Phys2-04 Current, Resistance etc

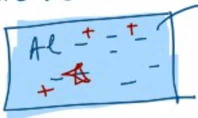
Monday, April 10, 2023 2:41 PM

Chapter 25: Current Resistance & Electromotive Force

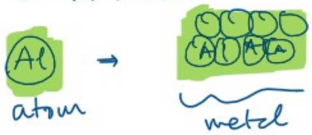
upto now electrostatics $\vec{E}, \vec{F}, V, C$ ....

→ after Ch 25 Electrodynamics charges are moving.

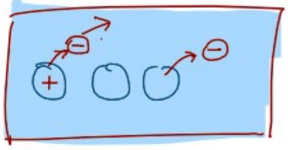
metal



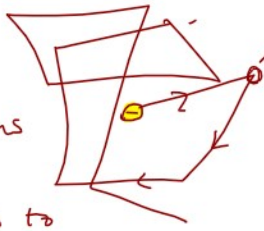
free e^- in metals



atom → metal



- abundance of free charges (−)
- free e^- collide with other e^-, and atoms



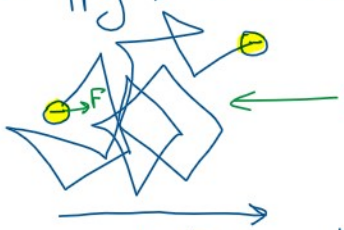
collision

Random motion collisions

- electrostatic = time average of an e^-, e^- seems to stay.

what if we apply $\vec{E}$ in the metal.

$F = eE$



E

Net displacement

$\vartheta \to \vartheta_d$ = drift velocity

$\vec{E}$ field drift (sürüklene) e^- in the metal.

collisions + eE force

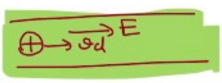
$\vartheta \sim 10^6$ m/s

$c = 3 \times 10^8$ m/s

$\vartheta_d \sim$ mm/s $\sim 10^{-3}$ m/s $\ll 10^6$ m/s

wire

$\Rightarrow$



$qE = F$

current is defined w.r.t. ⊕ charge definition.

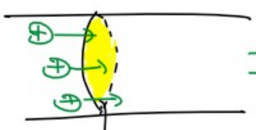
current $I \equiv \dfrac{Q}{t} = \dfrac{\Delta Q}{\Delta t} = \left[\dfrac{\text{Coulomb}}{\text{sec}} = \text{Ampere} \;;\; \dfrac{C}{s} = A\right]$

→ one of SI units

Phys I ⇒ kg, m, s

Phys II $A \sim \dfrac{\text{charge}}{\text{time}}$ $[C = A \cdot s]$

wire

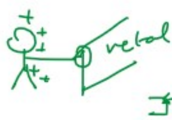


$I = ?$

Area = crossectional area

$I = \dfrac{Q \uparrow}{t \downarrow}$; $I \uparrow$

$\dfrac{q}{t}$ = Ampere



$\dfrac{\mu C}{ms} = mA$

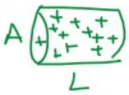
$\dfrac{\Delta Q}{\Delta t} = I$

$\Delta Q = q_e$ (number of charges)

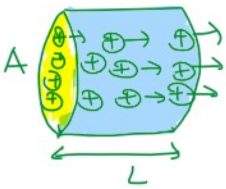
↳ $(1.6 \times 10^{-19} C)$

number of charges depends on the metal

$n = \dfrac{\text{number of charge}}{\text{Volume}} = \left[\dfrac{\text{number}}{m^3}\right]$



$AL = \text{volume}$



number of charges $\; n \text{ Volume} = nAL$

$$\frac{q_e(nAL)}{\Delta t} = \frac{\Delta Q}{\Delta t} = I$$

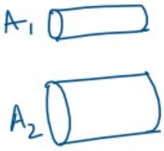
$$I = \frac{q_e n A L}{\Delta t}$$

velocity drift

$L = v_d \, \Delta t$

$$\boxed{I = q_e n A v_d}$$

$q_e = 1.6 \times 10^{-19}\, C$



$I \propto A$

$$\boxed{J} \equiv \text{current density} = \frac{I}{A} = \boxed{q_e n v_d} = \left[\frac{A}{m^2}\right]$$

$$\left[C \frac{1}{m^3} \frac{m}{s} = \frac{A}{m^2}\right]$$

ex) Cu wire diameter of 1.02 mm $\quad I = 1.67\, A$

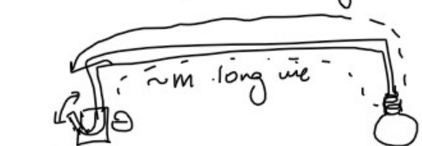
free e^- density $n = 8.5 \times 10^{28} \frac{1}{m^3}$ $\quad J = ? \quad v_d = ?$

$$J = \frac{I}{A} = \frac{1.67\, A}{\pi\left(\frac{1.02\times10^{-3} m}{2}\right)^2} = 2 \times 10^6\, A/m^2$$

$v_d = ?$ $\quad I = q_e n A v_d \quad ; \quad J = q_e n v_d$ $\quad (q_e \to 1.6\times10^{-19})$

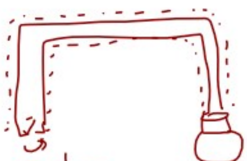
$$v_d = 0.15\, mm/s = 1.5 \times 10^{-4}\, m/s \quad (\text{very small})$$

Q) How come turning the lights on/off is much faster than v_d?



$$\frac{1m}{v_d} \sim \frac{1m}{0.1mm} \sim 10^3 s \sim 20 \text{ minutes}$$

there are e^- everywhere



$v \sim 10^6\, m/s$

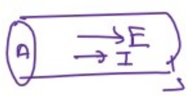
one ⊖ to pump from left to right on the switch is enough to turn on the light.

Resistivity (Direnç)

conductivity

$$J = \sigma E$$

conductivity



$\frac{J}{A} \propto I \propto \vec{E}$

$$\rho J = E$$

resistivity

$$\boxed{J = \sigma E} \qquad \boxed{\rho\sigma = 1}$$

$$\boxed{\begin{aligned} J &= \sigma E \\ J &= \frac{E}{\rho} \end{aligned}}$$

$\rho J = E$ — resistivity

$\boxed{\rho\sigma = 1}$

$\frac{I}{A} \propto I \propto E$

OHM's Law

$\int E\,dr = \Delta V$

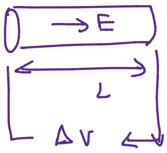
$E = \frac{\text{volt}}{\text{meter}} = \frac{V}{m}$

$J = \sigma E$

$\frac{A}{m^2} = \sigma \frac{V}{m}$

$$\left[\rho = \frac{Vm}{A}\right] \qquad \left[\sigma = \frac{A}{Vm}\right] \Rightarrow$$

$$\frac{I}{A} = J = \frac{E}{\rho} = \frac{\Delta V}{\rho L}$$



$\Delta V = EL$

$$\frac{\Delta V}{\rho L} = \frac{I}{A} \rightarrow \Delta V = \left(\rho \frac{L}{A}\right) I = RI$$

OHM's Law

$$\boxed{\Delta V = IR}$$

R = resistance $= \rho \frac{L}{A}$ $\qquad$ ρ = resistivity $\qquad$ $R \equiv [\Omega]$ = ohm

$\rho = \frac{RA}{L} = \left[\frac{\Omega m^2}{m}\right]$

Table 25.1 Resistivities at Room Temperature (20°C)

