PHYSICS I

Mechanics

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T-Th-F   8:30 – 12:30
9:00

5 x 3 gün = 15 gün x 4 saat

10-12 ünite

devamsizlik ; yoklama yok
siddetle tavsiye ederiz gelmenizi

Kitap: Serway / Physics ok
Sears & Zemansky 14th

Lecture Notes %80   questions
%20 → sorular

EXAMS   Classic (3-4)
↳ 3 Midterm 20+20+20
→ 1 Final 40
↳ (5-6 soru)

Mutlak notlar 60 ↑ CC

Standards & Units (birim)

Physics I

Time (seconds)
Length (meter)
Mass (kilogram)

SI (standard units) → 7

* amount of matter → mol
N_A chemistry
* charge electrical (Coulomb)
phys 2
* luminosity (candela)
* temperature (kelvin) → thermodi...

Time (seconds)

earth → year → day → hr → minutes → seconds

100m

$9.\boxed{73}$ second
100's

Egypt! = mod 12

modern definition of a second 1967

atomic clock

9.192.631 770 cycles two... vibration in a secon...

Length (meter)

~1875 a stick

~1960's **length**

meter established by using of speed of light

$c = 299\,792\,458$ m/s

t

1m

$t = \frac{1m}{c}$ ✓

atomic clock

$t = \frac{1}{299\,792\,458}$ s in **1m**

Mass (kilogram) (not gram!!)

weakest definition right now!

prototype definition still

but this definition will be upgraded!! youtube video

the most round object

Physics

mass	**length**	**time**
kg	m	s
[M]	[L]	[T]

Unit analysis

eqn = eqn

save units

---

distance = speed time

$[L] = v\,[T]$

speed $v = \left[\frac{L}{T}\right]$ unit.

$v = \frac{m}{s}$ ✓

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Length

$$[L = L \quad L \quad L]$$

$$x = a t^2$$

$$L = a\,T^2$$

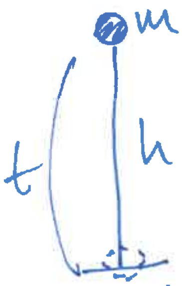
$$a = \left[\frac{L}{T^2}\right] = \frac{m}{s^2}$$

---

falling object

falling from **h** height

falling at **t** second



how **h** and **t** relate

$\uparrow h \sim t \uparrow$

$$h \sim t^{\alpha} m^{\beta} g^{\gamma}$$

g → gravitation constant

$$L^1 = \left[T^{\alpha} M^{\beta} \left(\frac{L}{T^2}\right)^{\gamma}\right] \quad (9.8\ m/s^2)$$

Length

$$L^1 = T^{\alpha} M^{\beta} \frac{L^{\gamma}}{T^{2\gamma}}$$

$\beta = 0$!

$$T^{\alpha - 2\gamma} = T^0$$

$B = 0 \quad M^0$

$T^{\alpha - \cancel{2}\gamma} = T^0$

$L^1 = L^\gamma \quad \underline{\underline{\gamma = 1}}$

$\alpha - 2\gamma = 0$

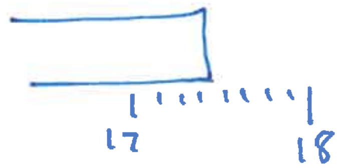
$\underline{\underline{\alpha = 2}}$

$h \sim t^\alpha m^\beta g^\gamma$

$\underline{\underline{L}} \sim \cancel{t^2} \cancel{m^0} \dfrac{L^1}{\cancel{T^2}}$

uncertainty & significant figures

$L = 17\,cm \pm 1\,cm$



$L = 17.3 \pm 0.1\,cm$

ALL measurements should HAVE uncertainties !! (error)

$G = 6.67428(67) \times 10^{-11} \dfrac{Nm^2}{kg^2}$

6.67428 ± 0.00067

gravitational constant.

significant figures

meaningful — number

$6.67 \rightarrow 3$ significant figures

$6.670 \rightarrow 3$ sf

$6.67428 \rightarrow 6$ sf

$6.607 \rightarrow 4$ sf

③ estimating orders of magnitude

"back of envelope" calculations

→ estimate roughly

→ calculate / operations

2 – 3 rows

→ eg. how many cars are there in Turkey?

~ 4 people → 1 car

80 million → 20 million car

→ how much money does the government TAX (vergi) from gasoline in each year?

→ estimate how many cars?

→ " how far they travel?

→ " how much " consume gas

→ amount of tax in 1 lt of gas.

👤👤👤👤 → 🚗 20 million 20×10^6 cars

1 month 1000 km

1 yr 12000 km

2 gas tanks → 100 lt ~ 1000 km

1 lt → 10 km

0.1 lt → 1 km

%50 TAX $\boxed{5\ TL}$ – 1 lt gas

total amount gas:

$20 \times 10^6 \times 12 \times 10^3 \cancel{km} \times 0.1 \dfrac{lt}{\cancel{km}}$

$24 \times 10^9\ lt \sim 2.4 \times 10^{10}\ lt$

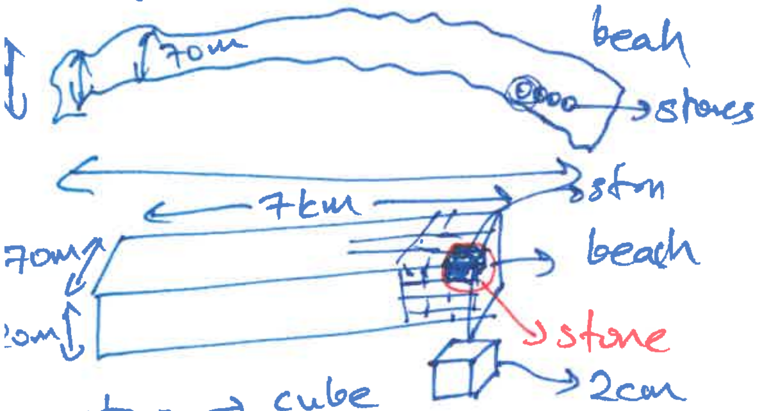
$2.4 \times 10^{10} \cancel{lt} \times 5 \dfrac{TL}{\cancel{lt}}$

$12 \times 10^{10}\ TL \Rightarrow$ TAX/YEAR

120 mil year TL

Eg: How many stones are there at Konyaaltı Beach?

4



stone → cube

beach → rectangular volume.

$$\# \text{ stones} = \frac{V_{beach}}{V_{stone}}$$

$$= \frac{7\times10^3 \times 70 \times 20 \; m^3}{(2\times10^{-2}\, m)^3}$$

$$\cong \frac{\cancel{100}\,10 \times 10^5 \; m^3}{\cancel{8}\,6 \times 10^{-6} \; m^3} \cong 10^{12}$$

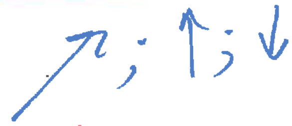
10^9 billion = milyar

1000 milyar tane

eg → how many cells do you have in your body?

→ how many breaths do a human take/give in a lifetime

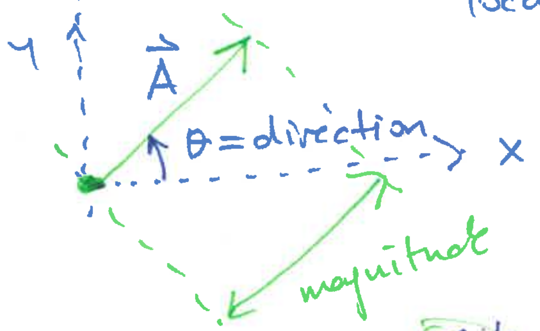
VECTORS



has a magnitude (length)

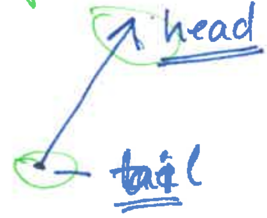
↳ has a direction (coordinate system)

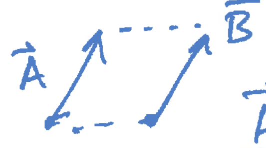
need vectors define motion location.



$\vec{A}$ ⇒ vector

A = length

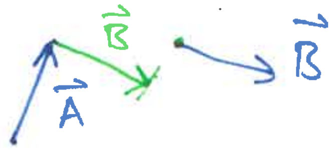




$\vec{A} = \vec{B}$ same direction, same length

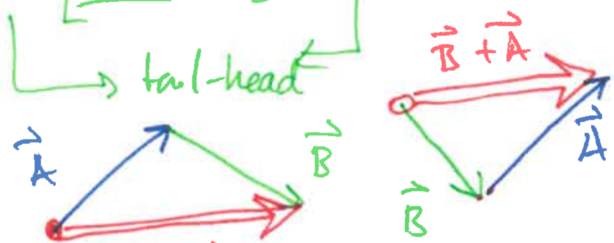
Summation

$\vec{A} + \vec{B}$



tail [head tail] head

↳ tail-head

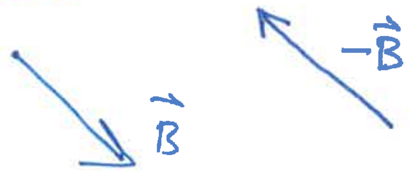


$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

commutative

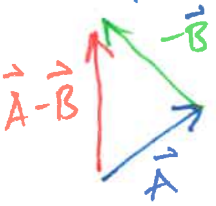
Subtraction of vectors

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

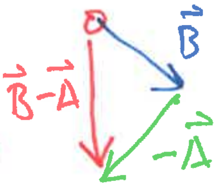


$-$ ⇒ flipping; rotating vector $\underline{180°}$

$\vec{A} - \vec{B} =$



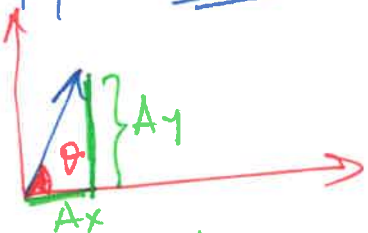
$\vec{B} - \vec{A}$



$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$

$\vec{A}$ ⇒ magnitude $|\vec{A}| = A$
→ direction θ

quantify ⇒ need coordinate sys.



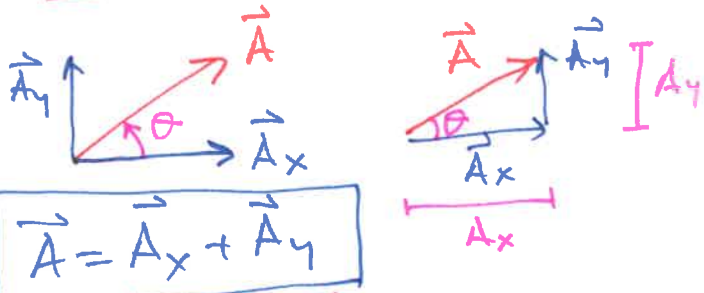
$$\tan\theta = \frac{A_y}{A_x} = \frac{\text{karşı}}{\text{komşu}}$$

$$A = \sqrt{A_x^2 + A_y^2} \equiv \text{magnitude}$$

$$= \sqrt{(A\cos\theta)^2 + (A\sin\theta)^2}$$

$$= \sqrt{A^2}\sqrt{\cos^2\theta + \sin^2\theta} = \underline{A}$$

Components of vectors



$$\boxed{\vec{A} = \vec{A_x} + \vec{A_y}}$$

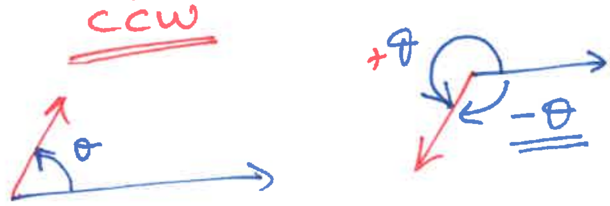
components (bileşen)

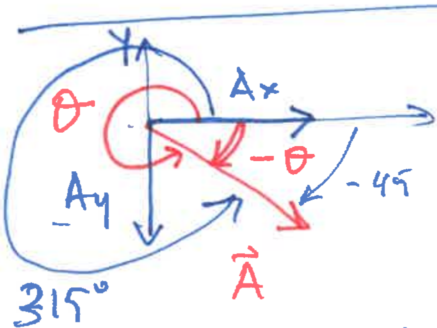
θ = direction

$$\tan\theta = \frac{A_y}{A_x}$$

$$\tan^{-1}\left(\frac{A_y}{A_x}\right) = \theta = \arctan\left(\frac{A_y}{A_x}\right)$$

$\theta \equiv$ always counter clockwise direction
(+)
ccw





$A_x \Rightarrow +$

$A_y \Rightarrow -$

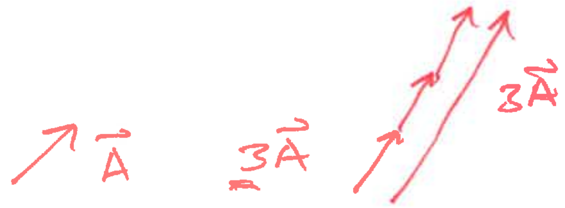
$A_x = 2m$ $A_y = -2m$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-2}{2}\right)$$

$$\theta = \tan^{-1}(-1) = -45°$$

$$= 315°$$

Multiplying vector with a scalar (number)

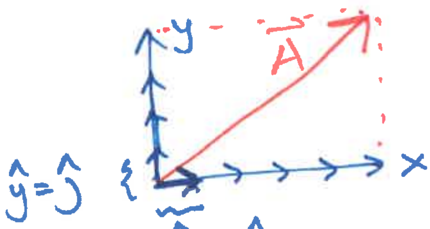


$0.5\vec{A}$ ↗

$-\vec{A} = (-1)\vec{A}$ ↙ $-\vec{A}$

unit vectors → has a magnitude of 1.

