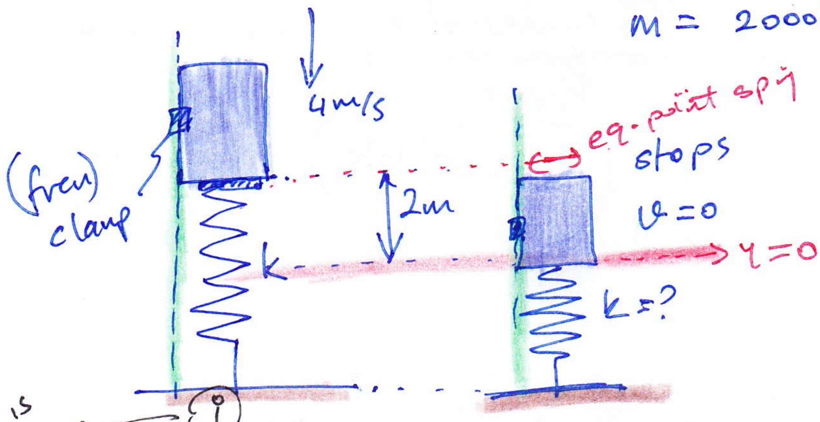


ex.) μ>0 ; U_g ✓ U_e ✓

A 2000kg elevator w/ broken cables is falling at 4 m/s when it contacts with spring in the bottom. the spring is stopping the elevator when it compressed 2m. During the motion a safety clamp applies 17000 N frictional force to the elevator.

What's $k = ?$ of the spring?

$m = 2000\,kg \Rightarrow mg \; 19600\,N$ ②



$f_{clamp} = f_{friction} = 17000\,N$

f is applied till it stops.

spring is not compressed yet.

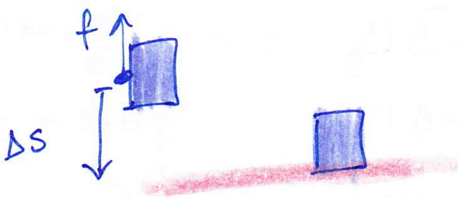
$$W_{other} + K_i + U_{g,i} + U_{e,i} = K_f + U_{g,f} + U_{e,f}$$

$$W_{other} + \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}k\Delta y_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}k\Delta y_f^2$$

$y_i;\ y_f$ are the locations w.r.t $y=0$ equilibrium point

$\Delta y_i;\ \Delta y_f \equiv$ compression values (due to) w.r.t

$W_{other} = W_f$



$$W_f = \vec{f}\cdot\vec{\Delta s} = f\,\Delta s \cos 180 = (17000)(2)(-1)$$

$f = 17000\,N$ $\quad m = 2000\,kg$

$\Delta S = 2m$ $\quad v_i = 4\,m/s$ $\quad y_i = 2m$ $\quad \Delta y_i = 0$ $\quad v_f = 0$ $\quad y_f = 0$ $\quad \Delta y_f = 2m$

$$-17000(2) + \frac{1}{2}(2000)(4^2) + 2000(9.8)(2) + 0 = 0 + 0 + \frac{1}{2}k(2^2)$$

$$k = 1.06\times10^4\ N/m$$ ✓

$$\underbrace{[-34000]}_{friction} + \underbrace{16000}_{K} + \underbrace{39200}_{U_g} = \underbrace{2k}_{U_{e,f}}$$

If there was no clamp force ≡ friction force ✓

$$55200 = 2k'$$

$$2.76\times10^4 = k'$$

(korunulu) **Conservative Forces** ; **Nonconservative Forces** (korunumsuz)

Conservative Forces

(reversible)

→ energy is conserved when they go back to initial location

→ mg gravitational

→ $-kx$ spring

Nonconservative Forces

(irreversible)

→ frictional forces.

properties of work done by conservative forces. (W_{mg}, W_{el})

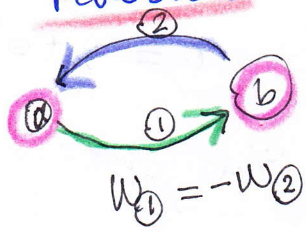
1. It can expressed as initial - final values of a potential function

$$W_{mg} = -\Delta U_g \quad ; \quad U_g = mgy$$

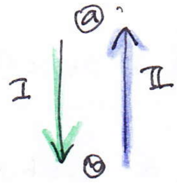
$$W_{el} = -\Delta U_{el} \quad \quad U_{el} = \frac{1}{2}kx^2$$

$$\boxed{W = -\Delta U}$$

2. It's reversible

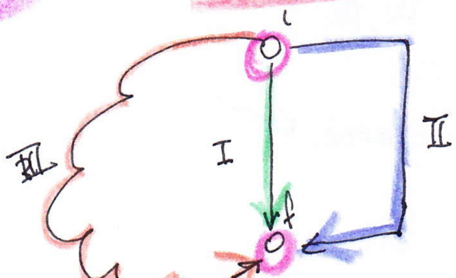


$W_{①} = -W_{②}$



$W_g(I) = -W_g(II)$

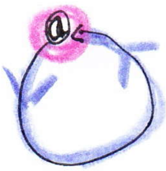
3. It's independent of the path.



$W_I = W_{II} = W_{III}$

4. When start = end points are same $W_{done} = 0$ (ZERO!)

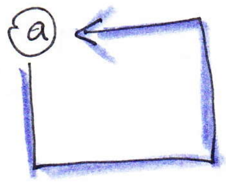
$\Sigma W = 0$



$W = 0$

$$W_{i \to f} = W_{a \to a} = 0$$

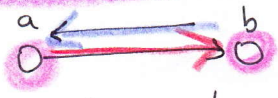
Nonconservative force ⇒ (frictional force.)



④ $W_f \neq 0$ $i \to f$

① $W_{friction} \neq -\Delta U$

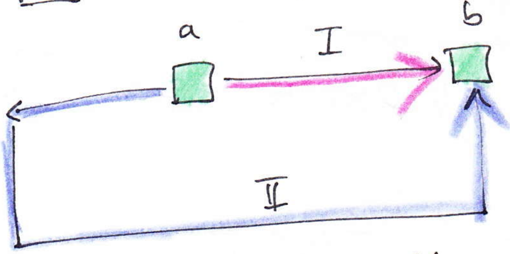
not reversible



[2] $W_{a\to b} \neq -W_{ba}$

$W_{a\to b} = W_{b\to a}$

[3] it's dependent of the path.



$W_{I} \neq W_{II}$

Longer the path $W_f \uparrow$

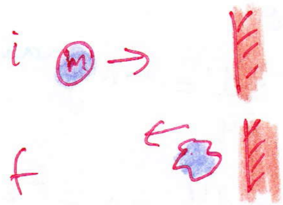
---

4 types of forces → grav. (c)
→ EM ⟷ friction
→ weak } nuclear } c c
→ strong

$F_g = G\,\frac{m_1 m_2}{r^2}$ (conservative)

$F_c = k\,\frac{q_1 q_2}{r^2}$ (conservative force)

frictional forces ⇒ (steal energy from the system) ⇒ W_f = energy is converted to heat; shape destruction; shape change



collusions ⟸

(conservative) FORCE & POTENTIAL ENERGY

F	U
mg	mgy
kx	$\frac{kx^2}{2}$

$W = -\Delta U = F_x \Delta x$ ⇒ $F = \frac{-\Delta U}{\Delta x}$ ≡ − derivative of potential w.r.t. displacement
force.

$W = -\Delta U = F_y \Delta y$ → $F = \frac{-\Delta U}{\Delta y}$

$F_x = -\frac{dU}{dx}$; $F_y = -\frac{dU}{dy}$; $F_z = -\frac{dU}{dz}$ } 1 dimensional forces.

$F_x = -\frac{\partial U}{\partial x}$ partial derivative

$F_y = -\frac{\partial U}{\partial y}$

$F_z = -\frac{\partial U}{\partial z}$

$U(x, y, z)$; x, y, z are independent of each other.

$\frac{dx}{dz} = 0$ $\frac{dx}{dy} = 0$; $\frac{dy}{dz} = 0$ ...

$\leftrightarrow x$

if $U = \frac{1}{2}kx^2$; $-\frac{\partial U}{\partial x} = -\frac{1}{2}k(2x) = -kx = F_x$

$-\frac{\partial U}{\partial y} = 0$; $-\frac{\partial U}{\partial z} = 0$

if $U = \frac{1}{2}ky^2$ $\updownarrow y$

$F_x = 0$

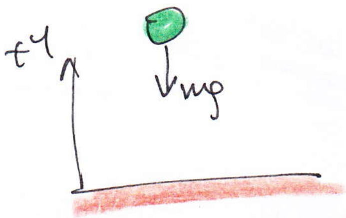
$F_z = 0$

$F_y = -ky = \frac{-\partial\left(\frac{1}{2}ky^2\right)}{\partial y}$

$$F = -\frac{\Delta U}{\Delta(\underbrace{\text{displacement}}_{\Delta x,\ \Delta y,\ \Delta z})}$$

$F_x = -\frac{\partial U}{\partial x}$

ex.) $mgy = U$



$-\frac{\partial U}{\partial x} = 0 = F_x$

$-\frac{\partial U}{\partial y} = -mg = F_y$

$-\frac{\partial U}{\partial z} = 0 = F_z$

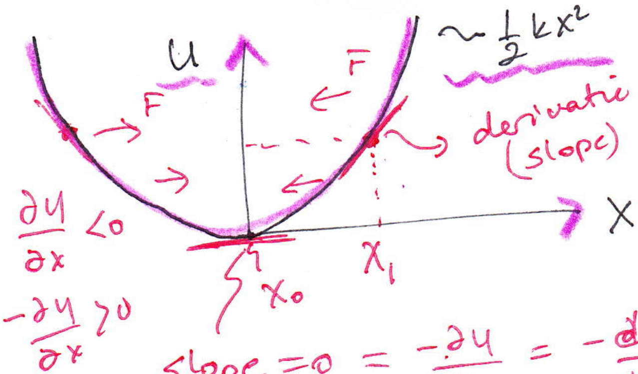
$F \Rightarrow$ derivative of pot. energy w.r.t. dimension (x, y, z) ...

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right) = -\vec{\nabla}U$$

$\nabla \equiv$ gradient

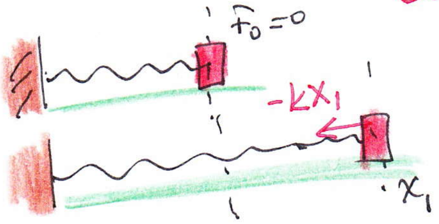
$$\vec{\nabla} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

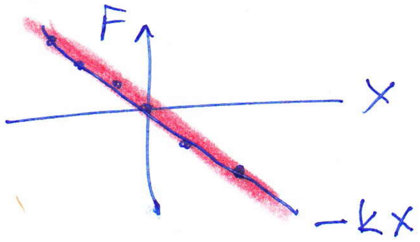


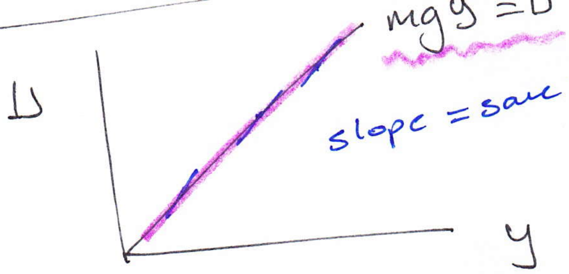
spring

$F_1 = -kx_1$

slope $= 0 = -\frac{\partial u}{\partial x} = -\frac{dU}{dx} \Rightarrow F_0 = 0$ at $x_0 \equiv$ equilibrium point

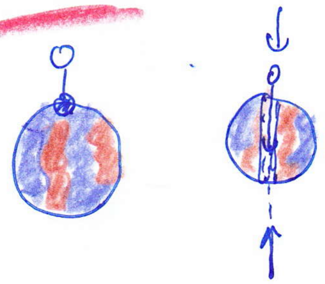


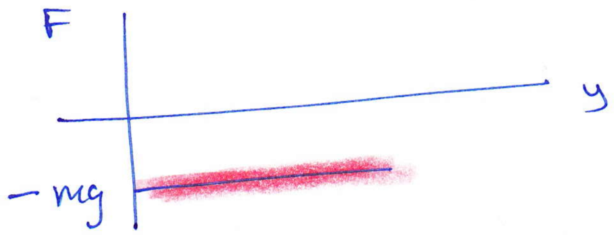




gravitational force.