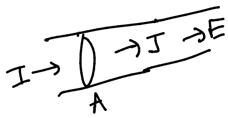
Substance		ρ ($\Omega \cdot$m)	Substance	ρ ($\Omega \cdot$m)
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

ρ

10^{-8} $\qquad$ 10^{16}

10^{24}

$$J = \frac{I}{A} = \frac{E}{\rho}$$



$R = \rho \frac{L}{A}$

$[\Omega\, m] = \rho$

$\frac{I}{A} = J = \frac{E}{\rho}$ OHM's LAW

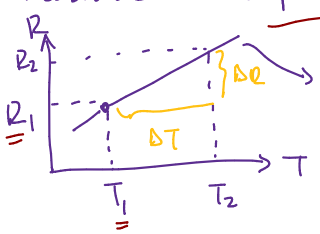
$E = \frac{\Delta V}{L}$

resistivity (özdirenç)

$$\Rightarrow \frac{I}{A} = \frac{\Delta V}{L}\frac{1}{\rho} \Rightarrow \Delta V = \underbrace{\rho \frac{L}{A}}_{R} I \Rightarrow \boxed{\Delta V = IR} \text{ OHM's LAW}$$

resistance (direnç)

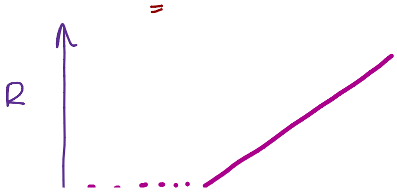
$\Rightarrow$ Resistance is Temperature dependent



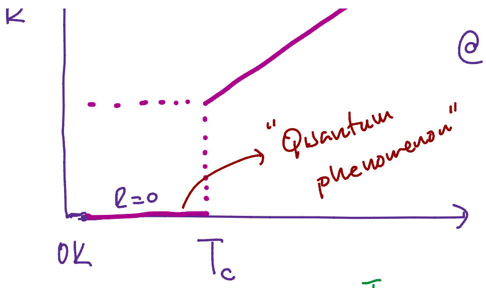
$$\text{slope} = \frac{\Delta R}{\Delta T} = \frac{R_2 - R_1}{T_2 - T_1} = \alpha$$

$$\boxed{R_2 = \alpha (T_2 - T_1) + R_1}$$

R_1, T_1 ✓ @ T_2 R_2?



@ T_c Resistance $= 0\ \Omega \Rightarrow$ no resistance



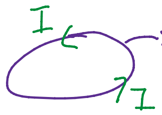
@ T_c Resistance = 0 Ω ⇒ no resistance

→ superconductivity

T_c = critical temperature

@ superconductivity

$T < T_c$



→ superconducting wire

current I flow FOREVER

$T_c \sim 4K$ $\quad$ $T_c \sim 140K$

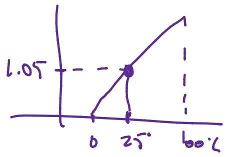
idea is to create materials that would be superconducting closer to room temperature.

→ MRI (magnetic resonance imaging) = "EMAR"

ex) $\alpha = 0.00393 \frac{1}{C^\circ}$ $\quad$ R @ 25°C = 1.05Ω

R @ 0°C = ?

R @ 100°C = ?



$$R_2 = R_1 + \alpha(T_2 - T_1)$$

$$R_2(0^\circ C) = 1.05 + 0.00393(0-25) = 0.97\,\Omega$$

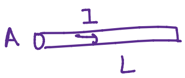
$$R_2(100^\circ C) = 1.05 + 0.00393(100-25) = 1.38\,\Omega$$

ex) $A = 8.2 \times 10^{-7}\ m^2$

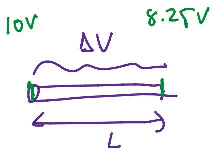
$L = 50m$

$I = 1.67\ A$

$\rho = 1.72 \times 10^{-8}\ \Omega m$



a) ΔV between two ends of wire?



$$\Delta V = IR = I\rho\frac{L}{A} = 1.75\,V$$

b) E field in the wire?

unit of $\vec{E}$

$$EL = \Delta V \Rightarrow E = \frac{\Delta V}{L} = \frac{1.75}{50}\frac{V}{m} = 0.035\,\frac{V}{m} = 0.035\,\frac{N}{C}$$

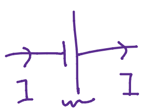
c) R of wire?

$$\rho\frac{L}{A} = R = 1.05\,\Omega$$

Electromotor Force (EMF) ≡ Battery

EMF makes current move from + side to − side

⇒



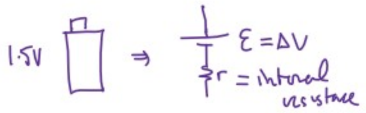
EMF source = Battery (pile)

EMF source

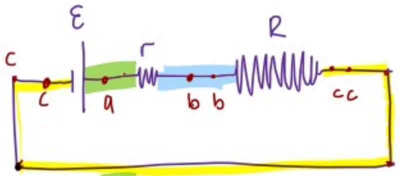
EMF (volts) ; "chemical reactions inside the battery gives out the energy to create current flow"

Battery ≡ active research area
- → smaller
- → more powerful
- → long lasting
- → short recharge time
- → lower internal resistance

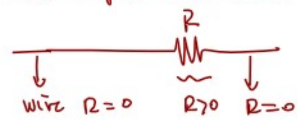
ALL of the batteries have internal resistance

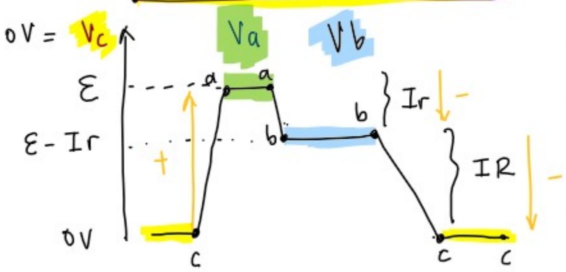


{ ideal battery $r = 0\,\Omega$ } → we will use it in our course.

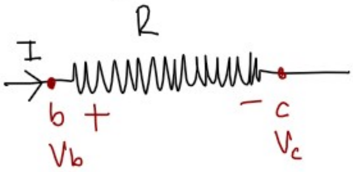


⇒ metal surface have same potential

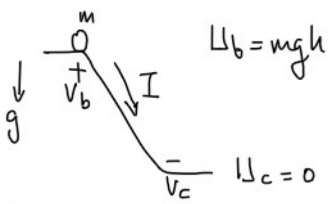




$$\boxed{\underset{V_a}{\mathcal{E}} - \underset{V_b}{Ir} - \underset{V_c}{IR} = 0}$$



$V_b > V_c$ ≡



current I flows towards lower potential

if $\mathcal{E} = 12V$, $Ir = 4V$, $IR = 8V$

$\mathcal{E} - Ir - IR = 0 \longrightarrow I = \frac{\mathcal{E}}{r+R} = \frac{12}{2+4} = 2A$

$12 - 4 - 8 = 0$

$r = 2\Omega \quad R = 4\Omega$

Power & Energy $\quad \Delta V = IR$

$\Delta V = V_f - V_i = V_- - V_+ < 0$

$P = \frac{\text{Energy}}{\text{time}} = \left[\frac{J}{s} = \text{watts}\right]$

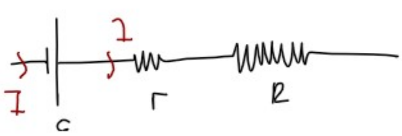
$[V = A\Omega]$

$\Delta V = IR$

$$P = \frac{\Delta U}{t} = \frac{q\,\Delta V}{t} = \underbrace{\frac{q}{t}}_{I} \Delta V = I\,\Delta V = \text{Power}$$