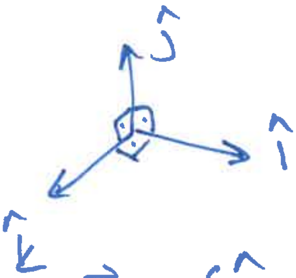
$\hat{a}$ hat $\equiv$ unit vector



$\hat{k} = \hat{z}$

$\vec{A} = \vec{A}_x + \vec{A}_y$

$\vec{A} = 5\hat{i} + 4\hat{j}$ unit vectors



$\vec{A} = 6\hat{i} + 3\hat{j} - 1\hat{k}$

$\vec{B} = 4\hat{i} - 5\hat{j} + 8\hat{k}$

$\vec{A} - 2\vec{B}$?

$3\vec{A} + 2\vec{B}$

$\vec{A} + \vec{B} = ?$

$\vec{A} + \vec{B} = 10\hat{i} - 2\hat{j} + 7\hat{k}$

magnitude of $\vec{A} + \vec{B} = ?$

$$|\vec{A} + \vec{B}| = \sqrt{10^2 + (-2)^2 + 7^2} = \sqrt{153}$$

products of vector

scalar product

$\vec{A} \odot \vec{B} = \underline{\underline{C}}$ (dot; number, scalar)

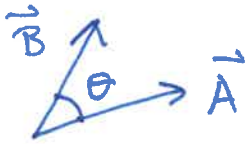
vector product

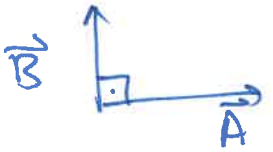
$\vec{A} \otimes \vec{B} = \underline{\underline{\vec{D}}}$ (cross; vector)

dot product

scalar product

$$\vec{A} \cdot \vec{B} = C = AB\cos\theta = |\vec{A}|\,|\vec{B}|\cos\theta$$





$$\vec{A} \cdot \vec{B} = AB\cos 90^{\circ} = \underline{\underline{0}}$$

$$\hat{i} \cdot \hat{j} = |\hat{i}|\,|\hat{j}|\cos 90 = 1 \; 1 \; 0 = \underline{\underline{0}}$$

$$\hat{i} \cdot \hat{i} = 1 \; 1 \cos 0 = \underline{\underline{1}}$$

$$\hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = \underline{\underline{1}}$$

$$\hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

9 terms ; 6 of them ZERO

$$= A_xB_x\underbrace{(\hat{i}\cdot\hat{i})}_{1} + A_yB_y\underbrace{(\hat{j}\cdot\hat{j})}_{1} + A_zB_z\underbrace{(\hat{k}\cdot\hat{k})}_{1}$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$\vec{A} = 2\hat{i} + 3\hat{j} + 1\hat{k}$

$\vec{B} = -4\hat{i} + 2\hat{j} - 1\hat{k}$

(a) $\vec{A} \cdot \vec{B} = ?$

(b) $|\vec{A}| = ?$ $\quad |\vec{B}| = ?$

(c) angle between $\vec{A}, \vec{B} = ?$

(a) $\vec{A} \cdot \vec{B} = 2(-4) + 3(2) + (1)(-1)$

$= -8 + 6 - 1 = \underline{\underline{-3}}$

(b) $|\vec{A}| = \sqrt{2^2 + 3^2 + 1^2}$

$|\vec{A}| = \sqrt{14}$

$|\vec{B}| = \sqrt{(-4)^2 + 2^2 + (-1)^2}$

$= \sqrt{21}$

(c) 

$\underline{\underline{\theta = ?}}$

$\underbrace{\vec{A} \cdot \vec{B}}_{\checkmark} = \underset{\checkmark}{A}\ \underset{\checkmark}{B}\ \underset{?}{\cos\theta}$

$-3 = \sqrt{14}\sqrt{21} \cos\theta$

$\cos\theta = \dfrac{-3}{7\sqrt{6}}$

$\theta = \cos^{-1}\left(\dfrac{-3}{7\sqrt{6}}\right) = \underline{\underline{100^\circ}}$

(7) vector product

$\vec{A} \times \vec{B} = \vec{C}$ → direction ↑ (inward)

↓ magnitude

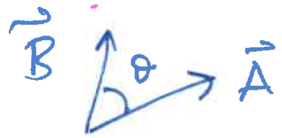
$|\vec{C}| = A B \sin\theta$

direction ⇒ right hand rule

$\vec{A} \times \vec{B} = \vec{C}$ → middle finger (orta)

$\vec{A}$ ↓ thumb (baş parmak)

$\vec{B}$ ↓ index finger (işaret)



$\underline{\underline{\vec{A}}} \times \vec{B} = \vec{C}$ (out)

$\vec{B} \times \vec{A} = \vec{D}$ (in)

$\boxed{\vec{D} = -\vec{C}}$

$|\vec{D}| = |\vec{C}| = AB\sin\theta$

6.7

last tie

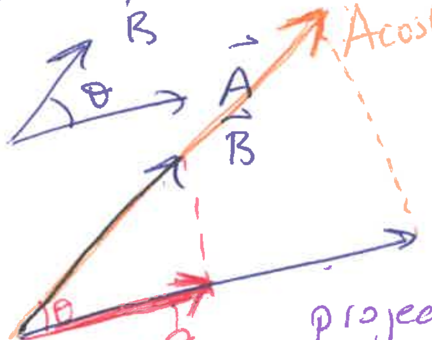
summation; subtraction
addition

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

scalar product (dot)

$\vec{A}\cdot\vec{B} = AB\cos\theta$

#; scalar

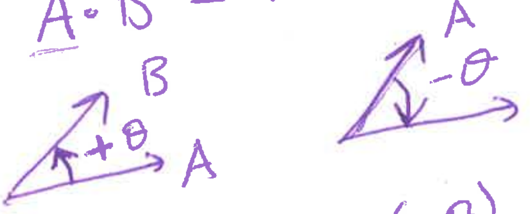


Acosθ

projection = izdüşüm

Bcosθ

$\vec{A}\cdot\vec{B} = AB\cos\theta$



$\cos\theta = \cos(-\theta)$

$\vec{A}\cdot\vec{B} < 0 \Leftarrow \theta > 90^\circ$

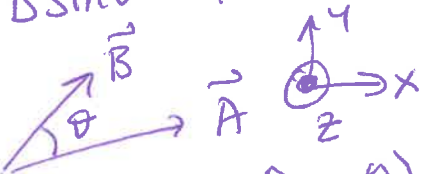
vector product (cross product)

$\vec{A}\times\vec{B} = \vec{C}$ (vector)

→ magnitude

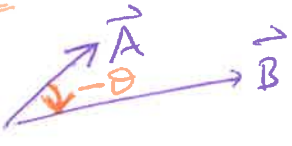
→ direction

$AB\sin\theta = |\vec{C}|$



Right hand rule

$\vec{A}\times\vec{B} = +\hat{k}\ (+\hat{z})$



$\vec{A}\times\vec{B} = -\hat{k}$

$\sin(-\theta) = -\sin\theta$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}\times\vec{B} =$

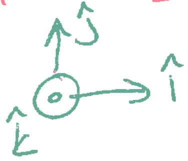
$A_xB_x\,\hat{i}\times\hat{i} + A_xB_y\,\hat{i}\times\hat{j} + A_xB_z\,\hat{i}\times\hat{k}$

$+ A_yB_x\,\hat{j}\times\hat{i} + A_yB_y\,\hat{j}\times\hat{j} + A_yB_z\,\hat{j}\times\hat{k}$

$+ A_zB_x\,\hat{k}\times\hat{i} + A_zB_y\,\hat{k}\times\hat{j} + A_zB_z\,\hat{k}\times\hat{k}$

$\hat{i}\times\hat{i} = |\hat{i}||\hat{i}|\sin 0 = 1\ 1\ 0 = \underline{0}$

$\hat{k}\times\hat{k} = \hat{j}\times\hat{j} = \hat{i}\times\hat{i} = \underline{0}$



$\hat{i}\times\hat{j} = +\hat{k}$

$\hat{j}\times\hat{i} = -\hat{k}$

$\hat{i}\times\hat{k} = -\hat{j}$ $\quad$ $\hat{k}\times\hat{i} = +\hat{j}$

$\hat{j}\times\hat{k} = \hat{i}$ $\quad$ $\hat{k}\times\hat{j} = -\hat{i}$

$\hat{k}\ \oplus\ \hat{j}$ → $\hat{i}$ $\quad$ $\hat{k}\ \circlearrowleft\ \hat{j}$ → $\hat{i}$

$\vec{A}\times\vec{B} = \hat{k}(A_xB_y - A_yB_x)$

$+\hat{j}(A_zB_x - A_xB_z)$

$+\hat{i}(A_yB_z - A_zB_y)$

*faster way possible

determinant!!

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}\,(A_yB_z - A_zB_y)$$
$$- \hat{j}\,(A_xB_z - A_zB_x)$$
$$+ \hat{k}\,(A_xB_y - A_yB_x)$$

---

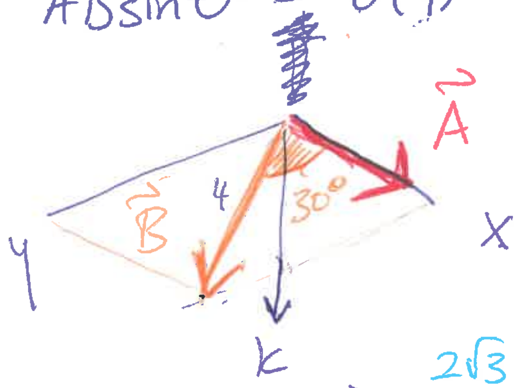
ex: vector $\vec{A}$ has a magnitude of 6 units. $\vec{B}$ " " of 4 units.

$\vec{A}$ is in $+x$ direction

$\vec{B}$ lies ~~in~~ xy plane makes 30° angle with x axis.

$\vec{A}\times\vec{B} = ?$

$AB\sin\theta = 6(4)\sin 30^\circ = \underline{12}$

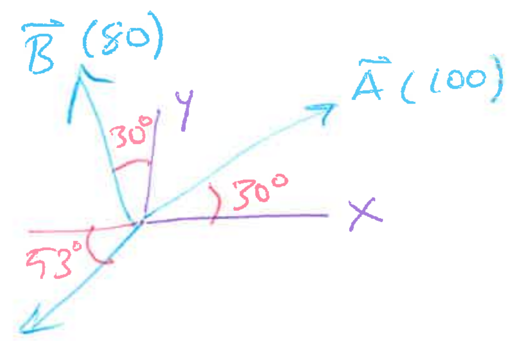


$$\boxed{\vec{C} = 12\hat{k}}$$

$\vec{A} = 6\hat{i}$; $\vec{B} = 4\cos 30\,\hat{i} + 4\sin 30\,\hat{j}$ $\quad (4\cos 30 = 2\sqrt{3},\ 4\sin 30 = 2)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 0 & 0 \\ 2\sqrt{3} & 2 & 0 \end{vmatrix} = \underbrace{\hat{i}\,(0) - \hat{j}\,(0) + \hat{k}\,(12-0)}_{12\hat{k}}$$

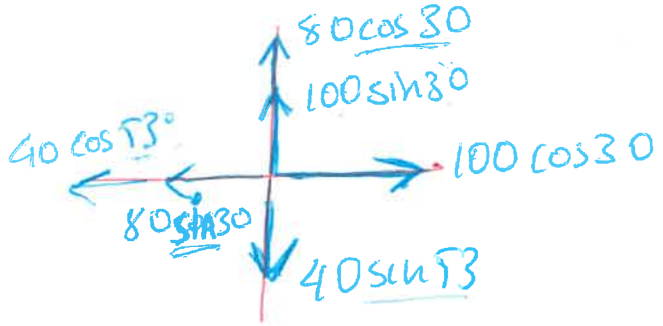
1.66 ②



$\vec{A}, \vec{B}, \vec{C}$ forces on an object.
what will be a 4th force that will cancel $\vec{A}+\vec{B}+\vec{C} = ?$

$$\vec{A}+\vec{B}+\vec{C}+\vec{D} = \underline{0}$$

$$\vec{D} = ?$$

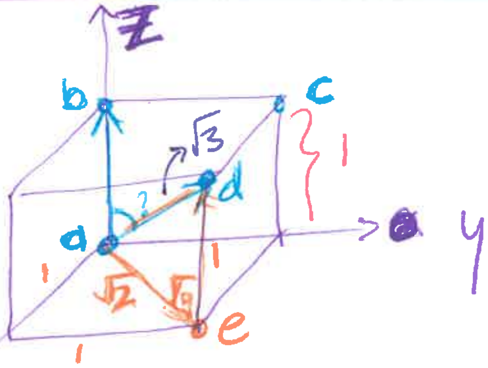


$$\vec{A}+\vec{B}+\vec{C} = \hat{i}\left(100\frac{\sqrt{3}}{2} - (80\frac{1}{2} + 40(0.\ldots\right)$$
$$+\hat{j}\left(100\frac{1}{2} + 80\frac{\sqrt{3}}{2} - 40(0.8)\right)$$

$$\vec{D} = -(\vec{A}+\vec{B}+\vec{C})$$ ✓

---

ex: 1.91
cube



→ angle between $\underline{ab}$ & $\underline{ad}$

$\vec{ab} = 1\hat{k}$ $\quad \vec{ad} = \vec{ae} + \vec{ed}$ (3)