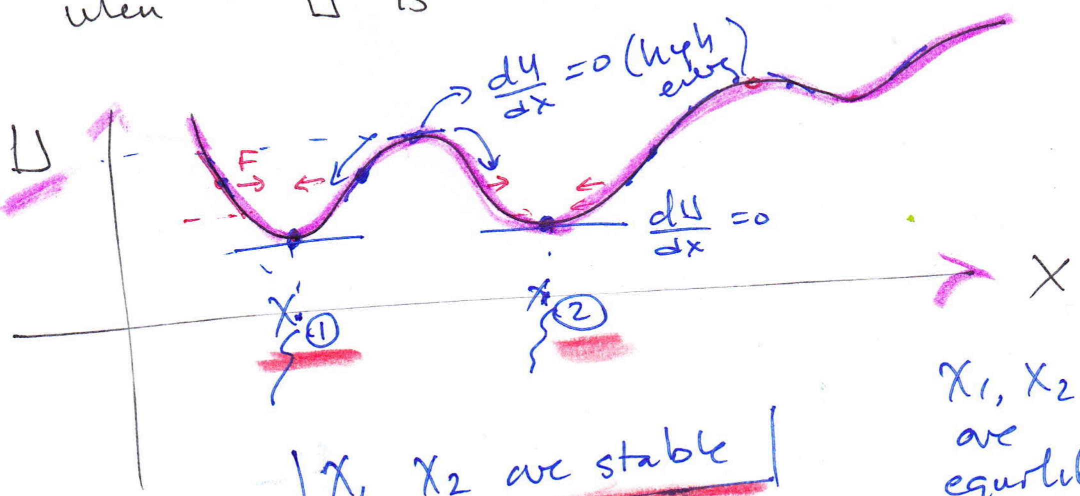
$-\frac{dU}{dy} = -mg = F$





---

general analysis of U is possible when U is more complicated.



$\boxed{x_1 \;\; x_2 \text{ are stable}}$

x_1, x_2 are equilibrium points

$F @ {x_1 \atop x_2} = 0$

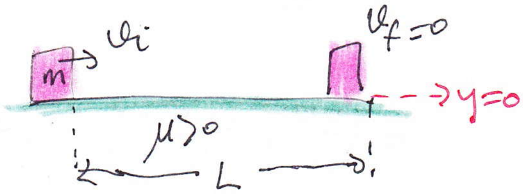
minimum potential value is the most stable point / state of a system.

$\left.\right\}$ $\frac{du}{dx} = 0$ and U is minimum!

$x_1 ; x_2$

23.12.20 (1)

ex)

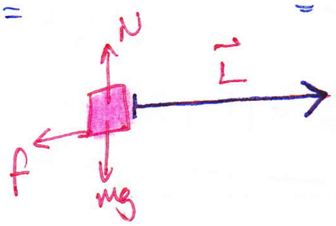


stops after L distance.

find L in terms of v_i, m, μ, g

$W_{other} + K_i + U_i = K_f + U_f$

U → grav.
  → spring



$f = \mu N = \mu mg$

$\overleftarrow{f}\ \overrightarrow{L}$

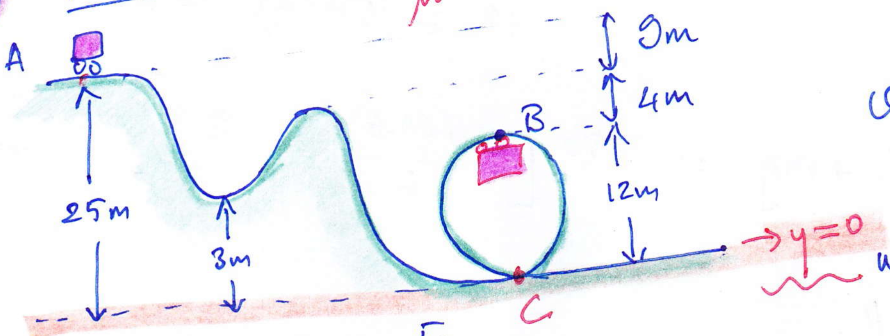
$\underbrace{\vec{f}\cdot\vec{L}} + \frac{1}{2}mv_i^2 + 0 = \frac{1}{2}m\overset{0}{v_f^2} + 0$

$-fL + \frac{1}{2}mv_i^2 = 0$

$\frac{1}{2}\cancel{m}v_i^2 = fL = \cancel{m}g\mu L$

$$\boxed{L = \frac{v_i^2}{2g\mu}}$$

7.41) Roller coaster



m = 350kg

$v_A = 0$

$v_B = ?$

what's the normal force applied to cart at point B? N_B?

$E_A = E_B$

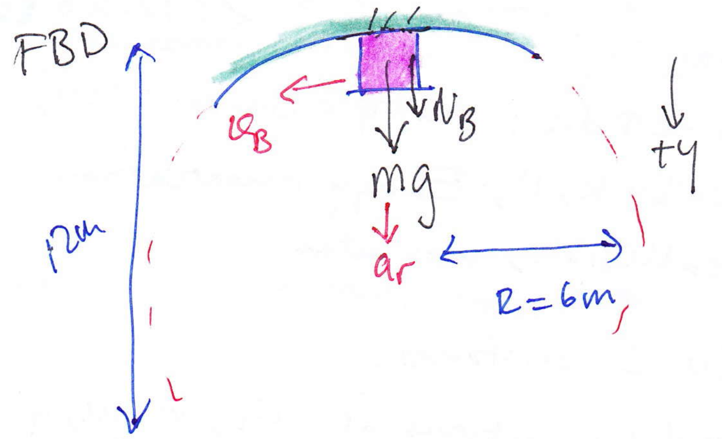
$\overset{0}{W_{other}} + K_A + U_A = K_B + U_B$

$\underbrace{\frac{1}{2}\cancel{m}v_A^2}_{0} + \cancel{m}g\,y_A = \frac{1}{2}\cancel{m}v_B^2 + \cancel{m}g\,y_B$

$y_B = 12m$

$g\,25 = \frac{1}{2}v_B^2 + g\,12$

$v_B = \sqrt{g(26)} = \sqrt{(9.8)26} = \underline{15.96\ m/s}$



$$\Sigma F_y = ma_y \qquad a_r = \frac{v_B^2}{R}$$

$$mg + N_B = m\frac{v_B^2}{R}$$

$$N_B = m\frac{v_B^2}{R} - mg$$

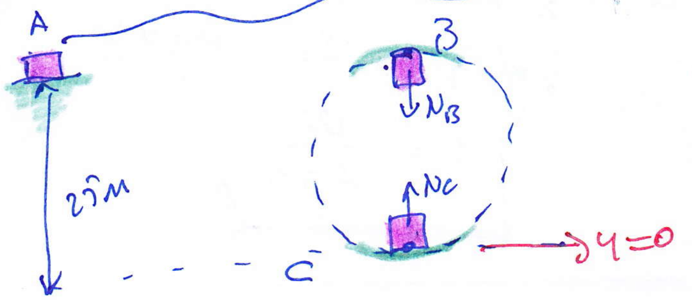
$$= \frac{(350)(9.8)26}{6} - 350(9.8)$$

$$= 350(9.8)\left(\frac{26}{6} - 1\right)$$

$$mg = 350(9.8) = 3430\,N$$

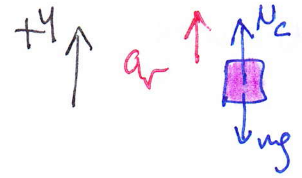
$$N_B = 11433.3\,N$$

$$N_B > mg$$



$N_c = ?$

$v_c = ?$



$$a_r = \frac{v_c^2}{R}$$

$$E_A = E_C$$

$$mg\,25 = \frac{1}{2}mv_c^2 + mg(0)$$

$$v_c = \sqrt{50(g)} = \sqrt{50(9.8)}\ m/s$$

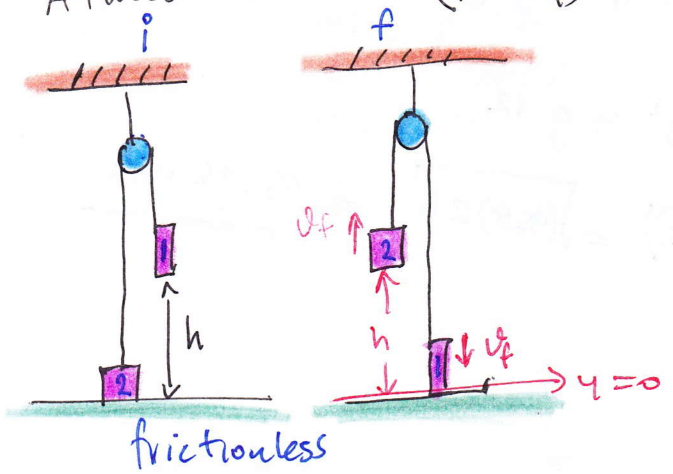
$$\Sigma F_y = ma_y$$

$$N_c - mg = m\frac{v_c^2}{R}$$

$$N_c = m\frac{v_c^2}{R} + mg = 350\left[\frac{50(9.8)}{6} + 9.8\right] = 32013.3\,N$$

$$N_c > N_B$$

7.51 Atwood machine ≡ (pulley)



$m_1 = 12kg \quad m_2 = 4kg \quad h = 2m$

if initially the masses are at rest ($v = 0$) what are their velocities just before m_1 hits the ground?

$W_{other} = 0$ ✓ $\qquad U_{spring} = 0$

$$U = mgy$$

$W_{other} + E_i = E_f$ — system has two masses

$$W_{other} + K_{1i} + U_{1i} + K_{2i} + U_{2i} = K_{1f} + U_{1f} + K_{2f} + U_{2f}$$

$$0 + \tfrac{1}{2} m_1 v_{1i}^2 + m_1 g y_{1i} + \tfrac{1}{2} m_2 v_{2i}^2 + m_2 g y_{2i} = \tfrac{1}{2} m_1 v_{1f}^2 + m_1 g y_{1F} + \tfrac{1}{2} m_2 v_{2f}^2 + m_2 g y_{2f}$$

$m_1 = 12kg$ $\quad v_{1i} = 0$ $\quad y_{1i} = 2m$

$m_2 = 4kg$ $\quad v_{2i} = 0$ $\quad y_{2i} = 0$

$\boxed{v_{1f} = v_f = v_{2f}}$

$y_{1F} = 0$

$y_{2F} = 2m$

$$0 + m_1 g 2 + 0 + 0 = \tfrac{1}{2} m_1 v_f^2 + 0 + \tfrac{1}{2} m_2 v_f^2 + m_2 g 2$$

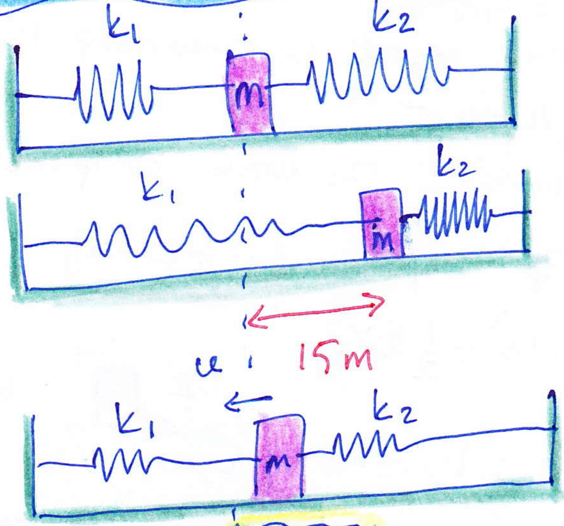
$$\frac{2\left[2g(m_1 - m_2)\right]}{(m_1 + m_2)} = v_f^2 \Rightarrow v_f = \sqrt{\frac{4(9.8)(8)}{16}} = \sqrt{2g}$$

$$\boxed{v_f = 4.4 \text{ m/s}}$$

7.62)

i ⇒

f ⇒



@ equilibrium

$k_1 = 25$ N/m | no friction

$k_2 = 20$ N/m

$m = 3$ kg

m is pushed 15m to the right and released from rest.

a) what's the max speed of m?