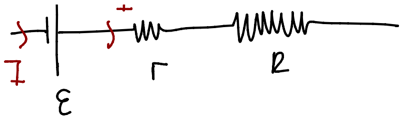
$\left[A\ V = \text{watts} = \frac{J}{s}\right]$

$$P = \underline{I\,\Delta V} = I\,(IR) = \underline{I^2 R} = \left(\frac{\Delta V}{R}\right)\Delta V = \underline{\frac{\Delta V^2}{R}}$$



Electric bills (kW sa) (kW hr)

paying



creates power = consume power

$$I\varepsilon = I^2 r + I^2 R$$

Electric bills (kW hr)

pay ↓ (kW hr)

↳ energy = power × time

$E = P\,t$

$J = W\,s$

$1 = \text{kW hr}$

$= 1000\,\text{W}\; 3600\,\text{s}$

energy $= 3.6\times10^{6}\,\text{J}$

If we run an oven 240V, I = 20A for 4 hrs. power consumed?

$$P = I\,\Delta V = (240\,\text{V})(20\,\text{A}) = 4800\ \text{Watt} = 4.8\,\text{kW}$$

Energy consumed $P\,t = (4.8\,\text{kW})(4\,\text{hrs}) = 19.2\ \text{kW hr}$

⟹

Heating the room with resistive heaters = "WFO"

⇒ Power WFO = 2000 watts

⇒ cold winter month ⇒ one day 6 hrs ⇒ ON OFF 3 hrs per day

30 days × 3 hrs = 90 hrs / month ≃ 100 hrs/month

2000 W × 100 = 200 kW hr ≡ 520 TL

2.6 TL

⟹ End of CH 25

↳ $I = \frac{q}{t}$ ⇒ $n q v_d A = I$

$J = \frac{I}{A} = \frac{E}{\rho}$

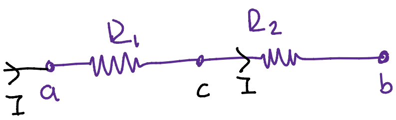
$P = I^2 R = I\Delta V = \frac{\Delta V^2}{R}$

$R = \rho \frac{L}{A}$; $\Delta V = IR$

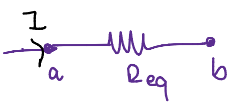
CH 26 DC circuits

$\Delta V = IR$;

Resistors in series connection



⟹



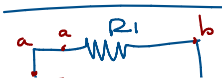
$V_{ac} + V_{cb} = V_{ab}$

$IR_1 + IR_2 = I\,R_{eq}$

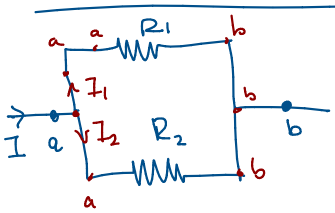
$$\boxed{R_{eq} = R_1 + R_2}$$

⇒ $\boxed{R_{eq} = R_1 + R_2 + \cdots R_N}$ SERIES.

Resistors in Parallel connection



Req



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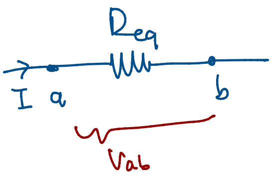
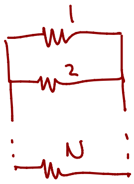


V_{ab} same for R_1 & R_2

$$I_1 + I_2 = I$$

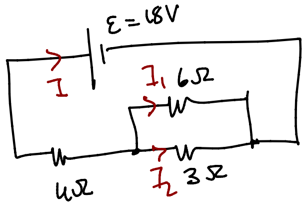
$$\frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} = \frac{V_{ab}}{R_{eq}}$$

$$\boxed{\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}}} \Rightarrow \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$



$$\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_N}\right]^{-1} = R_{eq}$$

ex)



R_{eq} ?

I =?

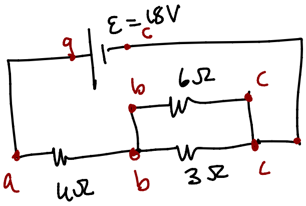
I_1 =?

I_2 =?

$V_{6\Omega}$ =?

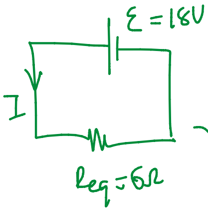
$P_{6\Omega}$ =?

power of the circuit?

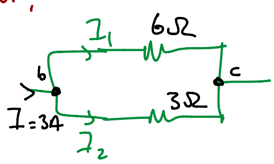


$$4 + (6 \,//\, 3)$$

$$4 + \frac{6(3)}{6+3} = 6\Omega = R_{eq}$$



$$I = \frac{18}{6} = 3A$$

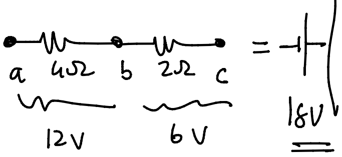


$$V_{6\Omega} = I_1 6\Omega = 1(6) = 6V$$

$$V_{3\Omega} = I_2 3\Omega = 2(3) = 6V$$

$$V_{4\Omega} = I(4\Omega) = 3(4) = 12V$$

$$\mathcal{E} = 18V$$



$$V_{bc} = V_{bc} = V_{6\Omega}$$

$$I_1 6 = I_2 3$$

$$I_1 + I_2 = 3A$$

$$I_1 + 2I_1 = 3A$$

$$\underline{I_1 = 1A} \quad \underline{I_2 = 2A}$$

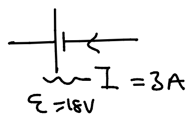
$$P_{6\Omega} = I^2R = 1^2(6) = 6W$$

$$P_{3\Omega} = I^2R = 2^2(3) = 12W$$

$$P_{4\Omega} = I^2R = 3^2(4) = 36W$$

$$+$$

$$54\ W = 54\ W$$

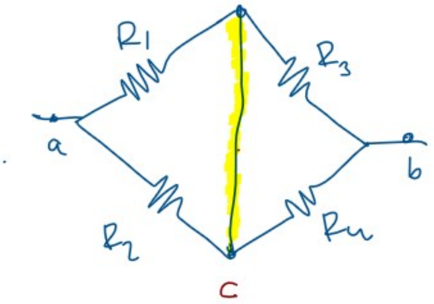


$$P_{\mathcal{E}} = I\mathcal{E} = 3(18) = 54W$$

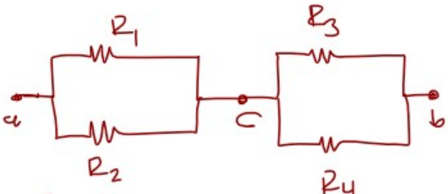
+

54 W consumed = 54 W created

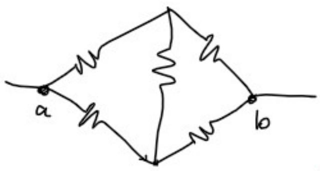
{54 W = power of circuit}



$R_{ab} = ?$



$(R_1 \parallel R_2) + (R_3 \parallel R_4)$



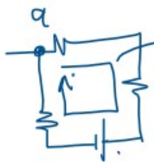
$R_{ab} = ?$ neither series nor parallel !!

KIRCHOFF RULES (LAWS)

- Node (düğüm)

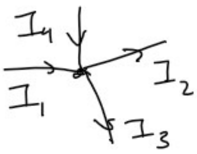
we use Kirchoff to calculate complicated circuits.