$\vec{ab} \cdot \vec{ad} = |\vec{ab}|\,|\vec{ad}| \cos\theta$

$1 \quad \sqrt{3}$

$(\vec{ab}) \cdot (\vec{ae} + \vec{ed}) =$

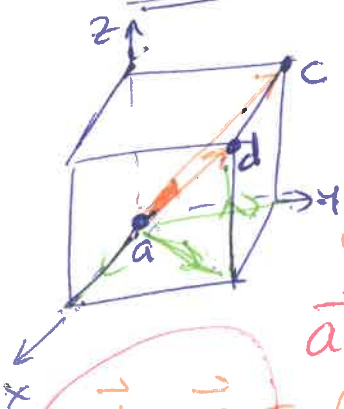
$\vec{ab} \cdot \vec{ae} + \hat{k}\,\vec{ab} \cdot \vec{ed}\,\hat{k} \} \underline{1}$

$\cos\theta = \cos 90 = 0$

$0 + 1 = 1\ \sqrt{3} \cos\theta$

$\cos\theta = \frac{1}{\sqrt{3}}$; $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = 54°$



$\vec{ad}$; $\vec{ac}$
angle between?

$\vec{ad} = \hat{i} + \hat{j} + \hat{k}$

$\vec{ac} = \hat{j} + \hat{k}$

$\vec{ad} \cdot \vec{ac} = (ad)(ac)\cos\theta$

$(\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{j} + \hat{k}) = \underline{1 + 1}$

$1 \quad 1$

$2 = \sqrt{1^2+1^2+1^2}\,\sqrt{1^2+1^2}\,\cos\theta$

$2 = \sqrt{3}\,\sqrt{2}\,\cos\theta$

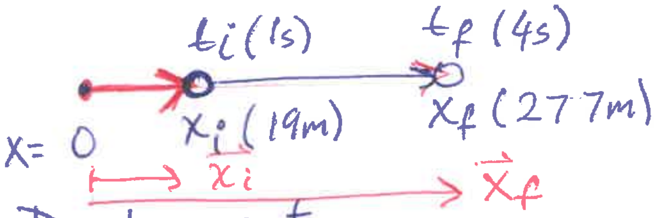
$\theta = \cancel{\ }\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

Chapter 2

motion along a straight line
$\equiv$ 1 dimensional motion

time (t) position (X)

$\Delta \equiv$ Delta $\equiv$ final − initial
son − ilk



Displacement

$\Delta X = X_f - X_i$; >0, <0, $=0$

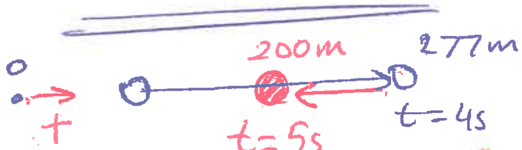
$\Delta t = t_f - t_i$ } >0

velocity $\equiv \frac{\text{displacement}}{\text{time}}$

$$\boxed{\bar{v} = \frac{\Delta x}{\Delta t} \equiv \text{average velocity}}$$

$\bar{v} = \frac{X_f - X_i}{t_f - t_i} = \frac{277 - 19}{4 - 1}\,\frac{m}{s}$

$\bar{v} = 86\ \frac{m}{s}$



final initial

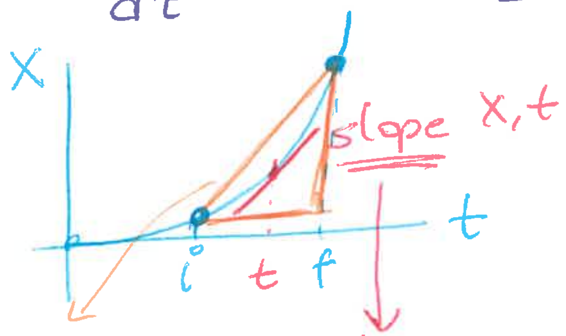
$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{200 - 277}{5 - 4}\,\frac{m}{s}$

$\bar{v} = -77\ m/s$

$\bar{v} \equiv$ average velocity (ortalama hız)

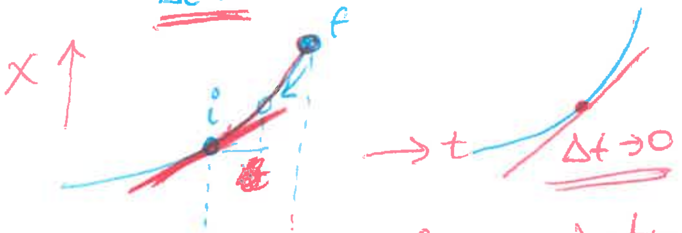
$v \equiv$ instantaneous velocity (anı hız)

$v = \frac{dx}{dt}$; $\bar{v} = \frac{\Delta x}{\Delta t}$



$\frac{\Delta x}{\Delta t} = \bar{v}$ $\quad \frac{dx}{dt} = v$

$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$



$\Delta t \to 0$ find → instant

ex: $x = 20m + 5\frac{m}{s^2} t^2$

Displacement ? $t_i = 1s$; $t_f = 2s$

$x_i = 20 + 5 \cdot 1^2 = 25m$

$x_f = 20 + 5(2^2) = 40m$

$\Delta x = x_f - x_i = 15m$

$\bar{v} = ? = \frac{\Delta x}{\Delta t} = \frac{15\,m}{2-1\,s} = 15\,m/s$

$v = ?$ at $t = 1s$

$\frac{dx}{dt} = 0 + 5(2)t^{2-1} = 10t$

$\frac{dx}{dt} = v = 10t$; $t = 1s$

$v = 10\,m/s$ (instant velocity)

$\bar{v}$; $t_f = 1.1s$ $\quad t_i = 1s$

$\Delta t = 1.1 - 1 = 0.1s$

$x = 20 + 5t^2$

$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{26.05 - 25}{1.1 - 1}$

$= \frac{1.05}{0.1} = 10.5\,m/s$

$\Delta t = 0.1s$

$\Delta t = 0.01s$ $\quad t_f = 1.01s$

$t_i = 1s$

$\bar{v} = \frac{\Delta x}{\Delta t} = 10.05\,m/s$ ($\Delta t = 0.0$

$\Delta t = 0.001s$; $\bar{v} = 10.005\,m/s$

$x = 20 + 5t^2$

$t_i = 1s$

$\bar{v}$	Δt	t_f	t_i
15 m/s	1 s	2	1
10.5	0.1	1.1	1
10.05	0.01	1.01	1
10.005	0.001	1.001	1

$\Delta t \to 0$

$\bar{v} \to v$

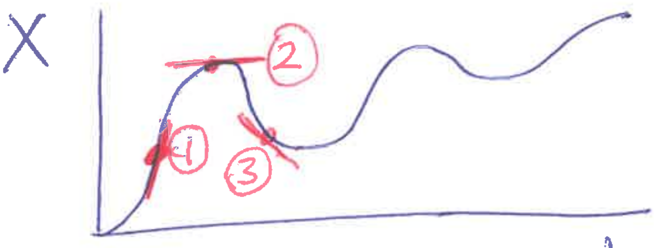
$v(t=1s) = 10\,m/s$

derivative of a polynomial

$f(t) = At^{n}$

$\frac{df}{dt} = A\,n\,t^{n-1}$

meaning of derivative on a graph. $\equiv$ SLOPE



① $\frac{dx}{dt} > 0$ ② $\frac{dx}{dt} = 0$

③ $\frac{dx}{dt} < 0$

position $X \rightarrow$ displacement ΔX

Δt velocity $\frac{\Delta x}{\Delta t} = \bar{v}$

$\frac{dx}{dt} = v$

ACCELERATION (live)

change of velocity over time

$\bar{a} = \frac{\Delta V}{\Delta t}$ $\Delta v > 0$, < 0, $= 0$

average acceleration $\bar{a}$ > 0, < 0, $= 0$

$a = \left[\frac{m/s}{s} = \frac{m}{s^2}\right]$

(5) $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

instantaneous acc. $a = \frac{dv}{dt}$

ex: $v = 60 + 0.5t^2$

$\left[\frac{m}{s}\right] v = 60\frac{m}{s} + 0.5\frac{m}{s^3}t^2$

(a) ; $t_i = 1s$ $t_f = 3s$

$\Delta v \equiv$ change in velocity?

$\Delta v = v_f - v_i$

$= (60 + 0.5\, 3^2)$

$- (60 + 0.5\, 1^2)$

$\Delta v = 64.5 - 60.5 = 4\, m/s$

(b) $\bar{a}$; $t_i = 1s$ $t_f = 3s$

$v = 60 + 0.5t^2$

$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{4\, m/s}{(3-1)s} = 2\, m/s^2$

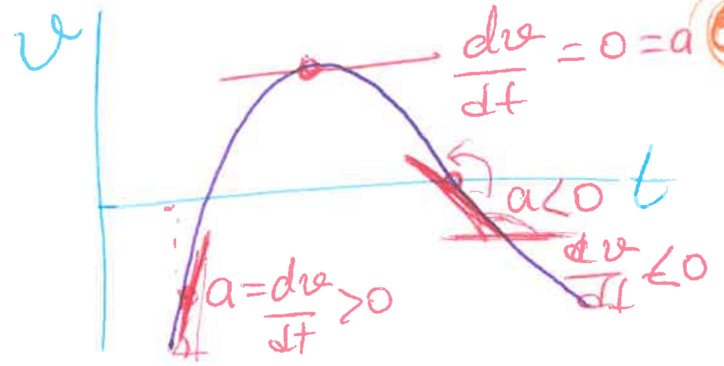
(c) $a = ?$ $t = 1s$

$a = \frac{dv}{dt} = 0 + 0.5(2)t^1$

$= t$

$a = 1\, m/s^2$ $(t = 1s)$ $\Delta t \Rightarrow 0$

$\bar{a}$	Δt	t_f	t_i
2	2	3	1
1.05	0.1	1.1	1
1.005	0.01	1.01	1



$a = \frac{dv}{dt} \equiv$ slope at that t

< 0

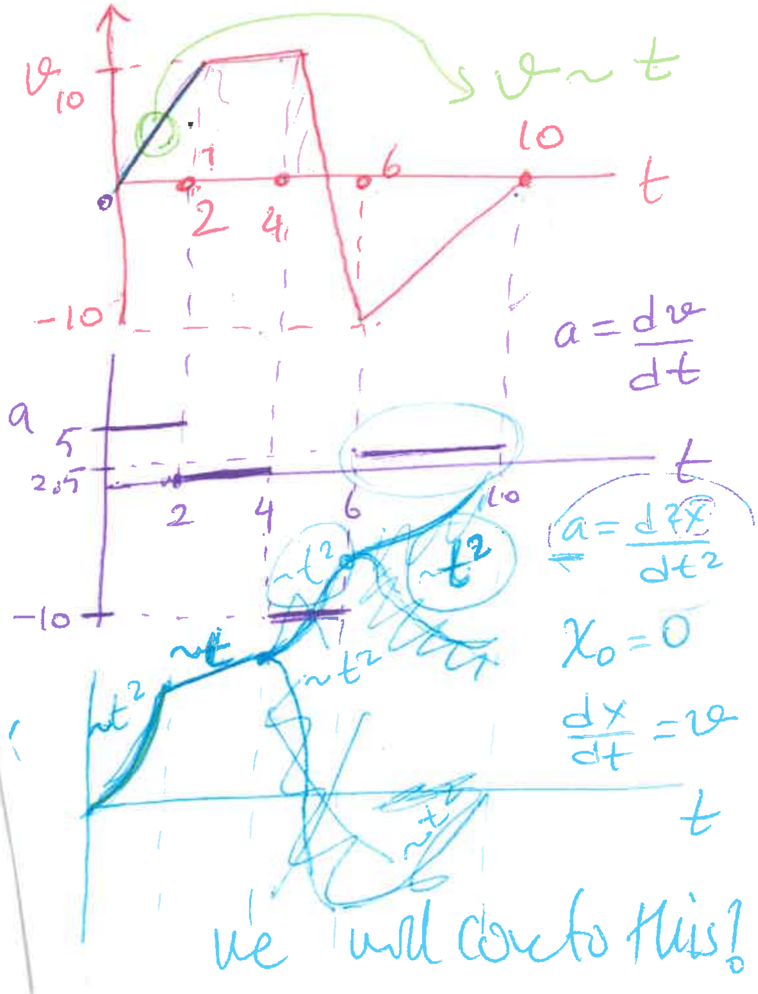
> 0

$= 0$

since $v = \frac{dx}{dt}$

$$a = \frac{d\left(\frac{dx}{dt}\right)}{dt} = \frac{d^2X}{dt^2}$$

acceleration is 2° degree derivative of X with respect to time.