U = elastic pot. energy $= U_{el}$

$E_i = E_f$:

$$K_i + U_{1i} + U_{2i} = K_f + U_{1f} + U_{2f} \qquad U = \tfrac{1}{2} k (\Delta x)^2$$

$$\tfrac{1}{2} m v_i^2 + \tfrac{1}{2} k_1 X_{1i}^2 + \tfrac{1}{2} k_2 X_{2i}^2 = \tfrac{1}{2} m v_f^2 + \tfrac{1}{2} k_1 X_{1f}^2 + \tfrac{1}{2} k_2 X_{2f}^2$$

0

$v_i = 0$; both springs compressed/strecthed same amount

$X_{1i} = X_{2i} = 15m$

$v_f = \max$ (want) $= ?$

$X_{1f} = X_{2f} = X_f = ?$

if $v_f = \max$; $U_{1F} = U_{2F} = 0$

$X_f = 0$

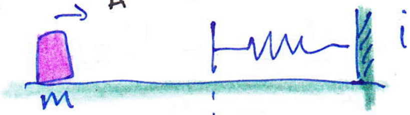
$$0 + \tfrac{1}{2} k_1 15^2 + \tfrac{1}{2} k_2 15^2 = \tfrac{1}{2} m v_{max}^2 + 0 + 0$$

$X_F = 0$ it is max speed at equilibrium point

$$(25 + 20) 15^2 = 3 v_m^2$$

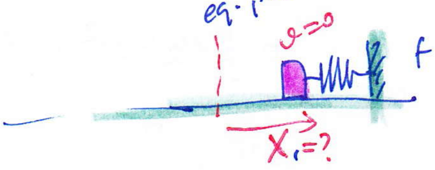
$$v_m = 15\sqrt{15} \text{ m/s} = 58.1 \text{ m/s}$$

ex) seesaw



what is the max compression in the spring?

(a)



$$E_i = E_f \qquad U_g = 0$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx^2$$

$\frac{1}{2}mv_f^2 \Rightarrow$ 0! $v_f = 0$; $\frac{1}{2}kx^2 \Rightarrow$ max

$$\frac{1}{2}mv_A^2 = \frac{1}{2}kX^2$$

$$\boxed{X_1 = \sqrt{\frac{m}{k}}\,v}$$

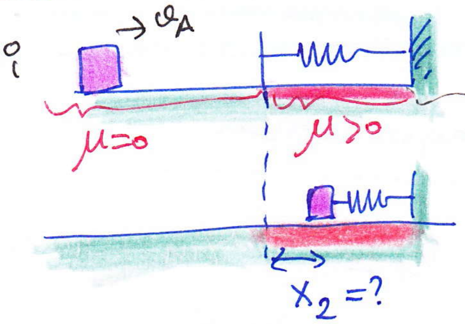
$m = 0.8\,kg$

$k = 50\ N/m$

$v_A = 1.2\,m/s$

$X_1 = 0.15m$

(b)



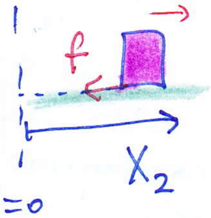
there is friction when object is compressing the spring.

$\mu = 0.5$

what is the max compression?

$$W_{other} + E_i = E_f \qquad \boxed{E = K + U}$$

$$-\mu mg X_2 + \frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_2^2 + \frac{1}{2}kX_2^2$$



$W_f = -fX_2$

$f = \mu N = \mu mg$

max compression $X_2 \uparrow$; $v_2 \Rightarrow$ min $= 0$

$$\boxed{-\mu mg X_2 + \frac{1}{2}mv_A^2 = \frac{1}{2}kX_2^2}$$

$$-(0.5)(0.8)(9.8)X + \frac{1}{2}(0.8)(1.2)^2 = \frac{1}{2}\,50\,X^2$$

$$0 = Ax^2 + Bx + C \to \frac{-B \mp \sqrt{B^2 - 4AC}}{2A}$$

$$0 = 25x^2 + 3.92X - 0.576$$

$X \to 0.092\,m$ ✓

$X \to \cancel{-0.25m}$

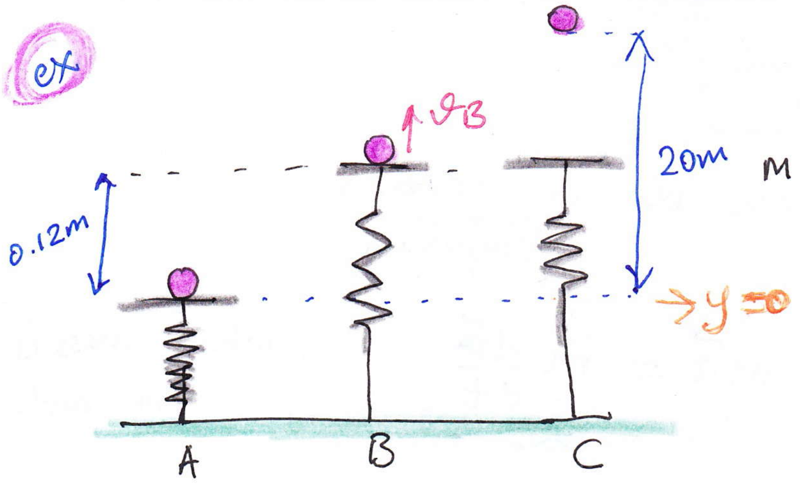
$X_2 = 0.092m$

compare with previous case

where no friction $\to X \to 0.15m$

$X \to 0.092m$ $(\mu = 0.5)$

ex



$m = 35\,gr$

A ⇒ m is on spring; spring is compressed 0.12m; released from rest!

C ⇒ m is 20m above A level when $v_C = 0$

$k = ?$

$$E_A = E_B = E_C$$

$$K_A + U_{gA} + U_{elA} = K_C + U_{gC} + U_{elC}$$

decide $y=0$?

$$\frac{1}{2} m \cancel{v_A}^2 + mg \cancel{y_A} + \frac{1}{2} k (\Delta y_A)^2 = \frac{1}{2} m \cancel{v_C}^2 + mg y_C + \frac{1}{2} k (\cancel{\Delta y_C})^2$$

$$\frac{1}{2} k (0.12)^2 = (0.035)(9.8)(20) + 0$$

$$\underline{k = 953\ N/m}$$

b) what's the velocity/speed of mass at point B?

$$E_A = E_B$$

$$0 + 0 + \frac{1}{2} k (0.12)^2 = \frac{1}{2} m v_B^2 + mg y_B + \frac{1}{2} k (\cancel{\Delta y_B})^2$$

$$\frac{1}{2}(953)(0.12)^2 = \frac{1}{2}(0.035) v_B^2 + (0.035)(9.8)(0.12)$$

$$\boxed{v_B = 19.7\ m/s}$$

always distinguish between y of mgy and Δy of $\frac{1}{2} k \Delta y^2$

becareful !! $y \neq \Delta y$

work energy chapters

end of ch 7; ch6, ch7 } W, K, U; $F = \frac{-dU}{dx}$

$W = -\Delta U$

Chapter 8

impulse (itme) & momentum & collisions (çarpışma)

Newton's 2nd law $\quad \Sigma\vec{F} = m\vec{a} = m\dfrac{d\vec{v}}{dt}$ } when mass is constant.

$$\Sigma\vec{F}\,dt = m\,d\vec{v} = d(m\vec{v})$$

$\Sigma\vec{F}\,dt = d\vec{P}$; $\boxed{\vec{P} \equiv m\vec{v}}$ ⟷ new thing called momentum.

$m\vec{a} = \Sigma\vec{F} = \dfrac{d\vec{P}}{dt}$; change in momentum wrt time is net force

2nd Law

when m is not constant.

ex) rocket + fuel system.

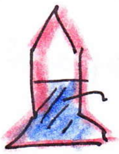
$\boxed{\Sigma\vec{F} = \dfrac{d\vec{P}}{dt} = \dfrac{d(m\vec{v})}{dt}}$ ***

$\Sigma\vec{F} = \dfrac{dm}{dt}\vec{v} + m\dfrac{d\vec{v}}{dt}$

most general case

but usually $m = $ const.

$$\Sigma\vec{F} = m\frac{d\vec{v}}{dt}$$



} $M = $ rocket iron + fuel

fuel ↓

we will discuss it soon!!

M is decreasing with time.

$$\vec{P} \equiv m\vec{v}$$

momentum $= \left[kg\,\dfrac{m}{s}\right]$ ***

$$\Sigma\vec{F} = \frac{d\vec{P}}{dt} \Rightarrow \Sigma\vec{F}\,dt = d\vec{P} \Rightarrow \Sigma\vec{F}\,\Delta t = \Delta\vec{P}$$

$\vec{J} =$ impulse (itme) $\left[\left(kg\,\dfrac{m}{s^2}\right)s = kg\,\dfrac{m}{s}\right]$

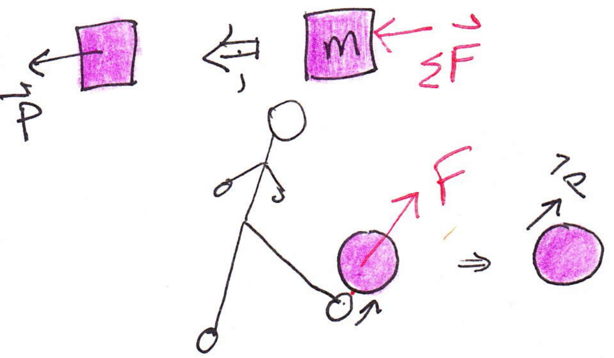
$$\boxed{\vec{J} = \sum \vec{F}\,\Delta t = \Delta \vec{P}}$$

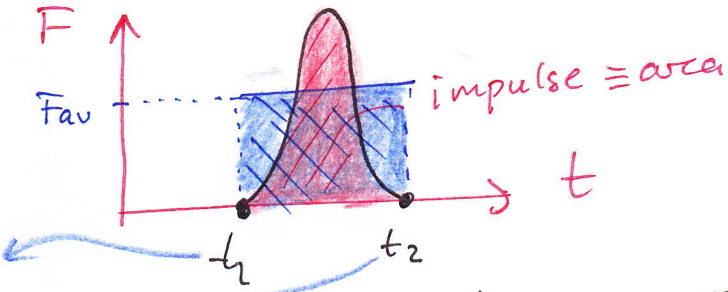
change in momentum is equal to impulse!

$\boxed{\vec{J} \equiv \text{impulse}}$

impulse = change in momentum.