- LOOP (Döngü)



KIRCHOFF CURRENT LAW (KCL)

The INCOMING and OUTGOING currents are EQUAL to eachother at the NODE.



$$\sum I_{in} = \sum I_{out}$$

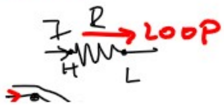
$$I_1 + I_4 = I_2 + I_3$$

KIRCHOFF VOLTAGE LAW (KVL)

→ the sum of all potential differences around any loop is ZERO

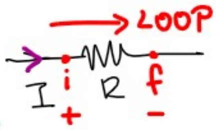
$$\Delta V_{a \to a} = 0 = V_a - V_a$$

HOW TO CALCULATE POTENTIAL DIFFERENCE



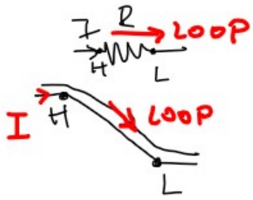
→ LOOP

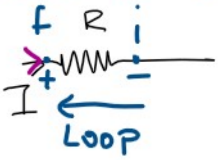
Resistors



$\Delta V = V_f - V_i < 0$

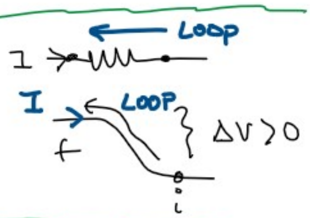
$\Delta V = -IR$



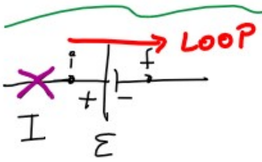


$\Delta V = V_f - V_i > 0$

$\Delta V = +IR$



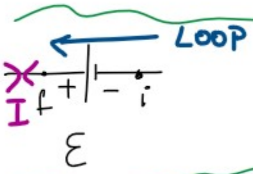
Battery



$\Delta V = V_f - V_i < 0$

$\Delta V = -\mathcal{E}$

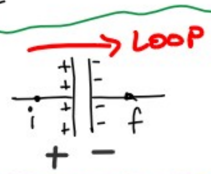
Direction of current does NOT matter!! for batteries



$\Delta V = V_f - V_i > 0$

$\Delta V = +\mathcal{E}$

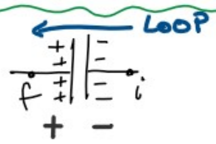
Capacitors



$\Delta V = V_f - V_i < 0$

$\Delta V = -\frac{Q}{C}$

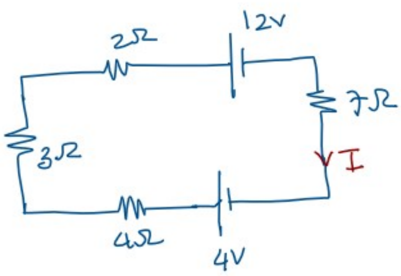
Direction of current does NOT matter. FIND + charges SIDE, that side has HIGHER Potential



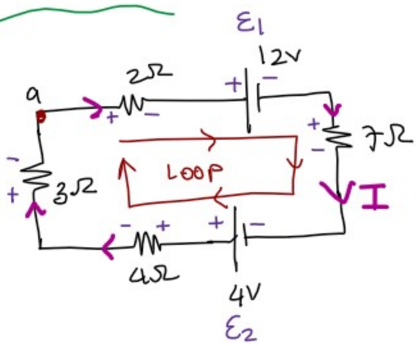
$\Delta V = V_f - V_i > 0$

$\Delta V = +\frac{Q}{C}$

ex)



$I = ?$



KVR

$a \to a$

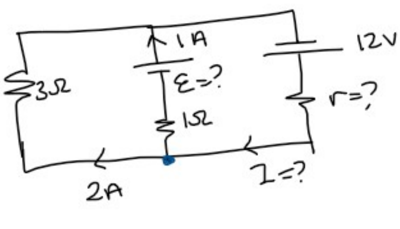
$$-V_{2\Omega} - \mathcal{E}_1 - V_{7\Omega} + \mathcal{E}_2 - V_{4\Omega} - V_{3\Omega} = 0$$

$$-2I - 12 - 7I + 4 - 4I - 3I = 0$$

$$-16I = 8 \qquad I = -\frac{1}{2}A = \underline{\underline{-0.5A}}$$ ✓ DONE!!

$\downarrow I = -0.5A \Rightarrow \uparrow I = 0.5A$

ex)



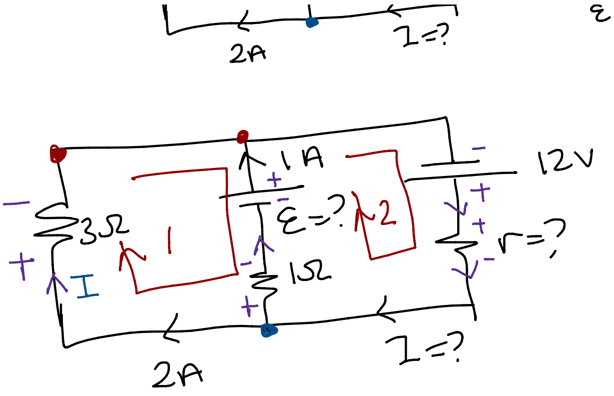
$I = ?$

$r = ?$

$\mathcal{E} = ?$

(KCL)

$I = 2 + 1 = 3A$ ✓



ε=?

$I = 2+1 = 3A$ ✓

1) $-\varepsilon + (1A)(1\Omega) - (2A)(3\Omega) = 0$

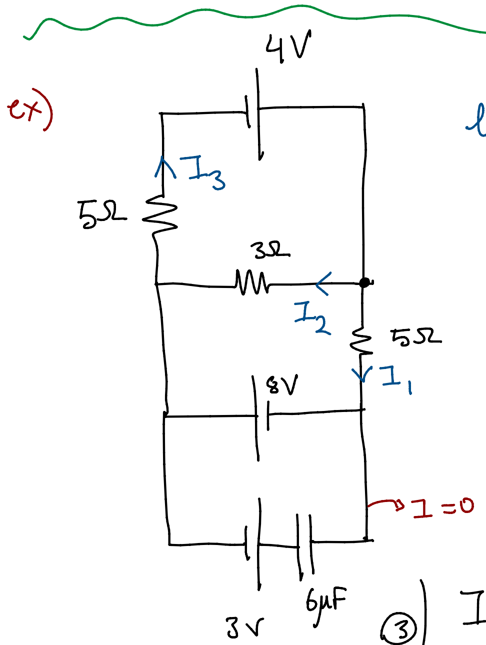
$-\varepsilon + 1 - 6 = 0$ $\qquad \varepsilon = -5V$

2) $+12V - Ir - 1A(1\Omega) + \varepsilon = 0$

$12 - 3r - 1 - 5 = 0$

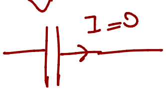
$3r = 6 \Rightarrow r = 2\Omega$

ex)



In this circuit we wait long enough for the capacitor to charged fully.

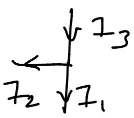
$I_1 = ?$
$I_2 = ?$
$I_3 = ?$

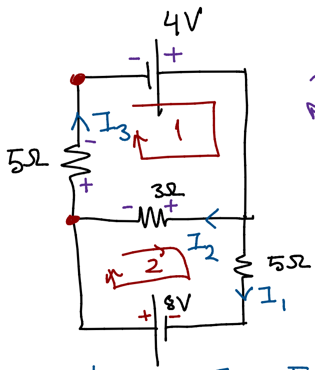


FULLY charged } I on capacitor = ZERO!