Speed $\neq$ velocity

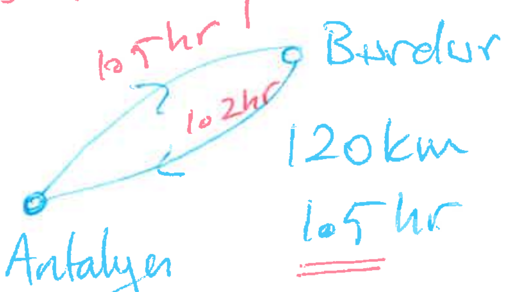
~~a~~ surat $\neq$ hız

$$\text{speed} = \frac{\text{total } \text{~~motion~~ distance}}{\text{time}}$$

we use this often.

speed > 0

speed $= 0$ no distance taken.
it's not moving



$$\text{speed} = \frac{120\,km}{1.5\,hr} = 80\,km/hr$$

overall $= \frac{120+120}{1.5+1.2}$

$$\text{speed} = \frac{240}{2.7}\,km/hr = 89\,km/h$$

$$\text{speed} = 89\,\frac{10^3\,m}{60\times60\,s} = 89\,\frac{1000}{3600}$$

$$\text{speed} = 89 \times \frac{1}{3.6}\,m/s = 24.7\,m/s$$

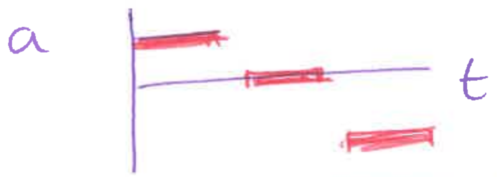
$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = 0$$

$\bar{v} = 0!!$

Motion w/ constant acceleration

(7)

$a = \text{const.}$ $g = \text{const} = 9.8\,m/s^2$



$* \; x \rightarrow \frac{dx}{dt} = v \rightarrow \frac{dv}{dt} = a$

$\frac{da}{dt} \rightarrow$?? not name this!

→ happening in real life.

$a = \text{const}$

$\frac{dv}{dt} = a \; ; \; \int_{v_i}^{v_f} dv = \int a\,dt$

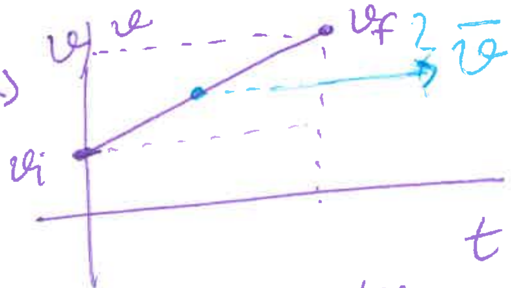
$v_f - v_i = \Delta v = a \Delta t \Big|_{t_i}^{t_f}$

$t_i = 0 \; ; \; t_f = t$

$v_f - v_i = a(t - 0)$

$* \; \boxed{v_f = v_i + at}$

$v_i = \pm \quad a \rightarrow +, \; \rightarrow -$



$\overline{v} = \frac{\Delta x}{\Delta t}$

$\frac{dv}{dt} = a \; ; \; v = \frac{dx}{dt}$

$\overline{v} = \frac{v_i + v_f}{2}$

$v_f = v_i + at$

$\overline{v} = \frac{v_i + v_f}{2} = \frac{\Delta x}{\Delta t}$

$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t - 0} = \frac{x_f - x_i}{t}$

$\frac{v_i + v_f}{2} = \frac{x_f - x_i}{t}$

$\frac{v_i + v_i + at}{2} = \frac{x_f - x_i}{t}$

$\frac{2v_i}{2} + \frac{at}{2} = \frac{x_f - x_i}{t}$

$\left(v_i + \frac{at}{2}\right)t = x_f - x_i$

(2) $\boxed{x_f = x_i + v_i t + \frac{at^2}{2}}$ **

(1) $\boxed{v_f = v_i + at}$

(1) and (2); make a 3rd equation that does not have "t" in it.

(1) $\frac{v_f - v_i}{a} = t$

$x_f = x_i + v_i\left(\frac{v_f - v_i}{a}\right) + \frac{a(v_f - v_i)^2}{2a^2}$

$= x_i + \frac{v_i v_f}{a} - \frac{v_i^2}{a}$

$+ \frac{1}{2a}\left(v_f^2 + v_i^2 - 2v_i v_f\right)$

8

$$x_f = x_i + \frac{v_i v_f}{a} - \frac{v_i^2}{a} + \frac{v_f^2}{2a} + \frac{v_i^2}{2a} - \frac{v_i v_f}{a}$$

$$x_f = x_i + \frac{v_f^2}{2a} - \frac{v_i^2}{2a}$$

$$v_f^2 - v_i^2 = 2a(x_f - x_i)$$

$$\boxed{v_f^2 = v_i^2 + 2a(x_f - x_i)}$$

***

$$\boxed{x_f = x_i + v_i t + \frac{1}{2}at^2}$$

**

$$\boxed{v_f = v_i + at}$$

*

7.7.17 Phys I ①

$a = \text{const.}$ $\quad v_f = v_i + at$

$$x_F = x_i + v_i t + \frac{1}{2}at^2$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

---

ex:  $\rightarrow a = 4\,m/s^2$

$t = 0s\,;\ x_i = 5m$

$v_i = 15\,m/s$

ⓐ $x_F = ?\ \ v_f = ?\ @\ t = 2s$

ⓑ when the velocity is $25\,m/s$ what is its location?

$$v_F = v_i + at$$
$$= 15 + 4(2) = 23\,m/s$$

ⓐ $x_F = x_i + v_i t + \frac{1}{2}at^2$

$$= 5 + 15(2) + \frac{1}{2}4(2^2)$$

$$x_F = 43\,m = 43m$$

ⓑ $x_F = ?\ \ v_F = 25\,m/s$

$$v_F^2 = v_i^2 + 2a(x_F - x_i)$$
$$25^2 = 15^2 + 2(4)(x_F - 5)$$
$$x_F = 55m$$

II way: $t = ?\quad v_F = 25\,m/s\quad v_i = 15\,m/s$

$$v_F = v_i + at$$

$t = 2.5s \Leftarrow 25 = 15 + 4(t)$

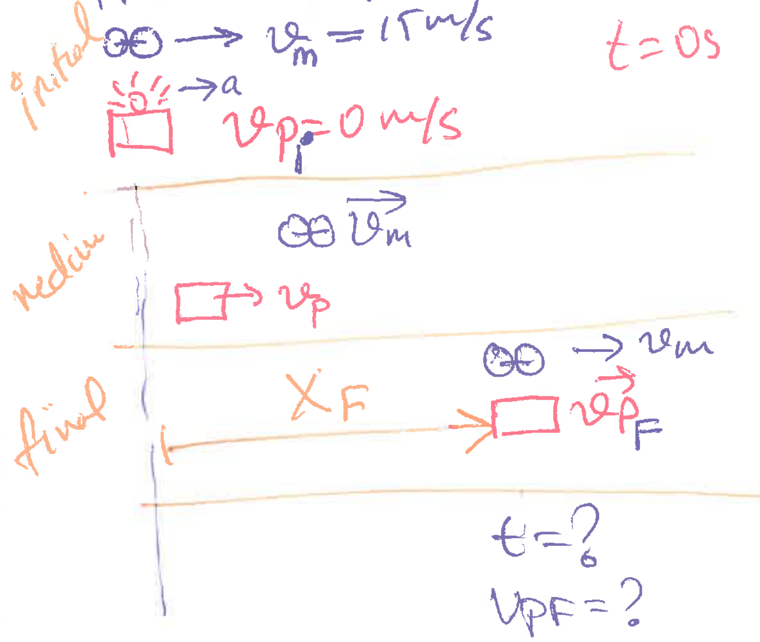
$$x_F = x_i + v_i t + \frac{1}{2}at^2$$
$$= 5 + 15\left(\frac{5}{2}\right) + \frac{1}{2}4\left(\frac{5}{2}\right)^2$$
$$= 5 + \frac{75}{2} + \frac{25}{2} = 55m$$

ex: a motorist traveling at a constant 15 m/s passes a police car where the speed limit is 10 m/s. Just after passing the police, police car tries to catch the motorist with $a = 3\,m/s^2$

ⓐ how much time passes before the police catch the motorist

ⓑ at that time; what's the police's speed.

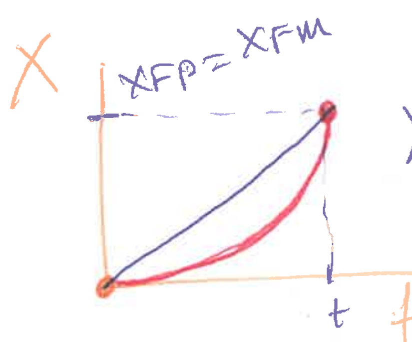
ⓒ how far has each vehicle travelled?



$t = ?$

$v_{PF} = ?$

$x_F = ?$



$x_{FM} = x_{FP}$ (motorist, pol)

$$x_{FP} = x_{iP} + v_{Pi}t + \frac{1}{2}at^2$$
$$x_{FM} = x_{iM} + v_{Mi}t + \frac{1}{2}at^2$$
$$\frac{1}{2}at^2 = v_{Mi}t$$

$\frac{1}{2}at^2 = v_{mi}t$

$\frac{1}{2}(3)\cancel{t^2} = 15\cancel{t}$

(a) $t = \frac{15(2)}{3} = \underline{\underline{10s}}$

(b) ~~x_{PF}~~ $v_{PF} = ?$

$v_{FP} = \cancel{v_{iP}} + a\,t$; $a \to (3)$, $t \to (10)$

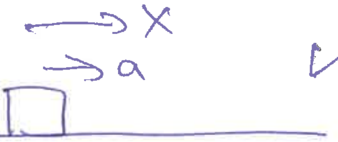
$= 30 m/s$

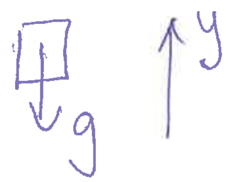
(c) $x_{PF} = x_{MF} = ?$

$x_{PF} = \frac{1}{2}at^2 = \frac{1}{2}\,3(10^2)$

$= 150m$

$a = const$; 1 dimensional motion ✓





$g = 9.8 m/s^2$

can also solve this question

Freely fallig objects ✓

@ $t=0s$; $v_i = 0 m/s$; g

what's its location and velocity at $t=1s$, $t=2s$, $t=3s$?