$\sum \vec{F}$ is applied for Δt seconds; m will gain momentum

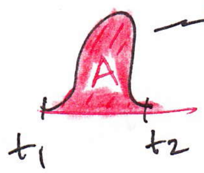




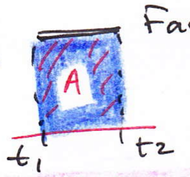
t_1 = contact starts

t_2 = " stops.

$\sum \vec{F}\, dt = \vec{J}$ = area under the curve.



→ Force complicated ⇒



Fave area = $F_{av}\,\Delta t$

check out cool videos online about slow motion crash; collisions!

30th ; 1430 short-exam
↳ wednesday
↳ work, energy, K, U, ...
↳ momentum; impulse, ... (end of today's)

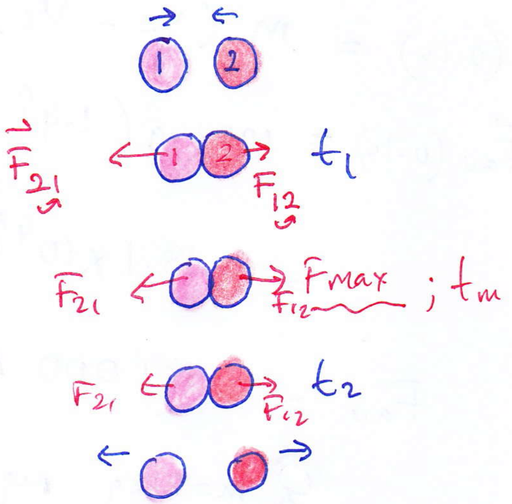
two objects are in contact during collision

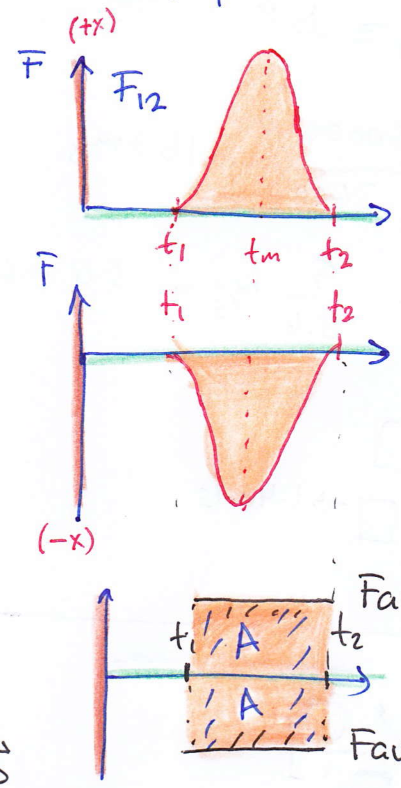
$\vec{P} \equiv m\vec{v}$; $\vec{J} \equiv \vec{F}_{av}\Delta t$; $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = m\frac{\Delta\vec{V}}{\Delta t}$

impulse is change in momentum

$\vec{J} = \Delta\vec{P}$

$\underbrace{\vec{F}_{av}\Delta t}_{\text{impulse}} = m\Delta\vec{V} = \Delta\vec{P}$





Force onto 2nd object

$\int F_{12}\,dt = A$ area

Force onto 1st object

$\int F_{21}\,dt = A$ area

Fave = F_{12}

Fave = $\vec{F}_{av,21}$

most of the time we can simply forces as

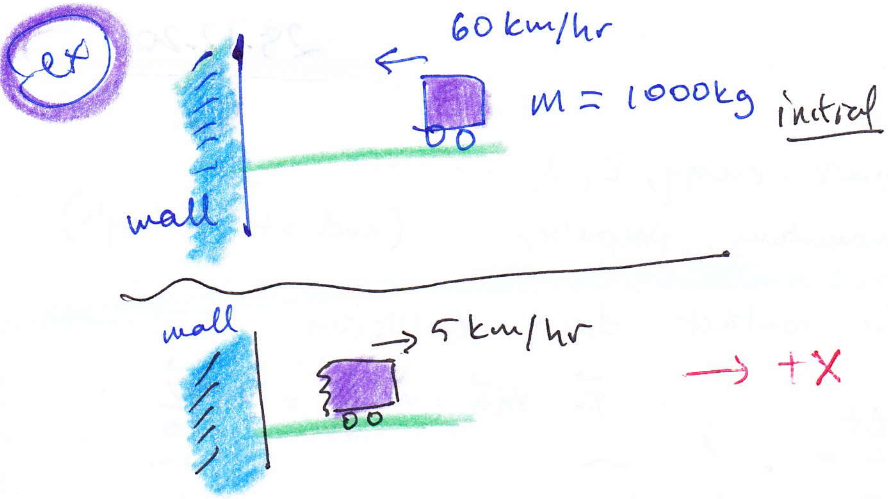
$$\boxed{\vec{J} = \int \vec{F}\,dt = \vec{F}_{av}\,\Delta t = \Delta\vec{P}}$$

↳ true for 2nd obj
↳ " " 1st obj

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$ action reaction pair during contact

↳ impulse on 2nd obj = change in momentum of 2nd object.
" " 1st " " " " " 1st ".

ex



a car is colliding with a wall
before its speed is 60 km/hr
after the collision its speed is 5 km/hr in opposite direction.
the collision took 0.1s.

a) F_{av} applied on the car?

$\vec{J} \equiv \Delta \vec{P} \Rightarrow \vec{F}_{av} \Delta t = \Delta \vec{P} = \vec{P}_f - \vec{P}_i$

$\vec{F}_{av}(0.1s) = m(\vec{v}_f - \vec{v}_i)$

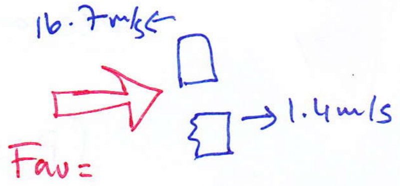
$v_i = 60 \frac{km}{hr} = \frac{60000\,m}{3600\,s} = 16.7\,m/s$

$v_f = 5 \frac{km}{hr} = \frac{5}{3.6}\,m/s = 1.4\,m/s$

$\vec{F}_{av}(0.1s) = 1000kg\left(1.4\hat{i} - (-16.7\hat{i})\right)\frac{m}{s}$

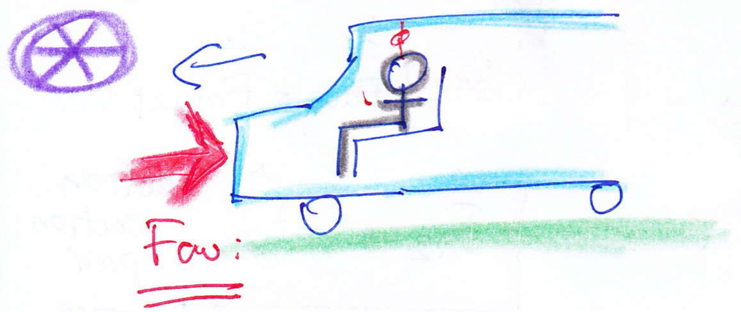
$= 18.1 \times 10^4 \hat{i}\,N$

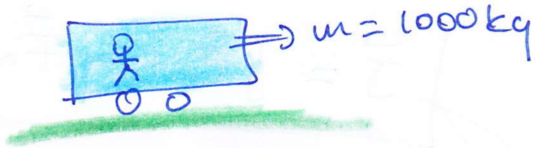
$F_{av} = 181000\,N$



m = 80kg mg ≃ 800N

F_{av} = (226 of 80kg person)
weight of





$\frac{F_{av}}{m_{car}} \times m_{person} = F_{av}$ onto person

$\frac{181000N}{1000kg}$ 80kg = 14480N

≃ 15000N

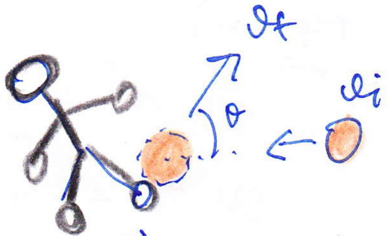
≃ weight of 18 person with 80kg

Fave onto person is 15000N ≃ 18 of  80kg

"Buckle up"
"emniyet kemerini tak"

children also!!!

ex



$v_i = 20\,m/s$ $\quad$ $\theta = 45°$

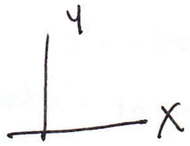
$v_f = 30\,m/s$ $\quad$ m of ball $= 0.4\,kg$

$\Delta t = 0.01\,s$

$\rightarrow +x$

$\vec{J} = ?$ impulse

$\vec{F}_{av} = ?$



$$\vec{F}_{av}\,\Delta t = \vec{J} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i = m\vec{v}_f - m\vec{v}_i$$

$$\vec{P}_i = m\vec{v}_i = (0.4)(-20)\,\hat{i}\ \left[\frac{kg\,m}{s}\right] = \left[-8\,\hat{i}\right]$$

$$\vec{P}_f = \vec{P}_{xf} + \vec{P}_{yf} = (0.4)\,30\cos 45\,\hat{i} + (0.4)\,30\sin 45\,\hat{j}\ \frac{kg\,m}{s}$$
$$= 6\sqrt{2}\,\hat{i} + 6\sqrt{2}\,\hat{j}$$

$$\vec{F}_{av}\,\Delta t = 6\sqrt{2}\,\hat{i} + 6\sqrt{2}\,\hat{j} - (-8\,\hat{i})$$

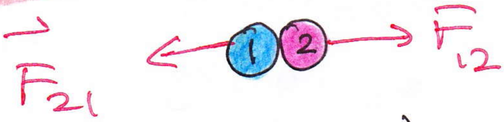
$$\vec{F}_{av}(0.01) = (6\sqrt{2} + 8)\,\hat{i} + (6\sqrt{2})\,\hat{j} \Rightarrow$$

$$\vec{F}_{av} = \frac{16.5\,\hat{i} + 8.5\,\hat{j}}{0.01} = 1650\,\hat{i} + 850\,\hat{j}$$

$$|\vec{F}_{av}| = \sqrt{1650^2 + 850^2} = \underline{\underline{1850\,N}}$$

$$\vec{F}\,\Delta t = \Delta\vec{P} = \vec{J} \quad ; \quad \underline{\vec{J} = (16.5\,\hat{i} + 8.5\,\hat{j})\ \frac{kg\,m}{s}}$$

Momentum conservation in collisions



$$\vec{F}_{12}\,\Delta t = \vec{J}_2 = \Delta\vec{P}_2$$

$$\vec{F}_{21}\,\Delta t = \vec{J}_1 = \Delta\vec{P}_1$$

{action reaction} $\quad \vec{F}_{12} = -\vec{F}_{21}$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$(\vec{F}_{12} + \vec{F}_{21})\,\Delta t = 0$$

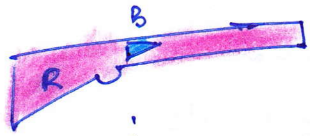
$$\Delta\vec{P}_2 + \Delta\vec{P}_1 = 0$$

$$(\vec{P}_{2f} - \vec{P}_{2i}) + (\vec{P}_{1f} - \vec{P}_{1i}) = 0$$

$$\boxed{\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f} \Rightarrow \sum\vec{P}_i = \sum\vec{P}_f}$$

momentum conservation in collisions !!

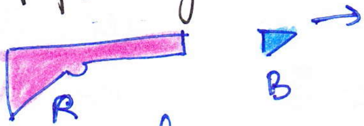
(ex) Recoil of a rifle (tüfeğin geri tepmesi)



i

bullet + rifle

"collision = explosion"



f

$\rightarrow +x$

$m_{bullet} = 5gr$

$m_{rifle} = 3kg$

$v_{BF} = 300 m/s$

final velocity of bullet.

what's recoil velocity of the rifle?