KCL (Law)

③ $\boxed{I_3 = I_2 + I_1}$





1) $4 - 3I_2 - 5I_3 = 0 \Rightarrow 4 - 3I_2 - 5I_2 - 5I_1 = 0$

$4 - 8I_2 - 5I_1 = 0$ ①

2) $3I_2 - 5I_1 + 8 = 0$ ②

$4 = 8I_2 + 5I_1$

$-1\,(8 = -3I_2 + 5I_1)$

+

$-4 = 11 I_2 \Rightarrow I_2 = -4/11\ A$

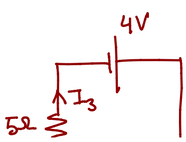
$4 = 8\left(\frac{-4}{11}\right) + 5I_1$

$\frac{44+32}{11} = 5I_1$

$I_1 = \frac{76}{55} A$??

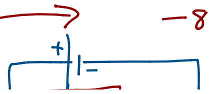
$I_3 = I_1 + I_2 = \frac{-4}{11} + \frac{76}{55} = \frac{56}{55} A$

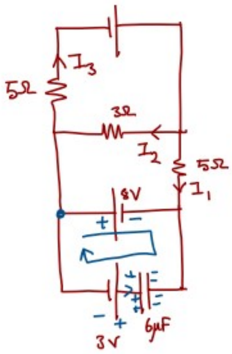
(5)



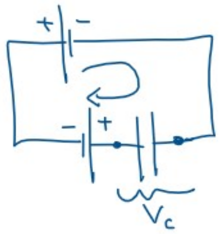
b) charge on the capacitor?

$-8 + V_c - 3 = 0$



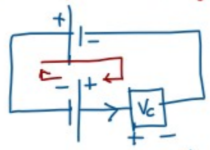


b) charge on the capacitor:



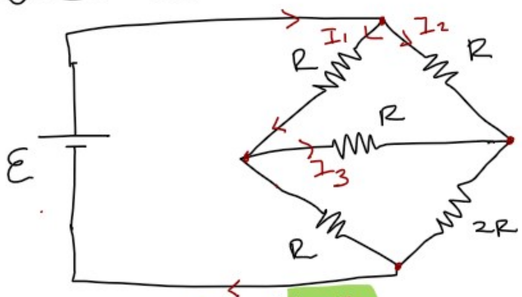
$8 + 3 - V_c = 0$

$-8 + V_c - 3 = 0$



$V_c = 11V \Rightarrow Q = CV = (6\mu F)\ 11V$

$Q = 66\mu C$



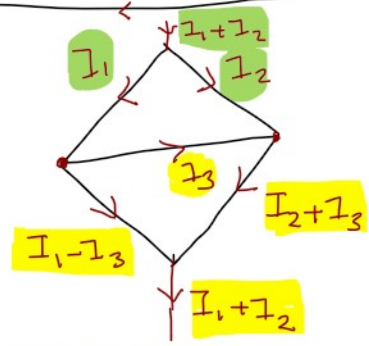
$R = 1\Omega$

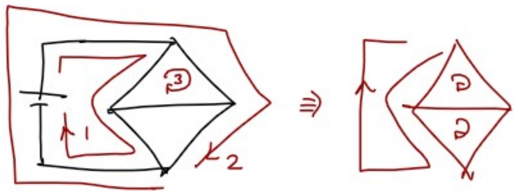
$\varepsilon = 13V$

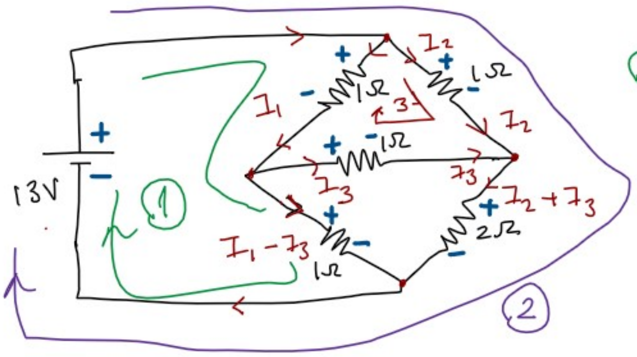
I_1, I_2, I_3 ?

R_{eq} of the circuit?

(KCL)







① $-I_1 - (I_1 - I_3) + 13 = 0$

② $-I_2 - 2(I_2 + I_3) + 13 = 0$

③ $-I_2 + I_3 + I_1 = 0$

① $13 = 2I_1 - I_3$ ⇒ $13 = 2I_1 - I_3 \quad (\times 5)$

② $13 = 3I_2 + 2I_3$ ⇒ $13 = 3I_1 + 5I_3$

③ $I_2 = I_1 + I_3$

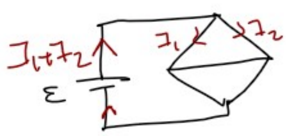
$+$

$13(6) = 13 I_1$ $\quad$ $I_1 = 6A$

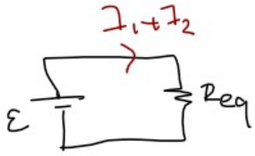
$I_2 = I_1 + I_3 = 6 - 1 = 5A$

$13 = 2(6) - I_3 \Rightarrow I_3 = -1A$

R_{eq} of the circuit?

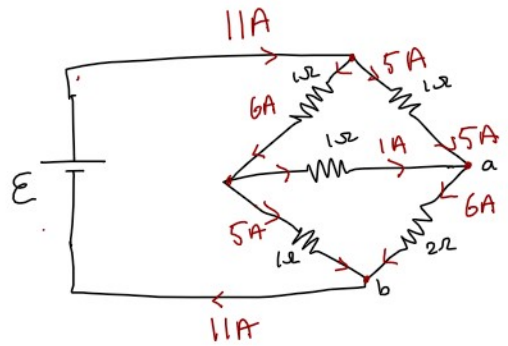


⇒



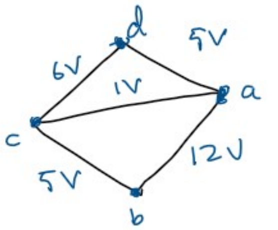
$\varepsilon = (I_1 + I_2) R_{eq}$

$13 = (6 + 5) R_{eq}$



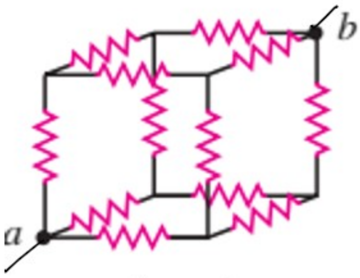
$R_{eq} = \frac{13}{11}\,\Omega$

$V_{ab} = (6A)(2\Omega) = 12V$



$V_{ac} = 1V$

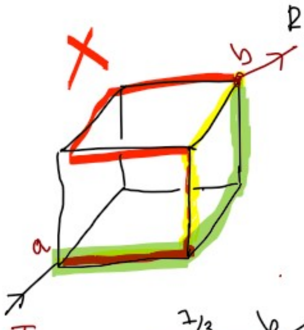
26.92)

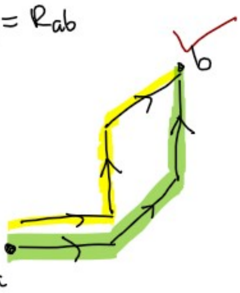


26.92 ••• Suppose a resistor R lies along each edge of a cube (12 resistors in all) with connections at the corners. Find the equivalent resistance between two diagonally opposite corners of the cube (points a and b in Fig. P26.92).