(2) $+y$; g

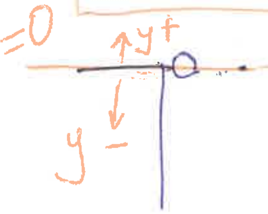
$v_F = v_i + at$

$\boxed{v_F = v_i - gt}$

$x_F = x_i + v_i t + \frac{1}{2}at^2$

$\boxed{y_F = y_i + v_i t - \frac{1}{2}gt^2}$

$\boxed{v_F^2 = v_i^2 - 2g(y_F - y_i)}$



$v_i = 0$ @ $t = 0s$

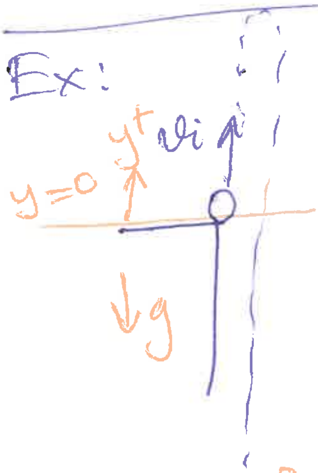
t	y_F	v_F
1	$-4.9m$	$-9.8 m/s$
2	-19.6	$-19.6 m/s$
3	-44.1	$-31.4 m/s$

$v_F = v_i - gt = 0 - (9.8)t$ $\begin{pmatrix}1\\2\\3\end{pmatrix}$

$y_F = \cancel{y_i} + \cancel{v_i t} - \frac{1}{2}gt^2$

$= -\frac{1}{2}(9.8)\begin{bmatrix}1^2\\2^2\\3^2\end{bmatrix}$ $\quad 4.9 \times 9 = 44.1$

Ex:



$v_i = 15 m/s$

(a) $t = 1s$

$y_F, v_F = ?$

$t = 4s$

$y_F, v_F = ?$

$y_F = \cancel{y_i} + v_i t - \frac{1}{2}gt^2$

$= 15(1) - \frac{1}{2}\,9.8(1^2)$

$y_F = 10.1\,m$ @ $t = 1s$

$v_F = v_i - gt$

$= 15 - (9.8)(1)$

$v_F = 5.2 \text{ m/s}$ @ $t = 1s$

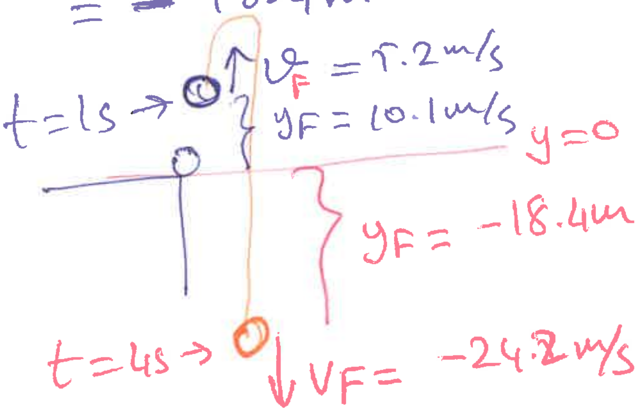
$v_F = 15 - 9.8(4)$ @ $t = 4s$

$= -24.2 \text{ m/s}$

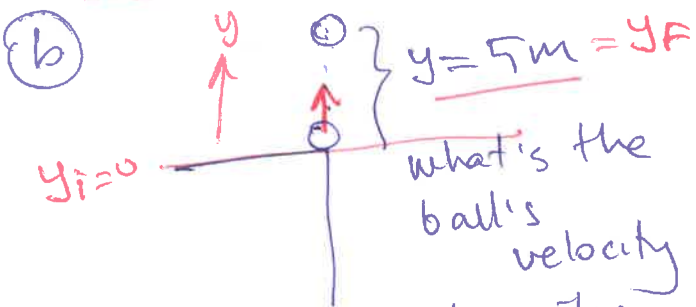
$y_F = \cancel{y_i}^{0} + v_i t - \frac{1}{2} g t^2$

$= 15(4) - \frac{1}{2}(9.8)4^2$

$= -18.4 m$



(b)



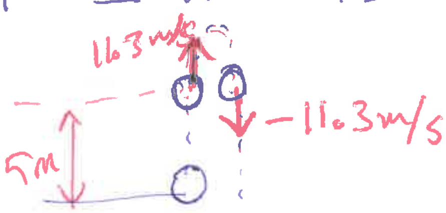
what's the ball's velocity when it's 5m above?

$V_F^2 = V_i^2 - 2g(y_F - y_i)$

$= (15)^2 - 2(9.8)[5-0]$

$V_F^2 = 127 (m/s)^2$

$V_F = \pm 11.3$ m/s



3 (c) ball's max height

$h_{max} = y_{max}$

$y_i = 0$

$v_F = 0 (?)$

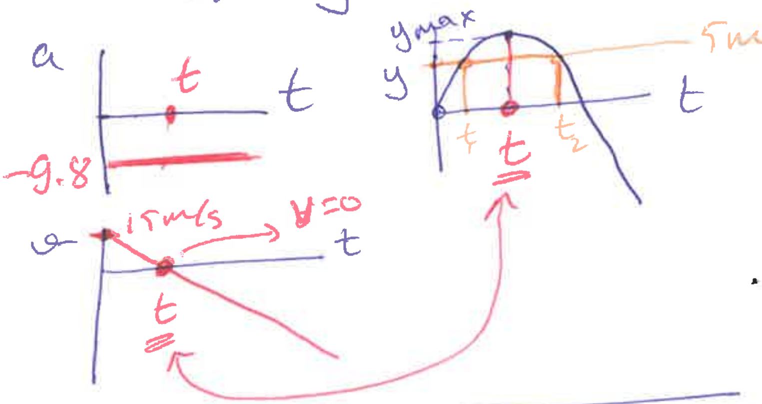
$v_F^2 = v_i^2 - 2g(y_F - y_i)$

$0 = 15^2 - 2(9.8) y_{max}$

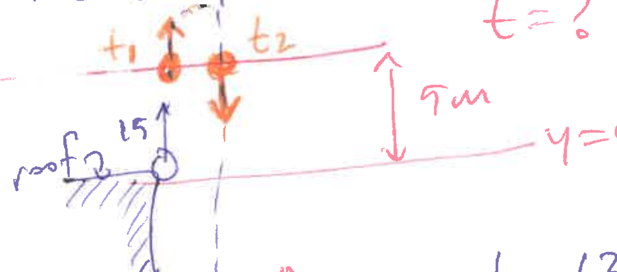
$y_{max} = \frac{15^2}{2(9.8)} = 11.5 m$

(d) what's ball's acceleration at maximum height?

$a = -g = -9.8 \text{ m/s}^2$



(e) ~~what~~ at what times the ball is 5m above the roof?



$y_F = \cancel{y_i}^{0} + v_i t - \frac{1}{2} g t^2$

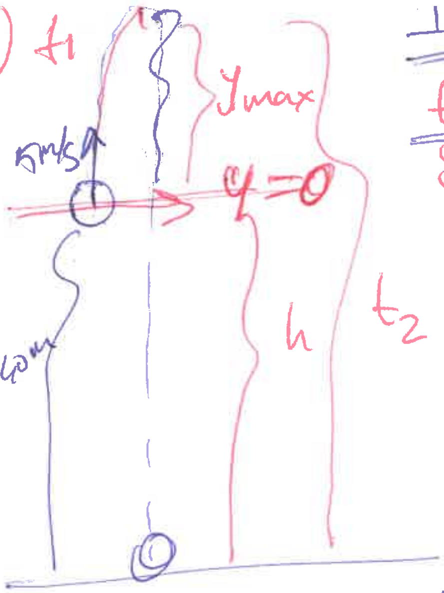
$5 = 15t - \frac{1}{2} 9.8 t^2$

$4.9t^2 - 15t + 5 = 0$

A B C

$At^2 + Bt + C = 0$

(4) t_1



1st way

t_1 ✓

y_{max} ✓

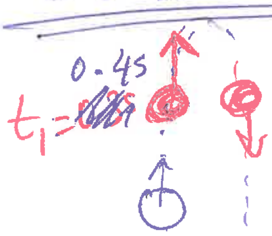
$h + y_{max} \Rightarrow t_2$

$t_1 + t_2$ ✓

$$t_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{15 \pm \sqrt{(-15)^2 - 4(4.9)5}}{2(4.9)}$$

$t_{1,2} =$ ~~...~~ ? ?



$$= 15 \pm \frac{\sqrt{225 - 20(9.8)}}{127}$$

$$t_{1,2} = \frac{15 \pm 11.3}{9.8} \begin{cases} \to 0.375 \\ \to 2.75 \end{cases}$$

$t_1 \Rightarrow$ tie to reach max height

$V_F = 0$

$V_F = v_i - gt$

$0 = 5 - 9.8t$

$t_1 = (5/9.8)s$

$y_F = y_i + v_i t - \frac{1}{2}gt^2$

$y_{max} = 0 + 5\left(\frac{5}{9.8}\right) - \frac{1}{2}(\cancel{9.8})\frac{5^2}{9.8^{\cancel{2}}}$

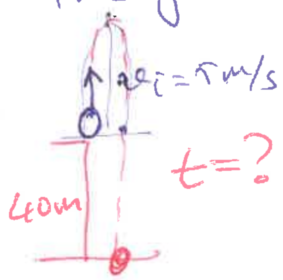
$y_{max} = \frac{25}{2(9.8)}$ m

Ex: 2.44

object released at these conditions.

$h = 40m$

when does the object hit the ground?



$$\boxed{40 + \frac{25}{2(9.8)}} \text{ m}$$

$y_F = y_i + \cancel{v_i t} - \frac{1}{2}gt^2$

$0 = y_i - \frac{1}{2}gt^2$

$y_i = \frac{1}{2}gt^2$

$40 + \frac{25}{2(9.8)} = \frac{1}{2} 9.8 t_2^2$

$41.3 = 4.9 t_2^2$

$t_2 = 2.9s$

$t_1 = \frac{5}{9.8} = 0.5s$ } 3.4

(5) Chapter 5

Motion in 2d & 3d

chapter 2 $\Rightarrow$ 1d

$x \rightarrow \Delta x$

$\bar{v} = \frac{\Delta x}{\Delta t}$

$v = \frac{dx}{dt}$

$\bar{a} = \frac{\Delta v}{\Delta t}$

$a = \frac{dv}{dt}$

$a = $ const

3 equ's of motion ✓

$x \rightarrow \vec{r}$ (position)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t}$ $\Rightarrow$ average velocity

$\vec{v} = \frac{d\vec{r}}{dt}$ $\Rightarrow$ instantaneous velocity

$\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t}$ $\Rightarrow$ average acceleration

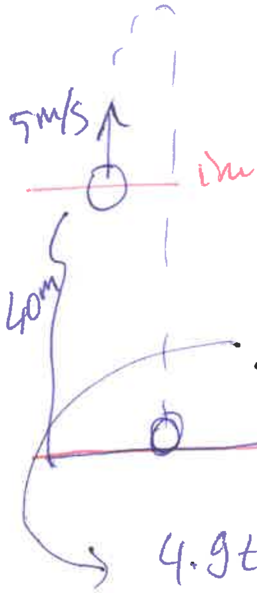
$\vec{a} = \frac{d\vec{v}}{dt}$ $\Rightarrow$ instantaneous acceleration

$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$

$$\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}}$$

$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$

$$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$$



$t = ?$ hit the ground.

initial

$y_F = y_i + v_i t - \frac{1}{2}gt^2$

$0 = 40 + 5t - 4.9t^2$

$y = 0 =$ final

$4.9t^2 - 5t - 40 = 0$

$At^2 + Bt + C = 0$

$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\frac{5 \pm \sqrt{25 - 4(4.9)(-40)}}{2(4.9)}$

$\frac{5 \pm \sqrt{809}}{9.8} =$

$\frac{5 \pm 28.4}{9.8} = \underline{\underline{3.41s}}$

ex: an object

$X = 2 - 0.25t^2$

$y = 1t + 0.025t^3$

$z = 0$

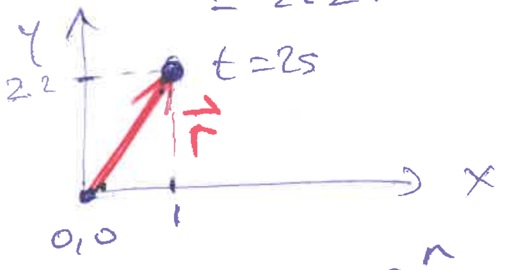
(a) what's the coordinates of the object at $t = 2s$?

how far is it from the beginning ($t = 0s$)

$X(t=2s) = ?$ $\quad y(t=2s) = ?$

$X(t=2s) = 2 - 0.25(2^2) = 1m$

$y(t=2s) = 1(2) + 0.025(2^3) = 2.2m$



$\vec{r} = 1\hat{i} + 2.2\hat{j}$

$|\vec{r}| = \sqrt{1^2 + 2.2^2} = 2.4m$

(b) what's $\Delta\vec{r} = ?$ $\quad \vec{\bar{v}} = ?$

between 0s and 2s.

$X_i = 0 \quad X_f = 1m$

$y_i = 0 \quad y_f = 2.2m$

$\Delta\vec{r} = \vec{r_f} - \vec{r_i} = (\hat{i} + 2.2\hat{j})\ m$

$\vec{\bar{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\hat{i} + 2.2\hat{j}}{2 - 0}\ \frac{m}{s}$

$\vec{\bar{v}} = 0.5\hat{i} + 1.1\hat{j}\ m/s$

6 (c) $\vec{v} = ?$ instantaneous velo.