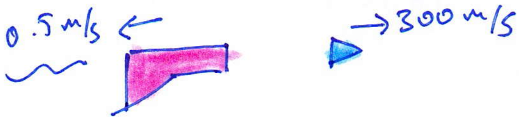
$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$\vec{P}_{Ri} + \vec{P}_{Bi} = \vec{P}_{Rf} + \vec{P}_{Bf}$$

$$m_R \vec{v}_{Ri} + m_B \vec{v}_{Bi} = m_R \vec{v}_{Rf} + m_B \vec{v}_{Bf}$$

$$0 + 0 = 3(\vec{v}_{Rf}) + (0.005)(300\hat{i})$$

$$\vec{v}_{Rf} = -0.5\hat{i} \text{ m/s}$$

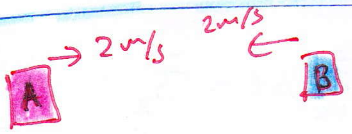


b) whats the kinetic energy of rifle & bullet finally?

$$K_R = \frac{1}{2} m_R v_{Rf}^2 = \frac{1}{2}(3)(0.5)^2 = 0.375 \text{ Joules}$$

$$K_B = \frac{1}{2}(0.005)(300)^2 = 225 \text{ Joules.}$$

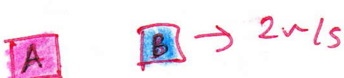
(ex)



$M_B = 0.3kg$

$m_A = 0.5kg$

$\rightarrow +x$



$v_{AF} = ?$

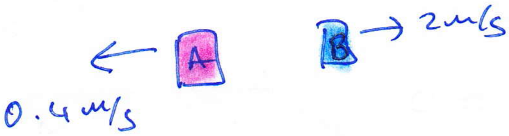
$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$$

$$(0.5)(2\hat{i}) + (0.3)(-2\hat{i}) = 0.5\vec{v}_{AF} + 0.3(2\hat{i})$$

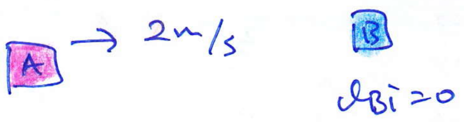
$$1\hat{i} - 0.6\hat{i} - 0.6\hat{i} = 0.5\vec{v}_{AF}$$

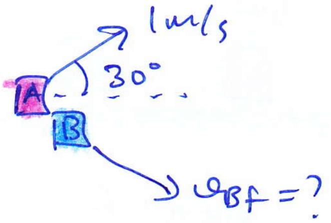
$$\frac{-0.2\hat{i}}{0.5} = \vec{v}_{AF} = -0.4 \text{ m/s}\,\hat{i}$$



ex. (3)

$m_A = 20\,kg$ $\quad$ $m_B = 12\,kg$





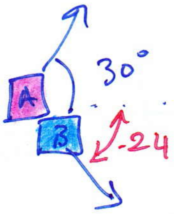
$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$$

$$20(2\hat{i}) + 0 = 20(1)\cos 30\,\hat{i} + 20(1)\sin 30\,\hat{j} + 12\,\vec{v}_{Bf}$$

$$40\hat{i} = 17.3\hat{i} + 10\hat{j} + 12\,\vec{v}_{Bf}$$

$$+22.7\hat{i} - 10\hat{j} = 12\,\vec{v}_{Bf}$$



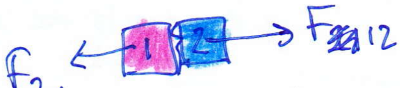
$$\vec{v}_{Bf} = \left[+1.89\,\hat{i} - 0.83\,\hat{j}\right]\ m/s$$

$$\tan\alpha = \left(\frac{-0.83}{1.89}\right) \Rightarrow \alpha = -24°$$

$$v_{Bf} = \sqrt{1.89^2 + 0.83^2} = 2.06\ m/s$$

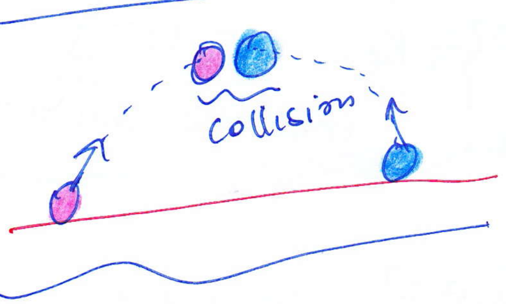
During collision

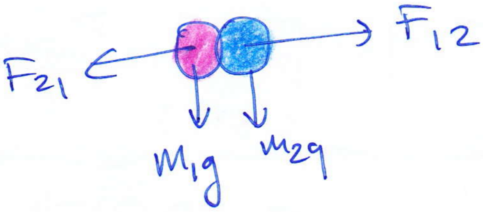
true only if there are no other forces acting on.



$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$\frac{\Delta \vec{P}_2}{\Delta t} + \frac{\Delta \vec{P}_1}{\Delta t} = 0 \Rightarrow \Sigma \vec{P}_i = \Sigma \vec{P}_f$$





$$\vec{F}_{12} + \vec{F}_{21} + \vec{m_1g} + \vec{m_2g} \neq 0$$

$\Sigma \vec{F}_{0f} \neq 0$ of the system.

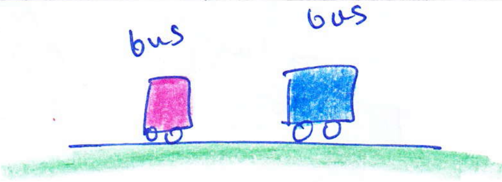
$F_{12} = F_{21} \gg m_1g, m_2g$

$1000N \gg 10N$

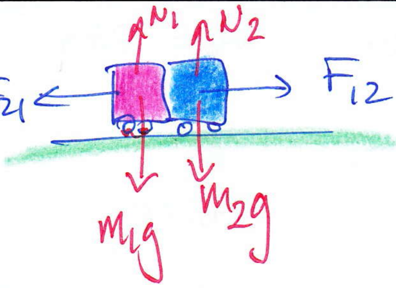
If m_1g ; m_2g are negligable then momentum is conserved.

neglect them!

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$



$\Rightarrow$



$\Rightarrow$

even if

$F_{12}, F_{21} \simeq m_1g, m_2g$

$\sum \vec{P}_i = \sum \vec{P}_f$

$\sum \vec{F}$ on system $= 0$ and

momentum conservation will be applicable

Elastic & Inelastic Collisions

Elastic

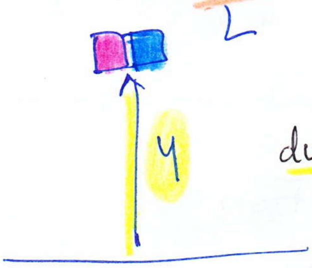
$\downarrow$

$\sum E_i = \sum E_f$

energy is conserved.

$E = K + U$

$U_i = U_f$ ✓



during collision the positions of the objects are almost always constant.

Kinetic Energy initially = Kinetic Energy finally } elastic collision

Inelastic

$\downarrow$

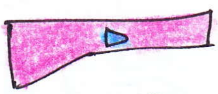
energy initial $\neq$ energy final

$K_i \neq K_F$

inelastic collision

---

ex)



$\sum K_i = 0$

0.375 J

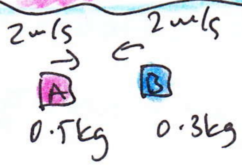
225 J

$\sum K_f = 225.375$ J

inelastic collision!

---

ex)



$\frac{1}{2}(0.5)\,2^2 + \frac{1}{2}(0.3)\,2^2$

1.6 J

$\underbrace{\frac{1}{2}(0.5)(0.4)^2}_{0.04\text{ J}} + \underbrace{\frac{1}{2}(0.3)\,2^2}_{0.6\text{ J}}$

K_i 1.6 J $\neq$ 0.64 J $= K_f$

$\sum K_i \neq \sum K_f$

inelastic collision

Collision Types

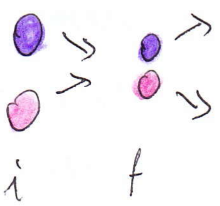
30.12.20

1

Elastic

KE conserved.

✓



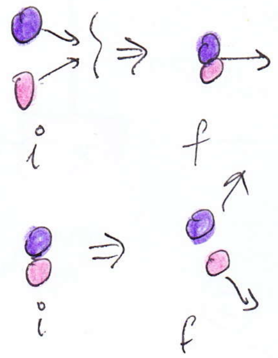
looking at the picture elastic? inelastic?

Inelastic

KE not conserved.

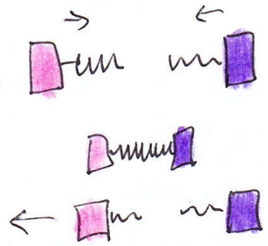
→ Total inelastic collisions

KE is not conserved.



objects are moving/staying together before/after the collision

Elastic collisions



during contact springs store energy

converts back to KE !!

at any instant momentum is always conserved!

Inelastic collisions

$$\frac{\Sigma K_f}{\Sigma K_i} = <1$$

calculate the percentage of energy lost!!

(ex) A → 2 m/s, 0.5 kg; B ← 2 m/s, 0.3 kg; → +x

after the collision A & B move together. what is their final velocity?

% K lost after collision?

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$(0.5)(2\hat{i}) + (0.3)(-2\hat{i}) = (0.8)\vec{v}_f$$

$$\frac{0.4\hat{i}}{0.8} = \vec{v}_f = 0.5\,\text{m/s}\,\hat{i}$$

$$\Sigma K_i = \frac{1}{2}(0.5)2^2 + \frac{1}{2}(0.3)\,2^2$$

$$= 1.6\text{ J}$$

$$\Sigma K_f = \frac{1}{2}(0.8)(0.5)^2 = 0.1\text{ J}$$

energy lost.

1.6 J → 0.1 J

$$\frac{1.6 - 0.1}{1.6} = \frac{\text{LOST } K}{\text{initial } K}$$

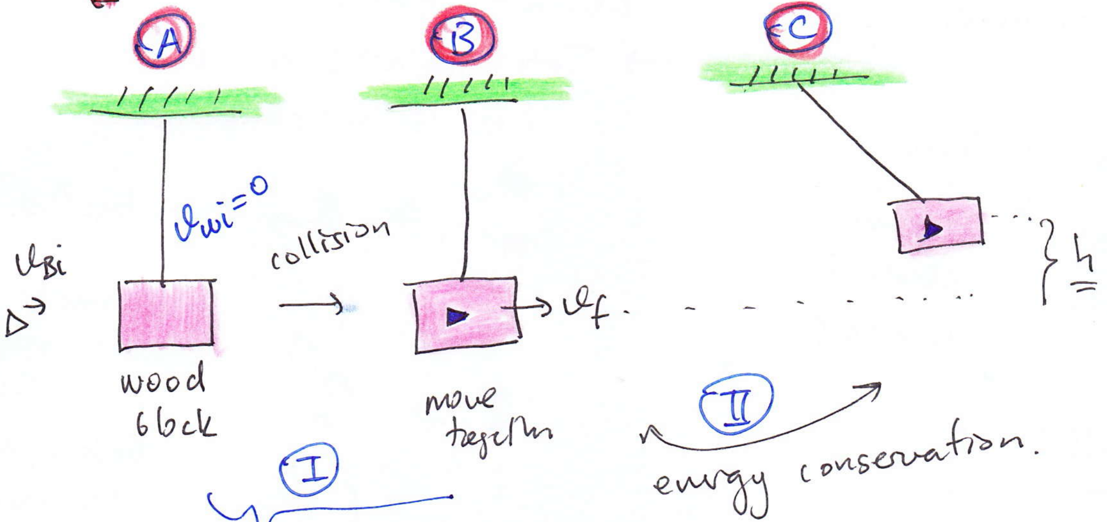
% LOST energy = 0.94

94 % lost.