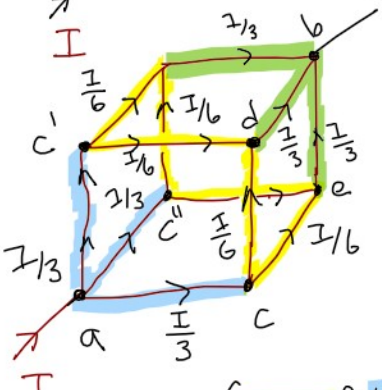
$R_{eq} = R_{ab}$

symetry





$a \to b$

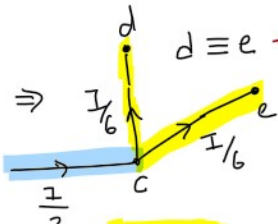


$\underbrace{c'' = c' = c}$ are equivalent $\Rightarrow$ likely to happen equally

divide I into 3

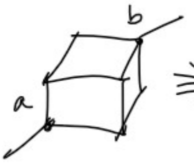
$d \equiv e \Rightarrow$ so $\frac{I}{3} \to I/6,\ I/6$

NODE C $\Rightarrow$

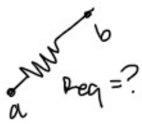


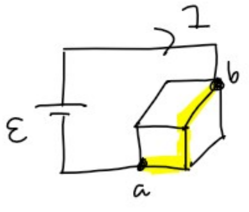
{green & blue lines are similar $I/3$}

{yellow lines $\frac{I}{6}$}

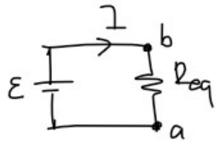


$\Rightarrow$

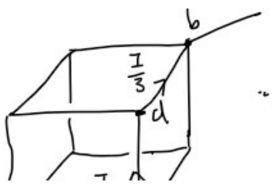




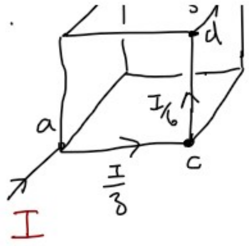
$\Rightarrow$



$\mathcal{E} = I R_{eq} = I R_{ab} = V_{ab}$



$V_{ab} = ?$

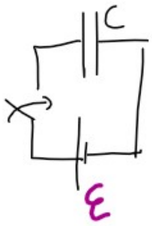


$$V_{ab} = V_{ac} + V_{cd} + V_{db}$$

$$V_{ab} = \underset{(2)}{\frac{I}{3}}R + \frac{I}{6}R + \underset{(2)}{\frac{I}{3}}R = I\,R_{ab}$$

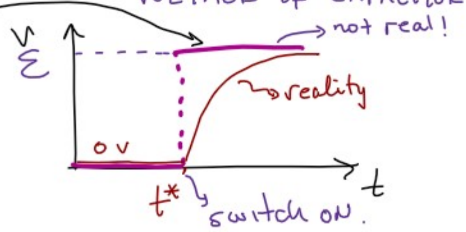
$$R_{ab} = \frac{5}{6}R \Leftarrow \frac{5IR}{6} = I\,R_{ab}$$

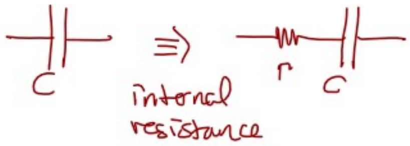
RC CIRCUITS



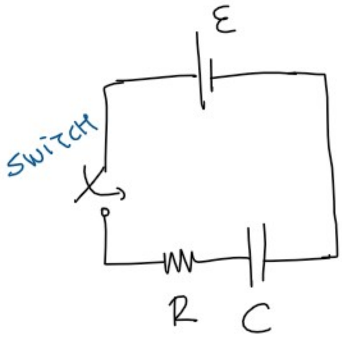
when switch ON we thought capacitor FILLS UP immediately!

VOLTAGE of CAPACITOR



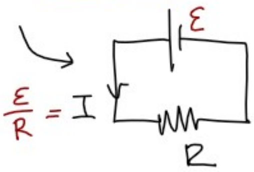


even if we don't use external resistance capacitor always charged with RC circuit because capacitors have internal resistance



switch ON charges are running through towards the capacitor

$t = 0^+_s$ (very early times after switch is ON)

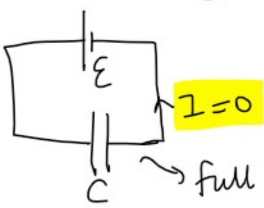


I max (as if no capacitor!!)

$t \gg 0_s$ (very late stage)

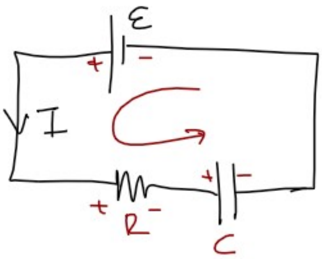
wait long enough; C is full

no current goes through R (as if no resistor!!)



we WILL CHECK THESE with FORMULAS!

$I \begin{cases} \to \max & 0^+_s \\ \to 0 & \infty \end{cases}$



LET'S SOLVE for intermediate t times;

$$\varepsilon - V_R - V_C = 0$$

$$\varepsilon - IR - \frac{q}{C} = 0 \qquad I = \frac{dq}{dt}$$

$$\varepsilon - \frac{dq}{dt}R - \frac{q}{C} = 0 \quad \text{(diff. eqn)}$$

$$\leftarrow \varepsilon - \frac{dq}{dt}R - \frac{q}{C} = 0 \quad \text{(diff. equ)}$$

$$\frac{dq}{dt} = \left(\varepsilon - \frac{q}{C}\right)\frac{1}{R} \Rightarrow \frac{dq}{\left(\frac{\varepsilon C - q}{RC}\right)} = dt \Rightarrow \int \frac{dx}{x} = \ln x$$

$$\int \frac{dq}{\varepsilon C - q} = \int \frac{dt}{RC} \Rightarrow -\ln(\varepsilon C - q)\Big|_{q_i}^{q_f} = \frac{1}{RC}\,\Delta t\Big|_{t_i}^{t_f}$$

$$\ln(\varepsilon C - q_f) - \ln(\varepsilon C - q_i) = -\left(\frac{t_f - t_i}{RC}\right) \quad \begin{Bmatrix} t_i = 0 \\ t_f = t \end{Bmatrix}$$

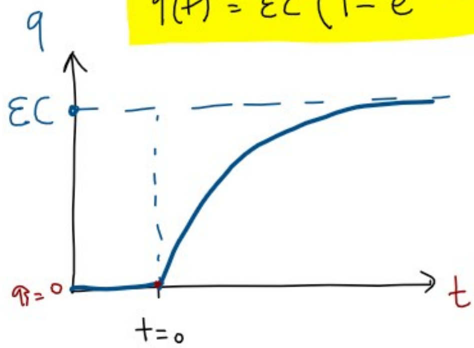
$$\ln(\varepsilon C - q(t)) - \ln(\varepsilon C) = -t/RC \quad \begin{Bmatrix} q_i = 0 \text{ (no charge at } t=0) \\ q_f = q(t) \end{Bmatrix}$$

$$\ln\left(\frac{\varepsilon C - q(t)}{\varepsilon C}\right) = -t/RC$$

$$\varepsilon C - q(t) = \varepsilon C e^{-t/RC} \Rightarrow q(t) = \varepsilon C - \varepsilon C e^{-t/RC}$$

CHARGE on the CAPACITOR at any time.