$\vec{v} = \frac{d\vec{r}}{dt}$

$\vec{r} = x\hat{i} + y\hat{j}$

$= (2 - 0.5t^2)\hat{i} + (1t + 0.025t^3)\hat{j}$

$\frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j}$

$\frac{d\vec{r}}{dt} = (-0.5)(2)t\,\hat{i} + (1 + 0.025(3)t^2)\hat{j}$

$\vec{v} = -t\hat{i} + (1 + 0.075t^2)\hat{j}$

$\vec{v}(t=2s) = ?$

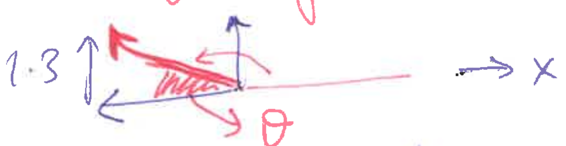
$\vec{v} = -2\hat{i} + (1 + 0.075(2^2))\hat{j}$

$\vec{v} = (-2\hat{i} + 1.3\hat{j})\ m/s$

what's the magnitude and direction of $\vec{v}(t=2s)$

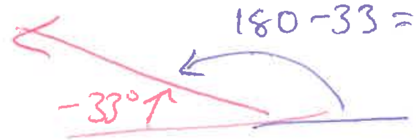
$|\vec{v}| = \sqrt{(-2)^2 + (1.3)^2} = 2.4\ m/s$

θ = angle between x, y, r



$\tan^{-1}\left(\frac{1.3}{-2}\right) = -33°$

$180 - 33 = 147°$

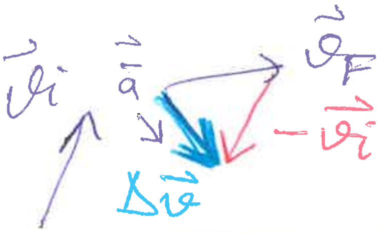


$\vec{r} \rightarrow \vec{v}$

$\vec{v} \rightarrow \vec{a}$

$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$



ex: $\vec{v} = \underbrace{-0.5t}_{v_x}\hat{i} + \underbrace{(1+(0.075)t^2)}_{v_y}\hat{j}$

$\bar{\vec{a}}$; between $0s$, $\underline{2s}$

$\vec{a}(t=2s) = ?$

$$\frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{\underbrace{[-\hat{i}+1.3\hat{j}]}_{v_f} - \underbrace{[\hat{j}]}_{v_i}}{2-0}$$

$$\bar{\vec{a}} = \left(-\frac{\hat{i}}{2} + \frac{0.3}{2}\hat{j}\right)\frac{m}{s^2}$$

$$|\bar{\vec{a}}| = \sqrt{\left(-\tfrac{1}{2}\right)^2 + \left(\tfrac{0.3}{2}\right)^2} \quad \text{magnitude}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$v_x = -0.5t$; $v_y = 1 + 0.075t^2$

$\vec{a} = -0.5\hat{i} + 0.15t\hat{j}$

$\vec{a}(t=2s) = -0.5\hat{i} + 0.15(2)\hat{j}$

$\vec{a} = (-0.5\hat{i} + 0.3\hat{j})\ m/s^2$

$|\vec{a}| = \sqrt{(-0.5)^2 + (0.3)^2}$

$\theta = \tan^{-1}\left(\frac{0.3}{-0.5}\right) = -31°$

7

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

curved; circular path

$\Delta\vec{v}$ pointing towards the center of the circle.

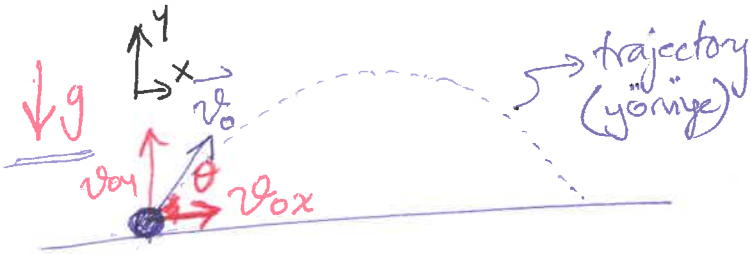
circular motion; you will get acceleration ~~point~~ in $(-\hat{r})$ direction.

motion in 2d

11.7.07

FIZI 1

Projectile Motion



$\hat{x}$ direction

$v_{ox} = v_f = v_i + \cancel{a}t \Rightarrow$ ①

velocity in $\hat{x}$ direction does NOT CHANGE

$v_0 \cos\theta = v_{ox}$

② $x_f = x_i + v_i t + \frac{1}{2}\cancel{a}t^2$

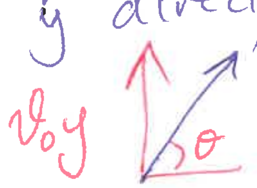
$x_i = 0$

$x_f = v_i t = v_{ox} t$

③ $v_f^2 = v_i^2 + 2\cancel{a}(x_f - x_i)$

$v_f = v_i = v_{ox}$

$\hat{y}$ direction



$v_{oy} = v_0 \sin\theta$

$v_f = v_i + at \quad (a = -g)$

g ↓ $+\hat{y}$ (0,0)

$$\boxed{v_{fy} = v_{oy} - gt}$$

$y_f = y_i + v_i t - \frac{1}{2}gt^2$

$y_i = 0$

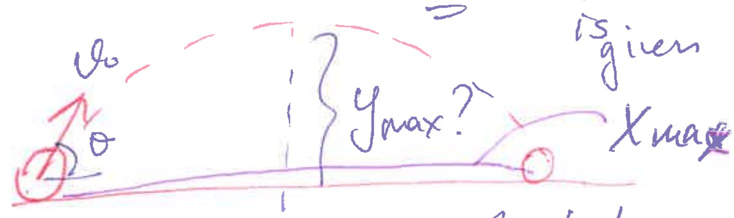
$$\boxed{y_F = y_i + v_{oy}t - \frac{1}{2}gt^2}$$

$v_f^2 = v_i^2 \cancel{+} 2g(y_f - y_i)$

$v_{fy}^2 = v_{oy}^2 - 2g(y_F - y_i)$

? ?

t? v_0, θ is given



① How long does it take to hit ground again?

t ?

② How high will it reach? y_{max}?

③ How far does it go? X_{max}?

motion in $\hat{y}$ direction

$v_{fy} = v_{oy} - gt \rightarrow$?

$|\vec{v}_{oy}| \overset{?}{=} |\vec{v}_{fy}|$

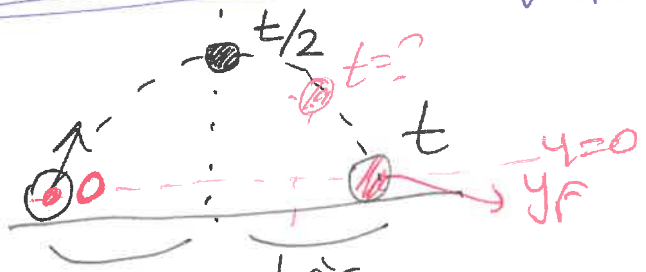
$\vec{v}_{oy} = -\vec{v}_{fy} \Rightarrow v_{fy} = -v_{oy}$

$-v_{oy} = v_{oy} - gt$

$gt = 2 v_{oy}$

$$\boxed{t = 2\frac{v_{oy}}{g} = \frac{2 v_0 \sin\theta}{g}}$$ (2)

time of flight for the projectile motion



symmetric

II. way

$y_F = \cancel{y_i} + v_{oy} t - \frac{1}{2} g t^2$

$0 = 0 + v_{oy} t - \frac{1}{2} g t^2$

$v_{oy} \cancel{t} = \frac{1}{2} g t^{\cancel{2}}$

time of flight

$$\boxed{t = 2\frac{v_{oy}}{g} = 2\frac{v_0 \sin\theta}{g}}$$

(2) v_{oy}

$v_{fy} = 0$ at max height.

I. way $\cancel{v_{fy}^2} = v_{oy}^2 - 2g(y_F - y_i)$

$v_{oy}^2 = 2g\, y_{max}$

$$\boxed{y_{max} = \frac{v_{oy}^2}{2g} = \frac{(v_0 \sin\theta)^2}{2g}}$$

II. way y_{max}

$y_F = y_i + v_{oy} t - \frac{1}{2} g t^2$

$v_{fy} = v_{oy} - g t$

$$t = \frac{v_{oy}}{g}$$

$y_{max} = v_{oy} \frac{v_{oy}}{g} - \frac{1}{2} \cancel{g} \frac{v_{oy}^2}{g^{\cancel{2}}}$

$$\boxed{y_{max} = \frac{v_{oy}^2}{2g} = \frac{(v_0 \sin\theta)^2}{2g}}$$

(3) How far it goes?

$\hat{X}$ direction

X max = ?

Range (menzil)

$X_F = \cancel{X_i} + v_0 x\, t \Rightarrow$ time of flight

from $\hat{y}$ direction

$t = 2\frac{v_{oy}}{g}$

$X_{max} = v_{ox} \frac{2 v_{oy}}{g}$

$X_{max} = \frac{2 v_0^2 \cos\theta \sin\theta}{g}$

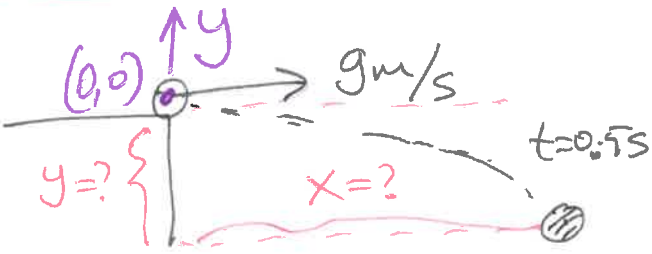
3

$$X_{max} = v_0^2 \frac{\sin 2\theta}{g}$$

10 m/s Range of the motion

$\theta = 35°$ $\quad X_{max} = \frac{10^2 \sin 70}{9.8}$

Ex



→ what's its location at $t = 0.5s$?

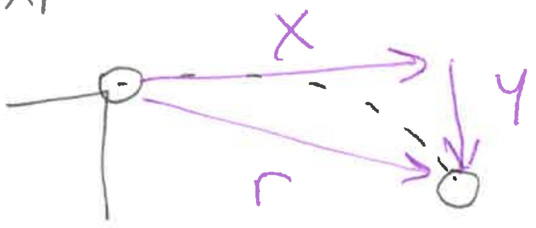
$$y_F = y_i + v_{oy} t - \frac{1}{2} g t^2$$

$$y_F = 0 + 0(0.5) - \frac{1}{2} 9.8 (0.5)^2$$

$$y_F = -1.2m$$

$$X_F = \cancel{X_i} + v_{ox} t$$

$$X_F = 9(0.5) = 4.5m$$



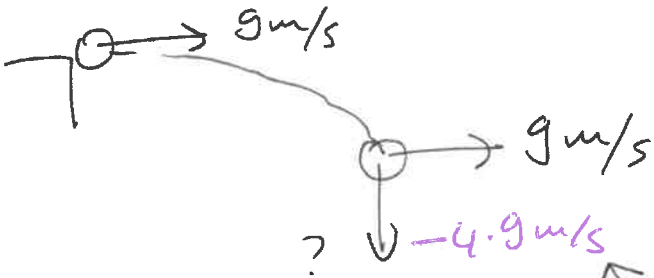
$$r^2 = x^2 + y^2$$

$$r = \sqrt{(4.5)^2 + (-1.2)^2}$$

$$r = 4.7m$$

(b) what's $V_{fy} = ?$ $\quad v_{fx} = ?$ at $t = 0.5s$

$$v_{fx} = v_{ox} = 9 m/s$$



$$v_{fy} = v_{oy} - gt$$

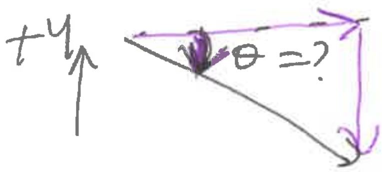
$$= -9.8(0.5)$$

$$v_{fy} = -4.9 m/s$$

$$\vec{v}_f = v_{fx} \hat{i} + v_{fy} \hat{j}$$

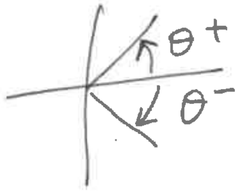
$$= 9\hat{i} - 4.9\hat{j}$$

direction of the $\vec{v}_f$ vector



$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$$

$$\theta = \tan^{-1}\left(\frac{-4.9}{9}\right) = -29°$$



position vector $\vec{r}$

$$\vec{r} = X_F \hat{i} + y_F \hat{j}$$

$$= 4.5\hat{i} - 1.2\hat{j}$$

$$\alpha = \tan^{-1}\left(\frac{-1.2}{4.5}\right)$$

(4)

ex:

$v_0 = 37\,m/s$

(a) position @ $t = 2s$

" ?

$\vec{v}$

$\vec{r} = \underset{?}{\underline{\underline{x}}}\,\hat{i} + \underset{?}{\underline{\underline{y}}}\,\hat{j}$

$\underline{\underline{X_F}} = \cancel{X_i}^{\,0} + v_{0x} t = v_0 \cos\theta\, t$

$y_F = \cancel{y_i}^{\,0} + v_{0y} t - \frac{1}{2} g t^2$

$v_0 \sin\theta$

$v_{0x} = (37)(\cos 53.1) = 22.2\,m/s$

$v_{0y} = 37 \sin 53.1 = \underline{29.6\,m/s}$

$X_F = (22.2)(2) = \underline{44.4\,m}$

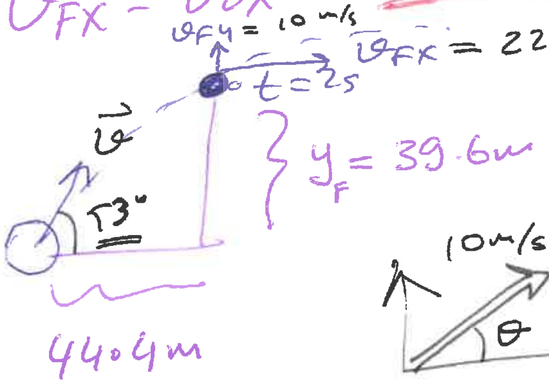
$y_F = (29.6)\,2 - \frac{1}{2}\, 9.8\; 2^2$

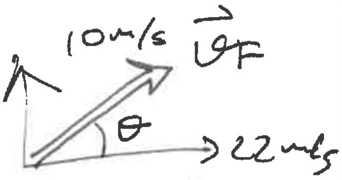
$y_F = \underline{39.6\,m}$

$\underline{v_{Fy}} = v_{0y} - gt = 29.6 - 9.8(2)$

$\underline{v_{Fy} = 10\,m/s}$

$v_{FX} = v_{0x} = \underline{22.2\,m/s}$

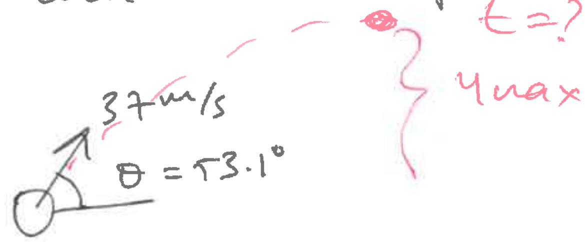




$\theta = \tan^{-1}\left(\frac{10}{22}\right) = 24°$

(b) $h_{max} = ? = y_{max}$

how long does it take to reach max height?