Ballistic Pendulum

the aim is to measure the initial velocity of a bullet.

$x, t \Rightarrow$ we need very fast camera!! (not feasible)



measure $\underline{h}$ to calculate v_{Bi} = ✓

m_B ; M_W

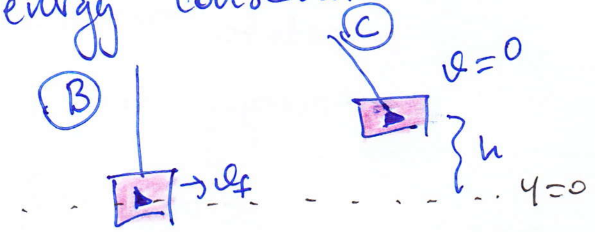
bullet mass ; wooden block mass.

I momentum conservation

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$m_B \vec{v}_{Bi} + M_w(0) = (m_B + M_w)\vec{v}_F \; ; \quad \boxed{v_F = \frac{m_B v_{Bi}}{(m_B + M_w)}} \quad (1)$$

II energy conservation



$$E_B = E_C$$

$$\frac{1}{2}(m_B + m_w) v_f^2 + 0 = 0 + (m_B + m_w) g h$$

$$K_B + U_B = K_C + U_C$$

$$\boxed{\frac{1}{2} v_f^2 = g h} \quad (2)$$

$$(1) \Rightarrow v_{Bi} = \frac{m_B + m_w}{m_B} v_f$$

$$\boxed{v_{Bi} = \frac{m_B + m_w}{m_B}\sqrt{2gh}}$$

$\Rightarrow$ just measure $\underline{h}$

v_{Bi} ✓

Don't forget collision here is inelastic. $\boxed{A \to B}$ mom. conserved.

from $\boxed{B \to C}$ $\Rightarrow$ energy conserved

2

ex) $m_B = 5\,gr$

$M_W = 2\,kg$

$h = 3\,cm$

$v_{Bi} = ?$

$v_f = ?$

how much energy LOST during collision?

$$v_{Bi} = \frac{2.005\ kg}{0.005\ kg}\sqrt{2(9.8)(0.03)}$$

$$\boxed{v_{Bi} = 307\ m/s}$$

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$(0.005)(307) + 0 = (2.005)\, v_f$$

or

$$v_f = \sqrt{2gh} =$$

$$\boxed{v_f = 0.76\ m/s}$$

$$\sum K_i = \frac{1}{2}(0.005)(307)^2$$

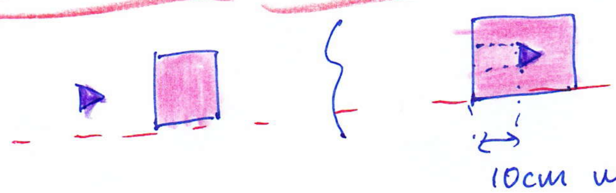
$$= 236\ J$$

$$\sum K_f = \frac{1}{2}(2.005)(0.76)^2 = 0.59 \simeq 0.6\ J$$

$$\frac{236 - 0.6}{236} = 0.997 \quad ; \quad 99.7\ \%\ \text{LOST}$$

where does this energy go?

bullet is making a hole.

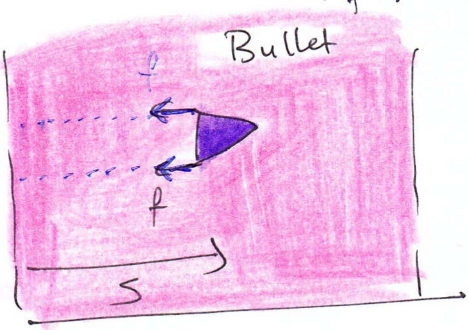


$y=0$ energy goes to shape/change also to friction, heat.

$$W_{other} + E_i = E_f$$

$$W_f + K_i + \cancel{U_i}^{\,0} = K_f + \cancel{U_f}^{\,0}$$

$$W_f + 236 = 0.6 \Rightarrow W_f = -235.4\ J$$



f is constant assume.

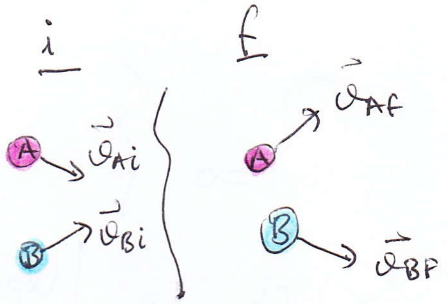
$$W_f = -f\underbrace{s}_{(0.1m)} = -235.4$$

$$f = 2354\ N$$

04.01.21

1

Momentum ⇒ Elastic collisions



(1) $\sum \vec{P}_i = \sum \vec{P}_f$

(2) $\sum K_i = \sum K_f$

$$\boxed{\begin{aligned} &\sum \vec{P}_i = \sum \vec{P}_f \\ &m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf} \quad * \\ &\sum K_i = \sum K_f \\ &\tfrac{1}{2} m_A v_{Ai}^2 + \tfrac{1}{2} m_B v_{Bi}^2 = \tfrac{1}{2} m_A v_{Af}^2 + \tfrac{1}{2} m_B v_{Bf}^2 \quad * \end{aligned}}$$

$\left\{ \begin{matrix} m_A & v_{Ai} \\ m_B & v_{Af} \\ & v_{Bi} \\ & v_{Bf} \end{matrix} \right\}$ 6 parameters

1 dimensional elastic collision

assume. $v_{Bi} = 0$

$$\begin{cases} m_A v_{Ai} = m_A v_{Af} + m_B v_{Bf} \quad ✓ \\ \cancel{\tfrac{1}{2}} m_A v_{Ai}^2 = \cancel{\tfrac{1}{2}} m_A v_{Af}^2 + \cancel{\tfrac{1}{2}} m_B v_{Bf}^2 \end{cases}$$

$m_B v_{Bf}^2 = m_A (v_{Ai}^2 - v_{Af}^2)$ --- (1)

$m_B v_{Bf} = m_A (v_{Ai} - v_{Af})$ -- (2)

$\frac{(1)}{(2)} \Rightarrow \boxed{v_{Bf} = v_{Ai} + v_{Af}}$ -- (3)

I know m_A m_B v_{Ai} ✓

v_{Af}?

v_{Bf}?

into (2)

$m_B (v_{Ai} + v_{Af}) = m_A (v_{Ai} - v_{Af})$

$m_B v_{Af} + m_A v_{Af} = m_A v_{Ai} - m_B v_{Ai}$

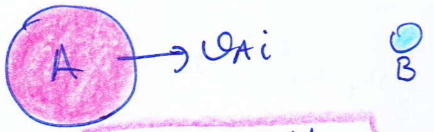
$$\boxed{v_{Af} = \cancel{m_A}\, v_{Ai} \frac{(m_A - m_B)}{(m_A + m_B)}}$$

→ into (3) to get v_{Bf} = ✓

$$v_{BF} = \left(1 + \frac{m_A - m_B}{m_A + m_B}\right) v_{Ai} = \boxed{\frac{2m_A}{m_A+m_B} v_{Ai} = v_{BF}}$$

$$\boxed{v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai}} \; ; \; \boxed{v_{BF} = \frac{2m_A}{m_A + m_B} v_{Ai}}$$

Two extremes:



$\boxed{m_A >> m_B}$

1000 kg >> 1 kg

$m_A + m_B \simeq m_A$

$m_A - m_B \simeq m_A$

$$v_{AF} = \frac{m_A}{m_A} v_{Ai} \cong v_{Ai}$$

$$v_{Bf} \cong \frac{2m_A}{m_A} v_{Ai} \simeq 2v_{Ai}$$

$\boxed{m_B >> m_A}$

1000kg >> 1kg $\quad m_A + m_B \simeq m_B$

$m_A - m_B \simeq -m_B$

$$v_{Af} = \frac{-m_B}{m_B} v_{Ai} \simeq -v_{Ai}$$

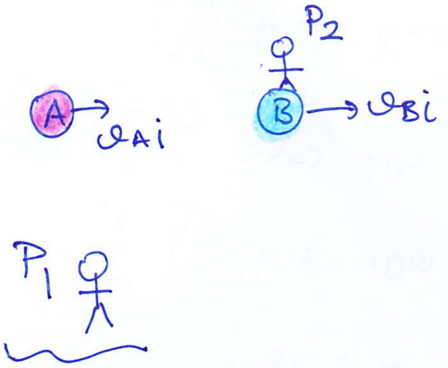
$$v_{Bf} = \frac{2m_A}{m_B} v_{Ai} \simeq \underline{\underline{0}}$$

what if $\boxed{m_A = m_B}$

$$v_{AF} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} = 0$$

$$v_{Bf} = \frac{2m_A}{m_A + m_B} v_{Ai} = \frac{2m_A}{2m_A} v_{Ai} = v_{Ai}$$

(2)



accordi to P_1

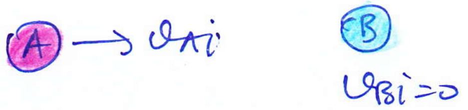
$$\boxed{v_{Ai}' = v_{Ai} - v_{Bi}}$$

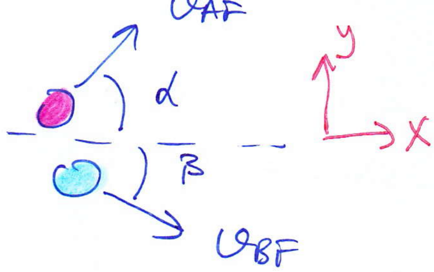
acc. to P_2

$$\left\{\begin{aligned} v_{Af}' &= \frac{m_A - m_B}{m_A + m_B} v_{Ai}' \\ v_{Bf}' &= \frac{2m_A}{m_A + m_B} v_{Ai}' \end{aligned}\right.$$

v_{Af}' , v_{Bf}' observed by $\underline{P_2}$

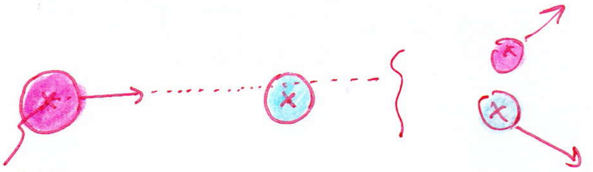
2dimensional elastic collision





m_A m_B v_{Ai} v_{Af} are given.

$$\boxed{\alpha, \beta, v_{Bf} = ?}$$



when cm are not aligned perfectly. they will go in 2d directions.

$$\sum \vec{P_i} = \sum \vec{P_f}$$

$$m_A v_{Ai} \hat{i} + 0 = m_A v_{Af} \cos\alpha \, \hat{i} + m_A v_{Af} \sin\alpha \, \hat{j} + m_B v_{Bf} \cos\beta \, \hat{i} + m_B v_{Bf} \sin\beta \, (-\hat{j})$$

2 eqn's. $\hat{i}$; $\hat{j}$

$$m_A v_{Ai} \, \hat{i} = (m_A v_{Af} \cos\alpha + m_B v_{Bf} \cos\beta) \, \hat{i}$$

$$0 \, \hat{j} = (m_A v_{Af} \sin\alpha - m_B v_{Bf} \sin\beta) \, \hat{j}$$

$$\sum K_i = \sum K_f$$

$$\cancel{\tfrac{1}{2}} m_A v_{Ai}^2 + 0 = \cancel{\tfrac{1}{2}} m_A v_{Af}^2 + \cancel{\tfrac{1}{2}} m_B v_{Bf}^2$$

α ?
β ?
v_{Bf} ?