$$\boxed{q(t) = \varepsilon C\left(1 - e^{-t/RC}\right)}$$

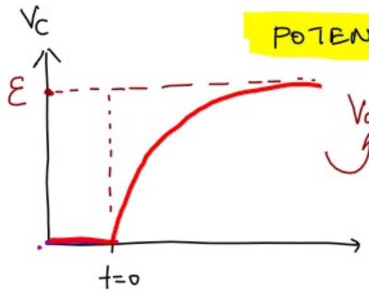


$t = 0\,s \quad q(0) = \varepsilon C(1 - e^{-0}) = 0$

$t \to \infty \quad q(t\to\infty) = \varepsilon C(1 - e^{-\infty}) = \varepsilon C(1-0) = \varepsilon C = q_{max}$

$$\underset{[C]}{q} = \underset{[C]}{\underbrace{\varepsilon C}_{VF}}\underbrace{(1 - e^{-t/RC})}_{\text{no unit}} \qquad e^{-t/RC} = e^{-\left[\frac{s}{s}\right]} \qquad RC = [\text{second}]^* \quad [\Omega F = s]$$

POTENTIAL of the CAPACITOR

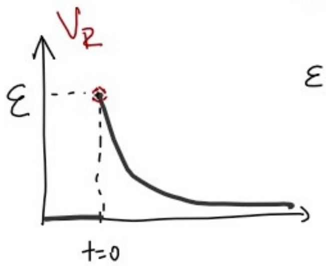


$$V_C = \frac{q(t)}{C} = \frac{\varepsilon C}{C}\left(1 - e^{-t/RC}\right) = \boxed{\varepsilon\left(1 - e^{-t/RC}\right)} = V_C$$

$t = 0\,s \quad V_C(t=0) = \varepsilon(1 - e^{-0}) = \varepsilon(1-1) = 0$

$t \to \infty\,s \quad V_C(t\to\infty) = \varepsilon(1 - e^{-\infty}) = \varepsilon(1-0) = \varepsilon$

POTENTIAL of the RESISTOR



$$V_R = IR = \left(\frac{dq}{dt}\right)R$$

$$\varepsilon = V_C + V_R \Rightarrow V_R = \varepsilon - V_C = \varepsilon - \left(\varepsilon\left[1 - e^{-t/RC}\right]\right)$$

$$\boxed{V_R = +\varepsilon e^{-t/RC}} = \frac{dq}{dt}R$$

$t = 0\,s \quad V_R(t=0) = \varepsilon e^{-0} = \varepsilon$

$t \to \infty \quad V_R(t\to\infty) = \varepsilon e^{-\infty} = 0$

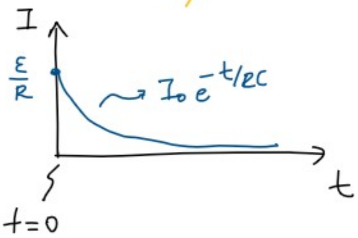
CURRENT on the CIRCUIT

CURRENT on the CIRCUIT

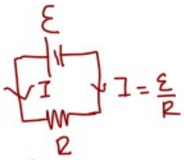
Since we know $q(t)$; $I = \frac{dq}{dt}$ (time derivative of charge)

$$I = \frac{d}{dt}\left(\mathcal{E}C\left[1-e^{-t/RC}\right]\right) = \mathcal{E}C\left[0-\left(-\frac{1}{RC}\right)e^{-t/RC}\right]$$

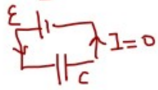
$$I = \frac{\mathcal{E}\cancel{C}}{R\cancel{C}}\, e^{-t/RC} = \boxed{\frac{\mathcal{E}}{R}\, e^{-t/RC} = I} = I_0\, e^{-t/RC}$$



$t=0$ $\quad I(t=0) = \frac{\mathcal{E}}{R} e^{-0} = \frac{\mathcal{E}}{R}$ }



$t \to \infty$ $\quad I(t\to\infty) = \frac{\mathcal{E}}{R} e^{-\infty} = 0$ }



$$q = \mathcal{E}C\left(1-e^{-t/RC}\right) \qquad \cancel{X} \quad I(t) = \frac{\mathcal{E}}{R}\, e^{-t/RC} \qquad V_R = IR \qquad V_C = \frac{\cancel{\emptyset}q}{C}$$

Dimensional analysis $\quad [\mathcal{E}C = VF = \text{Coulomb}] \qquad \left[\frac{\mathcal{E}}{R} = \frac{V}{\Omega} = \text{secon} = \text{Ampere}\right]$

$e^{-t/RC}$ = unitless $\quad e^{-\left[\frac{s}{s}\right]} \quad RC = \text{seconds}$ $\quad [\Omega F = s]$

$RC \equiv$ time const. of RC circuit $\quad RC = \tau$ (tau)

$$q(t) = \mathcal{E}C\left(1-e^{-t/\tau}\right) \quad ; \quad I(t) = \frac{\mathcal{E}}{R}\, e^{-t/\tau}$$

ex) $\mathcal{E} = 12V \quad R = 100\,\Omega \quad C = 20\mu F$

$$\tau = RC = 100\,(20)\times 10^{-6}\,s = 2000\times 10^{-6}\,s = 2\times 10^{-3}\,s = 2\,ms$$

$$q(t) = (12V)(20\mu F)\left[1-e^{-t/2ms}\right]$$

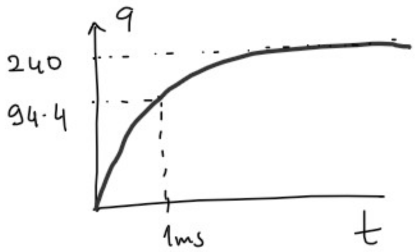
$$q(t) = 240\,\mu C\left[1-e^{-t/2ms}\right]$$

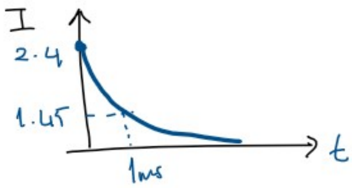
@ $t = 1ms$ what's the charge on C?
" " current of the circuit?

$$q(1ms) = 240\mu C\left[1-e^{-1ms/2ms}\right] = 240\mu C\left[1-\underbrace{e^{-1/2}}_{0.606}\right] = \underline{\underline{94.4\,\mu C}}$$

$$I(1ms) = ? \qquad I = I_0 e^{-t/\tau} = \frac{\mathcal{E}}{R} e^{-t/\tau} = \frac{240}{100} A\; e^{-1ms/2ms} = 2.4\, e^{-1/2}$$

$I(1ms) = 1.45\,A$





$$q(t^*) = \frac{Q_{max}}{2} = \varepsilon C\left(1 - e^{-t^*/RC}\right) \Rightarrow \frac{\cancel{\varepsilon C}}{2} = \cancel{\varepsilon C}\left(1 - e^{-t^*/RC}\right)$$