$v_{Fy} = v_{0y} - gt$

@ y_{max} $v_{Fy} = 0$!!

$t = \frac{v_{0y}}{g} = \frac{v_0 \sin\theta}{g}$

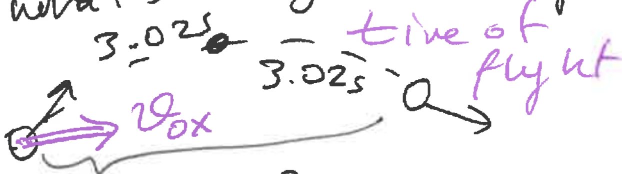
$\underline{\underline{t = 3.02s}} = \frac{(37)(\sin 53.1)}{9.8}$

$y_{max} = y_F = \cancel{y_i}^{\,0} + v_{0y} t - \frac{1}{2} g t^2$

$y_{max} = (29.6)(3.02) - \frac{1}{2}\, 9.8\, (3.02$

$y_{max} = 44.7\,m$

(c) what is horizontal Range?



$X_{max} = ?$

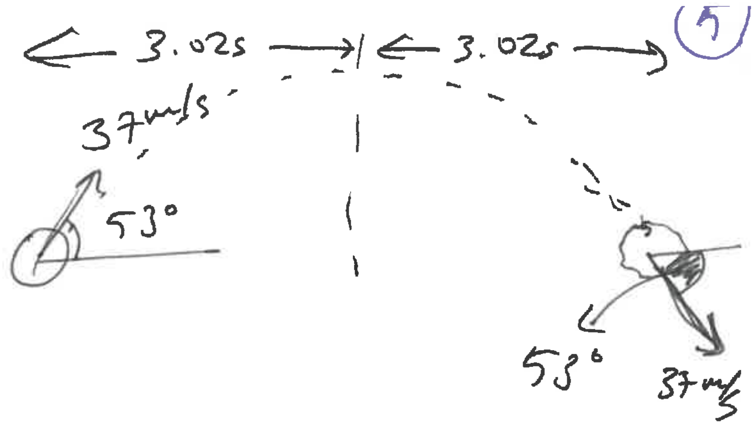
time of flight $= 2(3.02)s$

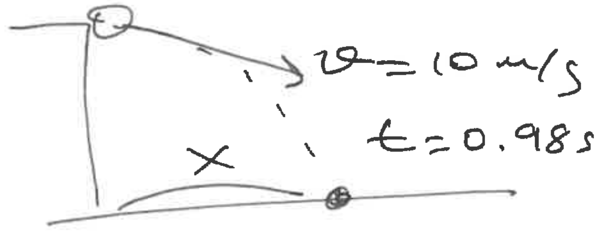
$= \underline{\underline{6.04s}}$

$X_{max} = v_{0x}\, t = (22.2)\, 6.04$

$= 134\,m$

5



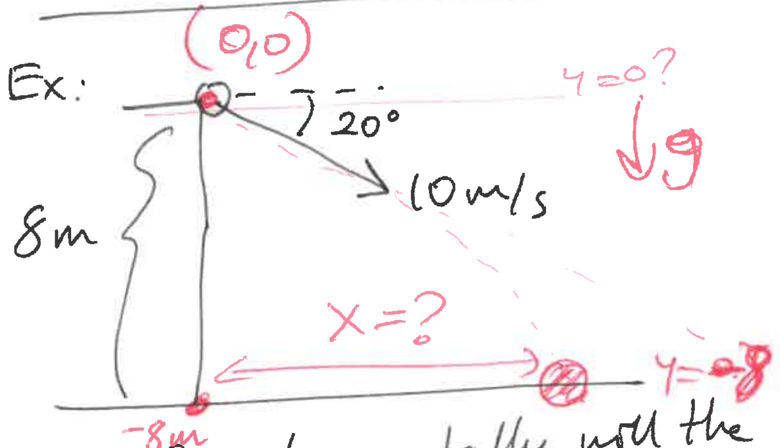


$X = v_{0x}\, t$

$= (9.4)(0.98)$

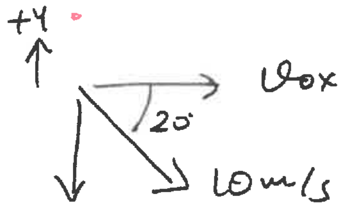
$X = 9.2\,m$

Ex:



How far horizontally will the ball hits the ground?

$X = v_{0x}\, t$? → $\hat{y}$ motion



$v_{0x} = 10\cos 20$
$= 9.4\,m/s$

$10\sin 20 = v_{0y} = 3.4\,m/s$

$y_F = y_i + v_{0y} t - \frac{1}{2} g t^2$

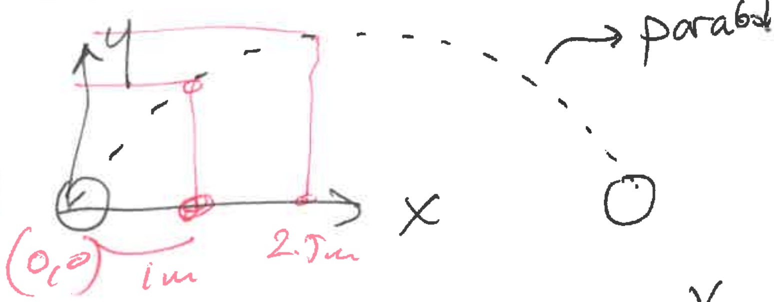
$-8 = 0 + (3.4)t - \frac{1}{2}(9.8)t^2$

$4.9t^2 + 3.4t - 8 = 0$

$At^2 + Bt + C = 0$

$t = \dfrac{-B \mp \sqrt{B^2 - 4AC}}{2A}$

$t = \cancel{-1.7s}$; $t = 0.98s$



y should be a function of X of X

$y(x) = ?$

$y_F = \cancel{y_i} + v_{yi}\, t - \frac{1}{2} g t^2$

$\hat{x} \Rightarrow x_F = x_i + v_{0x}\, t$

$x_F = v_{0x}\, t$

$t = \dfrac{x_F}{v_{0x}}$

$x_F \equiv x$
$y_F = y$

$y = v_{0y} \dfrac{x}{v_{0x}} - \frac{1}{2} g \dfrac{x^2}{v_{0x}^2}$

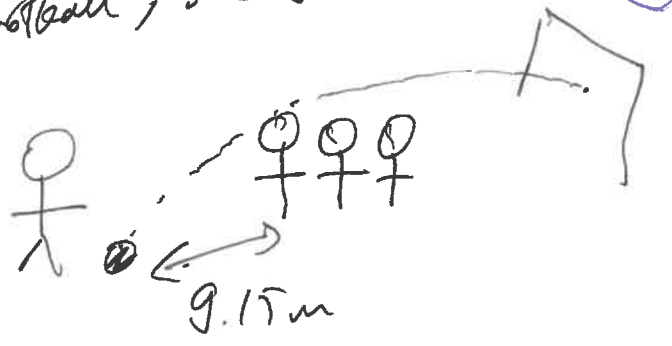
$y = \dfrac{\cancel{v_0}\sin\theta\, x}{\cancel{v_0}\cos\theta} - \frac{1}{2} g \dfrac{x^2}{v_0^2\cos^2\theta}$

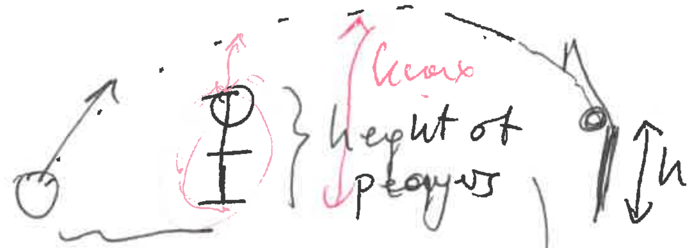
$y = \tan\theta\, x - \dfrac{g}{2v_0^2\cos^2\theta} x^2$

$y = (\text{const})\, x - (\text{const})\, x^2$

(6)

Football, soccer

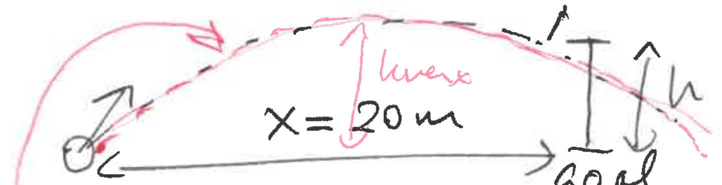




$X = 9.15\,m$

v_0, θ given

$y(X = 9.15\,m) > h$



$y(X = 20\,m) < h$ of goal

$$y = \tan\theta \, x - \frac{g}{2 v_0^2 \cos^2\theta} x^2$$

2d motion

↳ projectile motion

$a_y = -g \quad a_x = 0$

↳ motion in a circle

$a \neq 0$

Uniform Circular Motion

↳ radius = const

↳ speed = const.

speed ≠ velocity

sürat hız

number vector

$|\vec{velocity}| = speed$

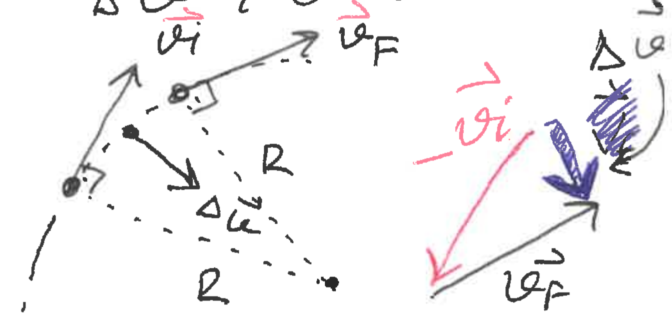
UCM (uniform circular motion)

→ magnitude of velocity (speed) does NOT change

→ BUT direction changes!!

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ } difference in vector

$\Delta \vec{v} \neq 0 \Rightarrow a \neq 0$

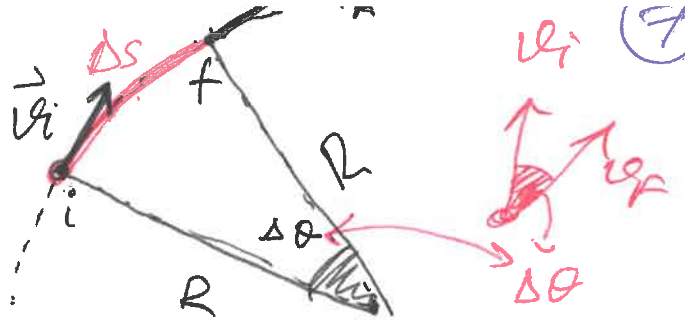


$\Delta \vec{v} = \vec{v_F} - \vec{v_i}$

$\Delta \vec{v}$ points towards the center!!

$\vec{a}$ points towards the center!!

(7)

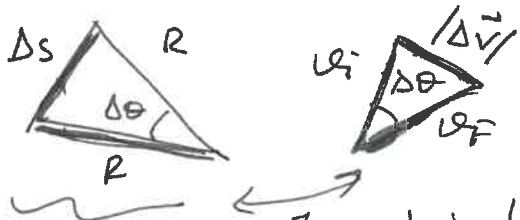


distance = arc (yay) = S

$R\Delta\theta = \Delta s$

if $\Delta\theta = 2\pi$

$\Delta s = (2\pi) R$



similar triangles

$$\frac{\Delta s}{R} = \frac{|\Delta\vec{v}|}{v_i}$$

$$\frac{1}{\Delta t}|\Delta\vec{v}| = \frac{\Delta s\, v_i}{R}\frac{1}{\Delta t}$$

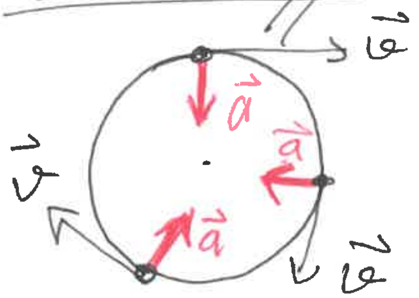
$$a = \frac{v}{R}\frac{\Delta s}{\Delta t}$$

$\Delta s \to$ disp.