$$\left\{\begin{aligned} m_A &= 0.5 \text{ kg} \\ m_B &= 0.3 \text{ kg} \end{aligned}\right. \qquad \left.\begin{aligned} v_{Ai} &= 4 \text{ m/s} \\ v_{Af} &= 2 \text{ m/s} \end{aligned}\right\} \text{given}$$

$$(0.5)\,4\,\hat{i} = \left[0.5\,(2)^{\cos\alpha} + 0.3\,v_{Bf}\cos\beta\right]\hat{i}$$

$$(2 = \cos\alpha + 0.3\,v_{Bf}\cos\beta)\,\hat{i} \longrightarrow (1)$$

$$(0 = \sin\alpha - 0.3\,v_{Bf}\sin\beta)\,\hat{j} \longrightarrow (2)$$

$$0.5\,(4^2) = (0.5)(2^2) + 0.3\,v_{Bf}^2$$

$$8 = 2 + 0.3\,v_{Bf}^2$$

$$v_{Bf} = \sqrt{\frac{6}{0.3}} = \sqrt{20}\ \text{m/s} \simeq 4.47\ \text{m/s}$$

into (1) & (2)

$$2 = \cos\alpha + 0.3\sqrt{20}\cos\beta$$

$$\sin\alpha = 0.3\sqrt{20}\sin\beta$$

$$\cos\beta = \left(\frac{2-\cos\alpha}{0.3\sqrt{20}}\right) \;; \quad \sin\beta = \left(\frac{\sin\alpha}{0.3\sqrt{20}}\right)$$

$$\boxed{\cos^2\beta + \sin^2\beta = 1}$$

$$\frac{4+\cos^2\alpha - 4\cos\alpha}{(0.3)^2(20)} + \frac{\sin^2\alpha}{(0.3)^2(20)} = 1$$

$$4 + 1 - 4\cos\alpha = (0.3)^2\,20$$

$$-\cos\alpha = \frac{1.8 - 5}{4}$$

$$\cos\alpha = \frac{5-1.8}{4} = \frac{3.2}{4} = 0.8$$

$$\alpha = \cos^{-1}(0.8)$$

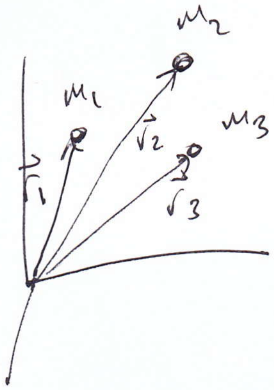
$$= 36.869 \simeq 37^\circ$$

$$\sin\beta = \frac{\sin\alpha}{0.3\sqrt{20}} = \frac{0.6}{0.3\sqrt{20}} = \frac{2}{\sqrt{20}}$$

$$\beta = \sin^{-1}\left(\frac{2}{\sqrt{20}}\right) = 26.56 \simeq 27^\circ$$

$$\boxed{\begin{aligned}\alpha &= 37^\circ\\ \beta &\simeq 27^\circ\\ v_{Bf} &= \sqrt{20}\ \text{m/s}\end{aligned}}$$

Center of Mass



$\Longleftrightarrow$

; $M = m_1 + m_2 + m_3$

$$M\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3$$

$$\boxed{\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}}$$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\boxed{\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}}$$

$$\vec{r}_{cm} = X_{cm}\hat{i} + Y_{cm}\hat{j} + Z_{cm}\hat{k}$$

$\frac{d}{dt}$

$$\frac{d\vec{r}_{cm}}{dt} = \vec{v}_{cm} = \frac{\sum m_i \frac{d\vec{r}_i}{dt}}{\sum m_i} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

$$\boxed{\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}}$$

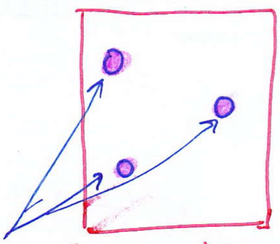
$$* \quad \boxed{\vec{v}_{cm} \sum m_i = \sum m_i \vec{v}_i}$$ mom. conservation

$$\boxed{\vec{a}_{cm} \sum m_i = \sum m_i \vec{a}_i}$$

mass of a system collides with each other

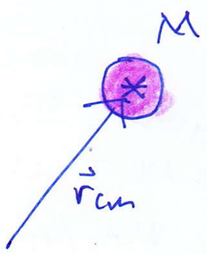
or $M \Rightarrow m_1 + m_2 + m_3$ explosion

net force on the system is zero!!

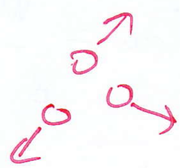


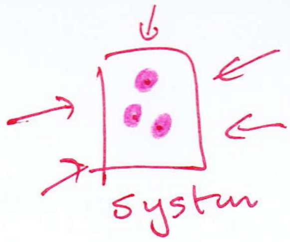
isolated system

$\Longleftrightarrow$



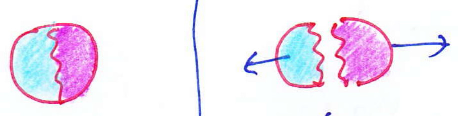
$\Longleftrightarrow$





$\Sigma F_{system/external} = 0$

$\Sigma \vec{F}_{external} = 0 \Rightarrow \Sigma \vec{P} = const.$



$t=0$ | $t>0$

$v=0$

$\Sigma P_i = 0$ | $\Sigma P_f = 0$

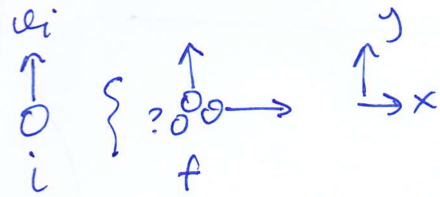
---

ex An object is thrown vertically at 1000 m above ground. Its velocity is $300\ m/s\ \hat{j}$; At this moment object splits into 3 equal parts.

1st part $450\ m/s$ upward $\hat{j}$

2 " $240\ m/s$ to west

$v_3 = ?$



$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

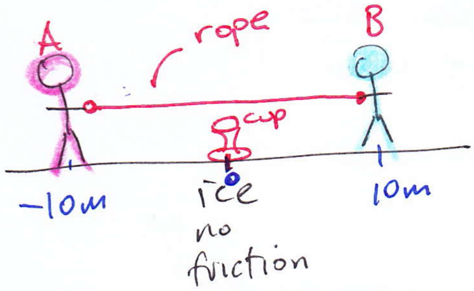
$$(3\cancel{m})\,300\hat{j} = \cancel{m}\,450\hat{j} + \cancel{m}\,240\hat{i} + \cancel{m}\,\vec{v}_3$$

$$\boxed{(900-450)\hat{j} - 240\hat{i} = \vec{v}_3} = -240\hat{i} + 450\hat{j}$$

$\nwarrow v_3$

---

ex TUG of WAR (halat çekme)



cup is in the middle point of two person.
they pull the rope; they are on ice
no friction

Distance btwen A & B = 20m

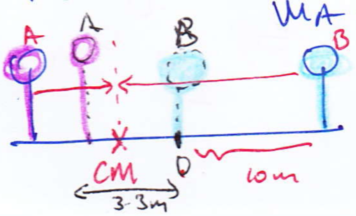
$m_A = 90kg$

$m_B = 60kg$

who reaches the cup first?

* A+B ⇒ system ; $\Sigma F_{ext} = 0$
cm does NOT change.

$$X_{cm} = \frac{m_A X_A + m_B X_B}{m_A + m_B} = \frac{90(-10) + 60(10)}{150} = -2m$$

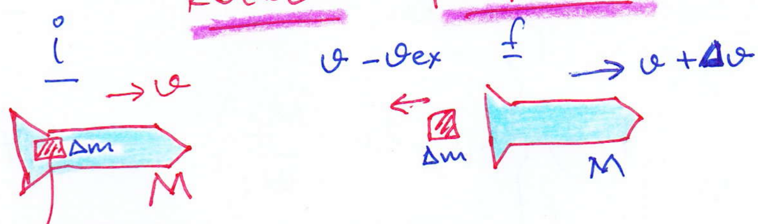


can NOT go further then cm.

If B moves 10m;

$$\frac{90(X_A) + 60(0)}{150} = -2 \quad X_A = \frac{-30\cancel{0}}{9\cancel{0}} = -3.3m$$

Rocket Propulsion



v_{ex} = exhaust speed of fuel.

push out the fuel at the same speed $= v_{ex}$

as the fuel is exhausted (pushed) the rocket will have new speed of $v + \Delta v$

v_{ex} with respect to the Rocket

M = mass of rocket

Δm = " of fuel exected!

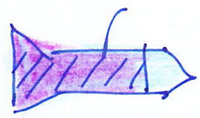
$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$(M + \Delta m)\vec{v} = \Delta m(\vec{v} - \vec{v}_{ex}) + M(\vec{v} + \Delta\vec{v})$$

$$Mv + \Delta m v = \Delta m v - \Delta m v_{ex} + Mv + M\Delta v$$

$$M\Delta v = \Delta m v_{ex}$$

%80 fuel



Technically total mass rocket $= M + \Delta m = M_T$

$M_T = M_B + M_{fuel}$

Total mass of rocket

t	M_T	M_B	Δm	M_{fuel}
0	10000	1000	0	9000
1	9900	1000	100	8900
2	9800	1000	$\boxed{100}$	8800

100 (between 8900 and 8800); 100 (between 9900 and 9800)

$\boxed{\Delta m = -\Delta M \quad M_B = \text{Bare weight of rocket w/o fuel}}$

$M_T = M_B + M_{fuel}$ $\Delta m = -\Delta M_T$

Δm

$\underline{\underline{M}}$: total mass of rocket.

as the fuel mass ejected; total mass change is negative

$\hookrightarrow dm = -dM$

$$\boxed{M\Delta v = -\Delta M v_{ex}}$$

$$\boxed{M\,dv = -dM\,v_{ex}}$$

**

$$\boxed{M\,dv = -dM\,v_{ex}} \quad ; \quad v_{ex} = \text{const.}$$

$$\int_i^f -\frac{dv}{v_{ex}} = \int_i^f \frac{dM}{M}$$

$$-\frac{\Delta v}{v_{ex}}\Big|_i^f = \ln M \Big]_i^f$$

$$\ln M_f - \ln M_i = \frac{-(v_f - v_i)}{v_{ex}}$$

$$-v_{ex} \ln\left(\frac{M_f}{M_i}\right) = v_f - v_i$$

$$\boxed{v_f - v_i = v_{ex} \ln \frac{M_i}{M_f}}$$

$M_i > M_f$; fuel is exhausted.

$$\frac{d}{dt}(M\,dv) = -\frac{d}{dt}(dM\,v_{ex})$$

$$\frac{d\vec{P}}{dt} = -\frac{d\vec{P}}{dt}$$

$$\left| M \overset{\vec{a}}{\frac{dv}{dt}} \right| = \left| v_{ex} \frac{dm}{dt} \right|$$

$$\vec{F} = \frac{dM}{dt} v_{ex} \quad ; \quad a = \frac{dv}{dt} = \frac{v_{ex}}{M}\left(\frac{dm}{dt}\right)$$

force on rocket

ex.) A rocket ejects fuel in the 1st second; it eject $\frac{1}{120}$ of its initial mass at a speed of 2400 m/s.

$\vec{a} = ?$ acc. of rocket.

$$\frac{dM}{dt} = \frac{\left(\frac{M}{120}\right)}{1\,\text{second}}$$

$$M\,dv = -dM\,v_{ex}$$

$$M\,a = -v_{ex} \frac{dM}{dt}$$

$$M\,a = (2400)\frac{M}{120} \Rightarrow \underline{a = 20\ m/s^2} \simeq 2g$$

2

b) if $v_i = 0$ m/s and the $\frac{3}{4}$ of the mass of rocket is fuel; fuel is consumed at const rate in 90seconds.