$Q_{max} \to \varepsilon C$

t^* = time to fill half (%50) $= T_{1/2}$

$$\frac{1}{2} = 1 - e^{-t^*/RC}$$

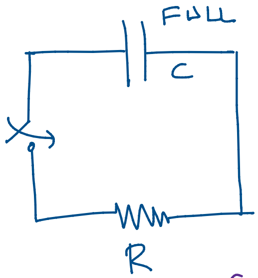
$$e^{-t^*/RC} = \frac{1}{2}$$

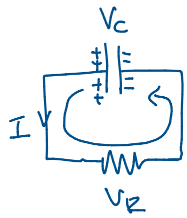
$$-t^*/RC = \ln\left(\tfrac{1}{2}\right) = -0.69$$

$$\Leftarrow \boxed{t^* = 0.69\, RC}$$

$\{0.69\ (2\text{ms}) = 1.39\text{ ms}\}$

DISCHARGING of CAPACITOR in RC circuit





$$V_C - V_R = 0$$

$$\frac{q}{C} - IR = 0$$

$$\frac{q}{C} - \frac{dq}{dt}R = 0$$

$$\Leftarrow \frac{q}{C} = \frac{dq}{dt}R$$

$$\int \frac{dt}{RC} = \int \frac{dq}{q}$$

$$\underbrace{\frac{t}{RC}} = \frac{1}{RC}\Delta t\Big|_{t_i}^{t_f} = \ln q\Big|_{q_i = Q_0}^{q_f} = \underbrace{\ln q(t) - \ln Q_0}_{(-)}$$

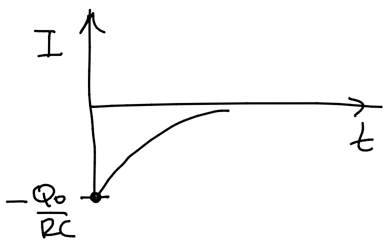
$$q(t) < Q_0$$

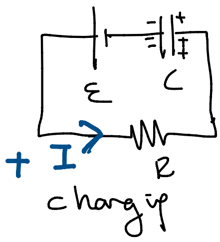
$$\ln\frac{q(t)}{Q_0} = t/RC \qquad \frac{q(t)}{Q_0} < 1\ ;\ \ln(\ldots) < 0$$

$$\ln\left|\frac{q(t)}{Q_0}\right| = -t/RC \Rightarrow \underline{q(t) = Q_0\, e^{-t/RC}}$$

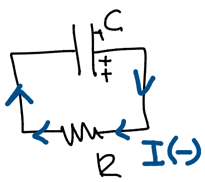
Discharging RC

$$I = \frac{dq}{dt} = \left(-\frac{1}{RC}\right)Q_0\, e^{-t/RC} = -\frac{Q_0}{RC}\, e^{-t/RC}$$





$\Rightarrow$

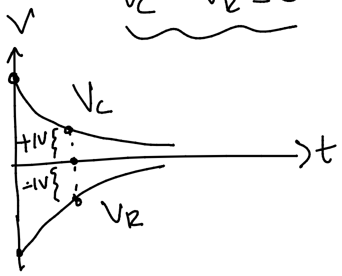


$$q(t) = Q_0\, e^{-t/RC} \qquad I = -\frac{Q_0}{RC}\, e^{-t/RC}$$

$$V_C = \frac{q}{C} = \frac{Q_0}{C}e^{-t/RC} \qquad V_R = IR = -\frac{Q_0}{\cancel{R}C}\cancel{R}\, e^{-t/RC} = -\frac{Q_0}{C}e^{-t/RC}$$

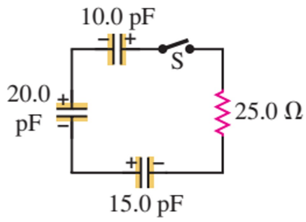
$\checkmark$ $\underbrace{V_C - V_R = 0}$

$V_c - V_R = 0$



26.47 •• **CP** In the circuit shown in Fig. E26.47 each capacitor initially has a charge of magnitude 3.50 nC on its plates. After the switch S is closed, what will be the current in the circuit at the instant that the capacitors have lost 80.0% of their initial stored energy?

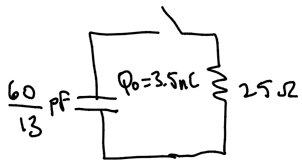
Figure **E26.47**



C_1, C_2, C_3 series

$\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{C_{eq}} \Rightarrow \frac{1}{10} + \frac{1}{15} + \frac{1}{20} \Rightarrow \left(\frac{13}{60}\right)^{-1} = C_{eq} \quad \frac{60}{13}\,\text{pF}$ pico $= 10^{-12}$

6 4 3



$25 \cdot \frac{60}{13}\,\text{ps} \simeq 115\,\text{ps}$

$q(t) = Q_0 e^{-t/RC}$

$U = \frac{q^2}{2C} \rightarrow$ LOST 80%

$I = ?$

$q(t) = 3.5\,nC\, e^{-t/115\,ps}$

$I(t) = -\frac{3.5\,nC}{115\,ps} e^{-t/115\,ps}$

$U = \frac{(3.5\,nC)^2}{2\left(\frac{60}{13}\right)pF} = \frac{159.25}{120} \times \frac{(10^{-9})^2}{10^{-12}} = 1.3 \times 10^{-6}\,J \Rightarrow \frac{20}{100}\, 1.3 \times 10^{-6}$

$\frac{U_f}{U_i} = \frac{20}{100} = \frac{(q_i^2)/\cancel{2C}}{(q_f^2)/\cancel{2C}} \Rightarrow q_f = \sqrt{0.2\, q_i^2} = \sqrt{0.2}\,(3.5\,nC)$

$q(t) = 3.5\,nC\, e^{-t/115\,ps} = \sqrt{0.2}\; 3.5\,nC \Rightarrow e^{-t/115\,ps} = \sqrt{0.2}$

$t = -\ln(\sqrt{0.2})\; 115\,ps$

$-(-0.8)\; 115\,ps$

$= \underline{\underline{92\,ps}}$

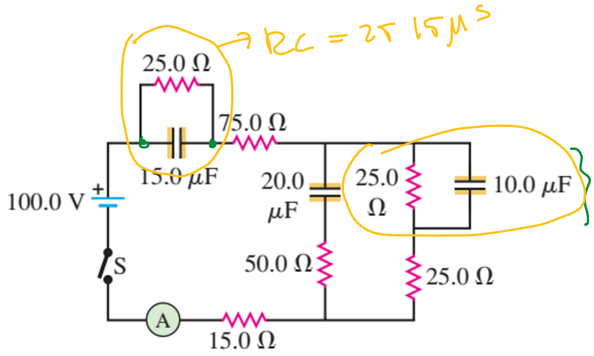
$I(t = 0.92\,ps) = I_0 e^{-t/RC}$

$-\frac{Q_0}{RC} e^{-t/RC} \Rightarrow 115$

$-\frac{3.5\,nC}{115\,ps} e^{-92/115} = \checkmark$

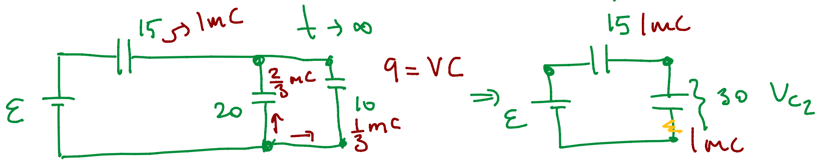
$RC = 25\; 15\,\mu s$

$\rightarrow RC = 25 \cdot 15\,\mu s$



$V_C =$

$t \to \infty$ $\quad q = VC \quad$ V_{C_1}



$$V_{C_1} + V_{C_2} = \mathcal{E}$$

$$C_{eq} = \frac{15(30)}{45} = 10\,\mu F$$

$$\frac{q}{C_1} + \frac{q}{C_2} = \mathcal{E} = \frac{q}{C_{eq}} \rightarrow q = \mathcal{E} C_{eq}$$

$$= (100\text{ V})\, 10\,\mu F$$

$$= 1000\,\mu C = 1\,mC$$

$$q_0 e^{-t/RC}$$

$$q_0 \left(1 - e^{-t/RC}\right)$$