$v_f = ?$ of the rocket.

$$M\,dv = -dM\,v_{ex} \Rightarrow v_f - \underbrace{v_i}_{0} = v_{ex} \ln \frac{M_i}{M_f}$$

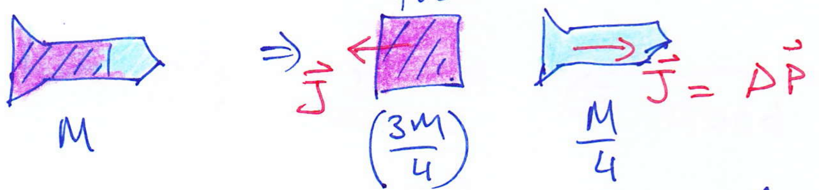
$$m_f = m_i - M_{fuel} = M - \frac{3}{4}M = \frac{M}{4}$$

$$v_f = 2400 \ln \frac{M}{\left(\frac{M}{4}\right)} = 2400 \ln 4$$

$$v_f = 3327.\ \text{m/s} \ @\ 90s$$

c) if $M = 1000$ kg fuel is ejected for 90s;

whats the propulsion force on the rocket?



$$\frac{\vec{J}}{\Delta t} = \vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

$$F = M\frac{dv}{dt} = v_{ex}\frac{dM}{dt} \Rightarrow F = (2400)\frac{\left(\frac{3M}{4}\right)}{90s} =$$

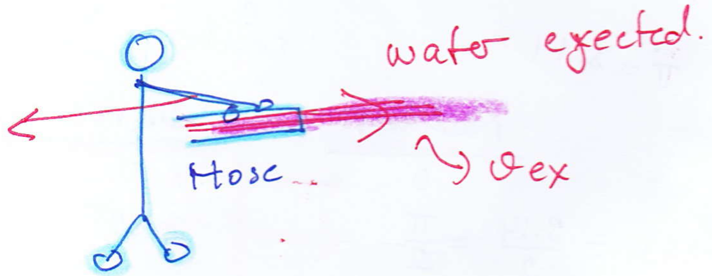
$$= 2400 \frac{\frac{3000}{4}}{90}$$

$$= 20\,000\ N$$

$$= 20\ kN$$

$$M\frac{dv}{dt} = F = v_{ex}\frac{dM}{dt}$$; Fire fighters (itfayei)

$$\frac{dM}{dt} = \left[\frac{kg}{s}\right] \cdot \underbrace{\left[\frac{m}{s}\right]}_{= v_{ex}}$$

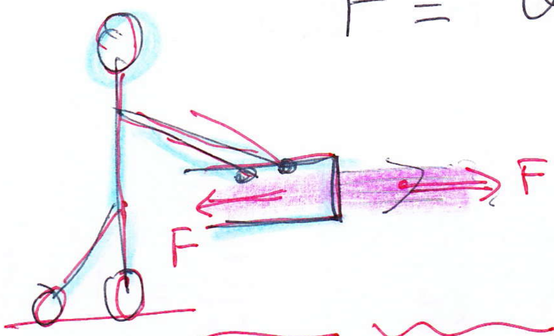


ex) if water is exhausted at $\frac{3600 L}{\text{minute}}$ from a fire hose; the force applied on the hose is 600 N; whats v_{ex} =?

$\frac{L}{min} \sim \frac{kg}{s}$ 1L ≃ 1kg of water

1 min = 60 s

$3600 \frac{L}{min} = \frac{dM}{dt} = \frac{3600\, kg}{60\, s} = 60 \frac{kg}{s}$

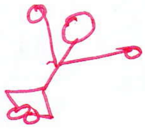


$F = v_{ex} \frac{dM}{dt} \Rightarrow 600\, N = v_{ex}\, 60 \frac{kg}{s}$

$v_{ex} = 10\, m/s$

600 N 600N ⇒ 10 m/s = v_{ex}

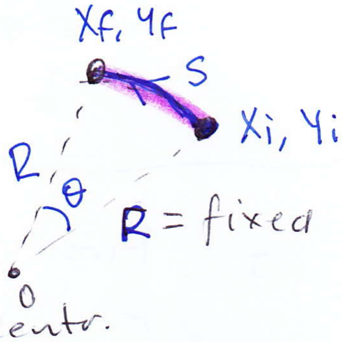
Ch8 Finished



Chapter 9 Rotation of Rigid Objects

Rotation

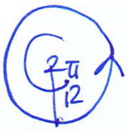
2 dimentional motion



arclength of path

$\boxed{S = R\theta}$

θ ≡ in terms of radians.



$R(2\theta)$ = circumfrence of circle

π = 3.14

radians		degree
0		0
1.67 = $\frac{3.14}{2}$ = $\frac{\pi}{2}$		90°
π	(3.14)	180
$\frac{3\pi}{2}$	($\frac{3}{2}$ 3.14)	270
2π	(2 3.14)	360°

180° = π = 3.14 rad.

$1^\circ = \frac{3.14\, rad}{180}$

$\frac{180^\circ}{3.14} = 57.3^\circ = 1\, rad$

rad ⇒ unitless number

$$\boxed{1\ \text{rad} = \frac{180}{\pi} = 57.3^\circ}$$

angular displacements will be interms of radians.

Linear motion		Rotational motion	
X	$\longleftrightarrow$	θ	= angular displacement
v	$\longleftrightarrow$	ω	= angular velocity
a	$\longleftrightarrow$	α	= angular acceleration

$\frac{d}{dt}$: $X \to v \to a$; $\frac{d}{dt}$: $\theta \to \omega \to \alpha$

α is related to a_t = tangential acceleration.

$\theta \Rightarrow \omega = \frac{d\theta}{dt}$ (instantanous angular velocity)
(omega)

$\bar{\omega} = \omega_{av} = \frac{\Delta\theta}{\Delta t}$ (averge ang. velocity)

$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (instantaneous angular acc.)

$\bar{\alpha} = \alpha_{av} = \frac{\Delta\omega}{\Delta t}$ (averge ang. accelration)
(alpha)

$\theta \Rightarrow [\text{rad}]$; $\omega = \left[\frac{\text{rad}}{s}\right]$; $\alpha = \left[\frac{\text{rad}}{s^2}\right]$

⇒ 2π
θ
6.28 rad

1 rotation
1 revolution

10 rpm = 10 rotation per minute = ω

10 rpm = $\frac{10 \times 2\pi}{60\ s}$ rad

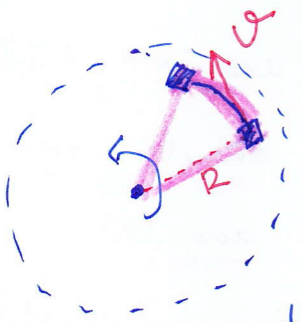
$$\boxed{10\ \text{rpm} = 1.05\ \frac{\text{rad}}{s}}$$

when $a = const$ rotation $\alpha = const$

$$x_f = x_i + v_i t + \frac{1}{2} a t^2 \longrightarrow \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$v_f = v_i + at \longrightarrow \omega_f = \omega_i + \alpha t$$

$$v_f^2 = v_i^2 + 2a\,\Delta x \longrightarrow \omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta \checkmark$$



$R = const$

$$x \simeq s = R\theta$$

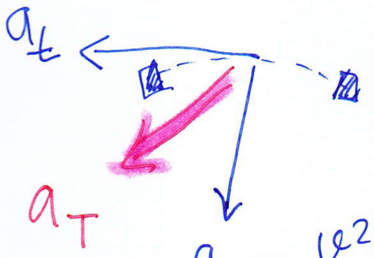
$$\frac{dx}{dt} = \cancel{\frac{dR}{dt}}^{\,0}\,\theta + R\frac{d\theta}{dt}$$

$$\boxed{v = R\omega} \; ; \; \boxed{x = R\theta}$$

$\frac{d}{dt}$

$$\frac{dv}{dt} = R\frac{d\omega}{dt}$$

$$\boxed{a_t = R\,\alpha} \text{ (tangential acceleration)}$$



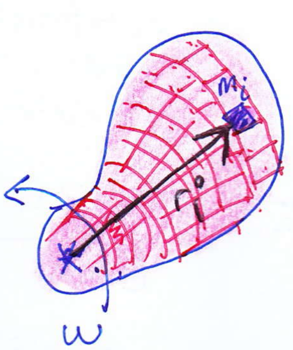
$$a_r = \frac{v^2}{R} \text{ (radial acc.)}$$

$$a_t \neq a_r = \frac{v^2}{R}$$

$$\boxed{\vec{a}_T = \vec{a}_r + \vec{a}_t}$$

non uniform circular motion

ENERGY in ROTATIONAL MOTION



KE = ? of rotating object. $\frac{1}{2} m v^2 = K$

$$K_i = \frac{1}{2} m_i v_i^2 \; ; \; \sum K_i = K \checkmark$$

every little piece will have K_i kinetic energy.

~~it will~~ all K_i will be different.

$\frac{1}{2} m_i v_i^2 = K_i$; $v_i = r_i w = w r_i$

every m_i will have SAME w; different r_i value.

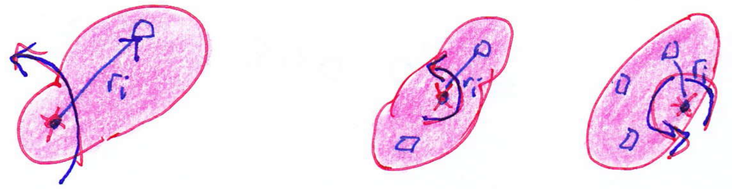
$K_i = \frac{1}{2} m_i (w r_i)^2$; $K = \sum K_i$

$$K \equiv \frac{1}{2} I w^2 \Longleftarrow = \sum_i \frac{1}{2} m_i r_i^2 w^2$$

I (moment of inertia) (eylemsizlik momenti)

$$\boxed{I = \sum m_i r_i^2}$$

moment of inertia.



moment of inertia depends on the rotation axis

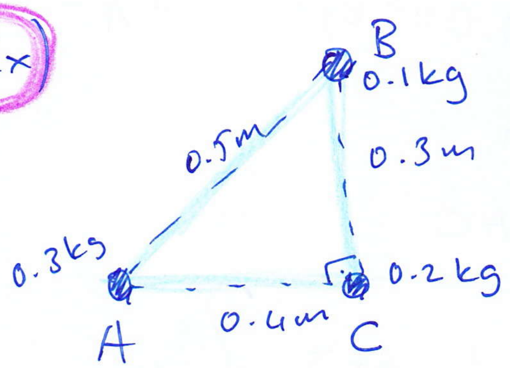
$K = \frac{1}{2} I w^2 \;\langle \simeq \rangle\; K = \frac{1}{2} m v^2$

$\underset{\text{linear}}{m} \rightsquigarrow \underset{\text{(rotation)}}{I}$

r is the $\perp$ distance to rotation axis.

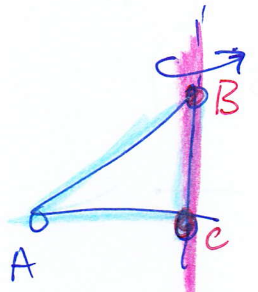
$\perp$ perpendicular

Ex



system is composed of 3 masses and distributed as in the figure.

$I_{BC} = ?$ when the system is rotated along BC Line?



$$I_{BC} = \sum m_i r_i^2 = m_A r_A^2 + m_B r_B^2 + m_C r_C^2$$

$$= 0.3(0.4)^2 + (0.1)\,0^2 + (0.2)\,0^2$$

$$\boxed{I_{BC} = 0.012 \text{ kgm}^2}$$