$\Delta t \to$ time

v

$$a = \frac{v^2}{R} \qquad v_i = v_f = v$$



$\vec{a}$ in $(-\hat{r})$ direction

UCM $\quad v = \text{const}$, $R = \text{const}$

$$|\vec{a}| = \frac{v^2}{R}; \quad \vec{a} = \frac{v^2}{R}(-\hat{r})$$

1 rotation

$\Delta s = 2\pi R$

time it takes 1 rotation is called T (Period)

$$T v = \Delta s = 2\pi R$$

$$a = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R}$$

$$\boxed{a = \frac{4\pi^2 R}{T^2} = \frac{v^2}{R}}$$

ex: 1 rotation in 4s of radius 5m.

$$a = \frac{4(3.14)^2 5}{4^2} = 12\ m/s^2$$

$$v = \frac{2\pi R}{T} = \frac{2(3.14)5}{4}$$

$v = 7.9\ m/s$

What happens if v (speed) is not constant in a circular motion?

Non uniform circular motion

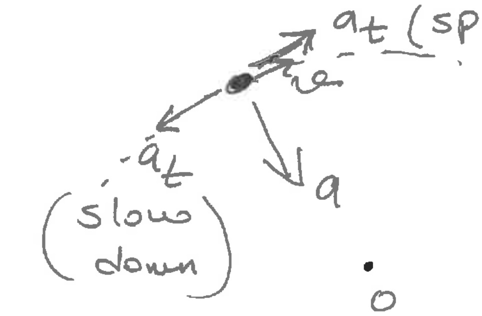
Nonuniform Circular Motion (8)

v NOT CONSTANT

direction changes

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_F - \vec{v}_i}{\Delta t}$$

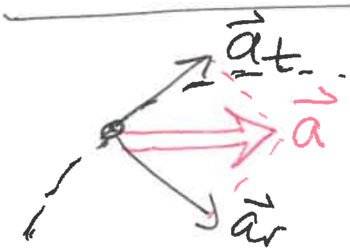
$$\vec{v}_F \neq \vec{v}_i$$

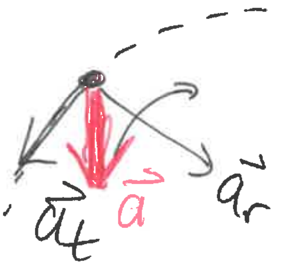


$$a_{radial} = a_{rad} = \frac{v^2}{R}$$

$$a_{tangential} = a_{tan} = \left| \frac{d\vec{v}}{dt} \right|$$

$$\boxed{\vec{a} = \vec{a}_{rad} + \vec{a}_{tan}}$$

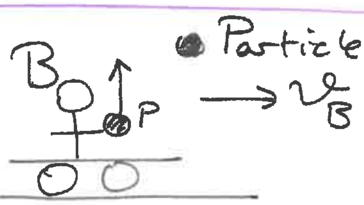




if $v = $ const $a_t = 0$!

Motion in 2d

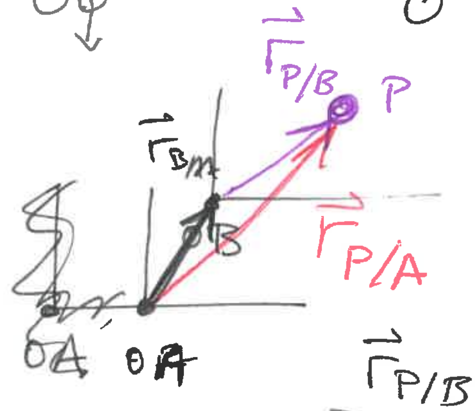
Relative motion



B, A describe Particle motion differently

what B sees

what A sees



$\vec{r}_{P/B}$: position of P wrt B

$\vec{r}_{P/A}$: " " P " A

$\vec{r}_{B/A}$: " " B " A.

$$\boxed{\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A}}$$

d/dt

$$\frac{d\vec{r}_{P/A}}{dt} = \frac{d\vec{r}_{P/B}}{dt} + \frac{d\vec{r}_{B/A}}{dt}$$

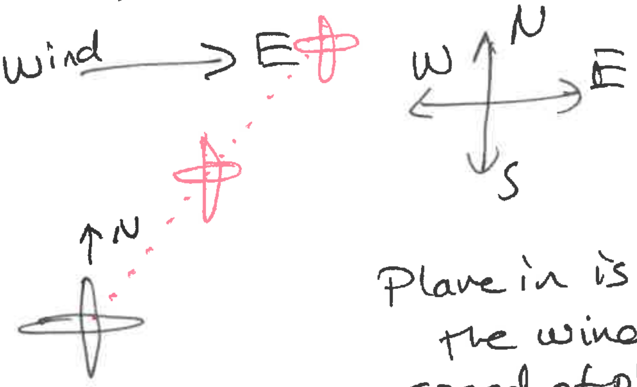
$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

$$\frac{P}{A} = \frac{P}{\cancel{B}} \cdot \frac{\cancel{B}}{A}$$

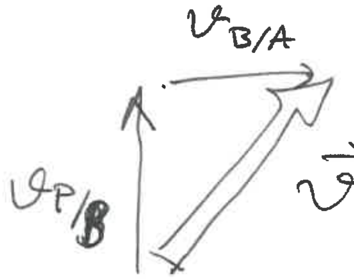
ex: plane's compass shows North direction its speed: 240 km/h. if there is a wind from west to East with a speed of 100 km/h what is the velocity of plane wrt earth?

Earth ⇒ A
Wind ⇒ B
Plane ⇒ P

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$



Plane in is the wind
speed of plane wrt wind



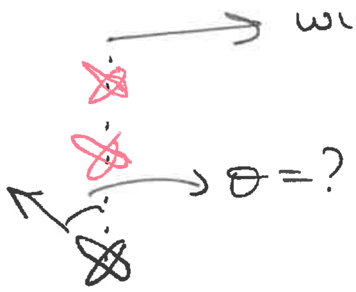
$$\vec{v}_{P/A} = 240\hat{j} + 100\hat{i}$$

$$|\vec{v}_{P/A}| = \sqrt{240^2 + 100^2}$$

$$= 260 \text{ km/h}$$

$$\theta = 23^\circ$$

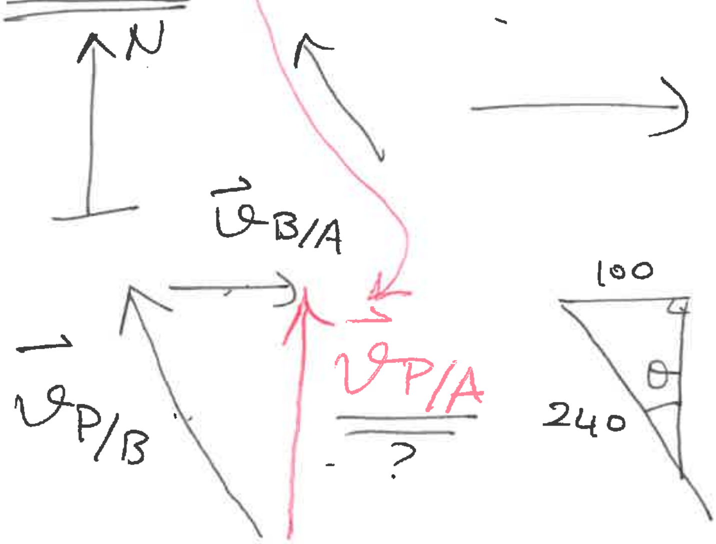
(4) ex: plane speed 240 km/hr
wind speed 100 km/hr
Wind W → E
what would be the direction of the plane so that it flies towards North wrt to earth



A ≡ earth
B = wind
P = plane

$$\boxed{\vec{v}_{P/A}} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

plane velocity wrt earth; plane vel. wrt wind; wind vel. wrt. earth

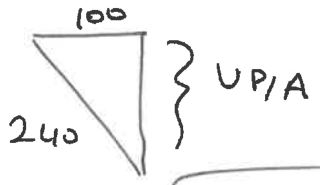


$$\sin\theta = \frac{100}{240}$$

$$\theta = \sin\left(\frac{100}{240}\right)$$

$$\theta = 25^\circ$$

$V_{P/A}^2 = v_{P/B}^2 - v_{B/A}^2$ (10)



$= \sqrt{240^2 - 100^2}$

$\vec{v}_{P/A} = 218$ km/hr

$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$

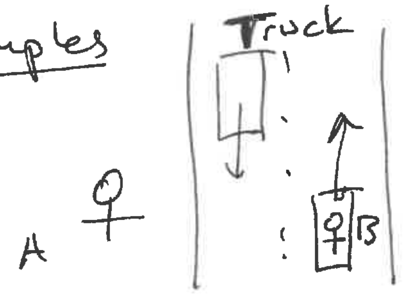
$A \equiv$ earth; not moving

$B \equiv$ moving coor. wrt earth

$P \equiv$ object

---

examples



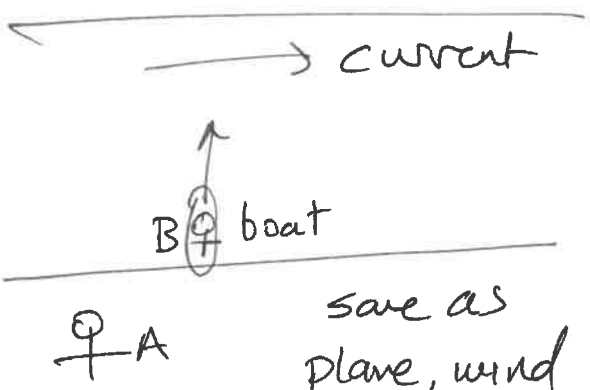
$v_{T/A}$; $v_{B/A}$; $\Rightarrow v_{T/B}$

$\vec{v}_{T/A} = \vec{v}_{T/B} + \vec{v}_{B/A}$

---

examples array on the river

boat

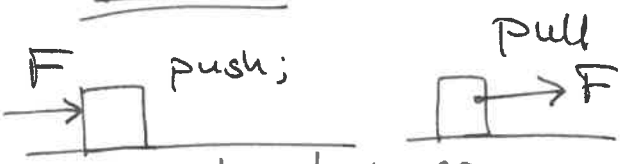


same as plane, wind problem.

13.7.17 PHYS I (1)

Chapter 4
Newton's Law of Motion

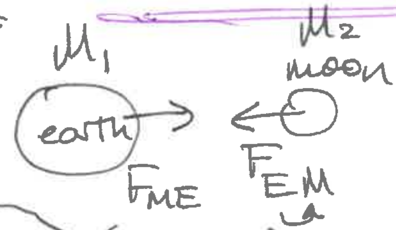
Force



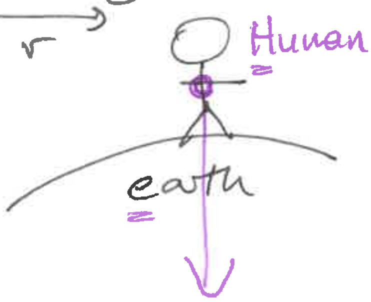
object moves
"contact"

4 types of force

1st Gravitational force



$$\boxed{F = G\frac{M_1 M_2}{r^2}}$$



const.

$$g = G\frac{M_E M_H}{R^2} = F_{EH} = mg$$

g → 9.8 m/s²

R ↳ radius of earth

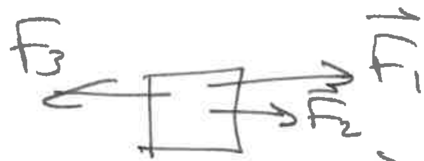
2nd Electro Magnetic Forces

$\vec{F}_{21}$ ⟵ $+q_1$ ⟵ r ⟶ $+q_2$ ⟶ $\vec{F}_{12}$

$$F = k\frac{q_1 q_2}{r^2}$$; force between magnets

3 ↳ weak
4 ↳ strong
Nuclear Forces

Force ⇒ Vector
↳ magnitude
↳ direction

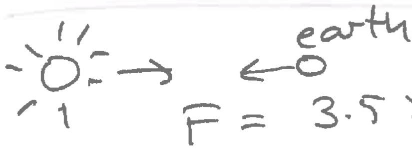


$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$|\Sigma\vec{F}| = |\vec{F}_1| + |\vec{F}_2| - |\vec{F}_3|$$

Unit of Force $F = [\text{Newton}]$
N

$F = mg$
$F = 80 \times 9.81$
$F = 784\ N$

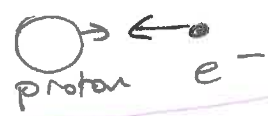


$F = 3.5 \times 10^{22}\ N$

blue whale weight = $10^6\ N$

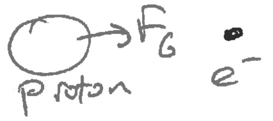
weight apple ⇒ 1N

H atom $F = 8 \times 10^{-8}\ N$



$\bullet\ e^-$ weight ⇒ $F = 10^{-30}\ N$

H atom

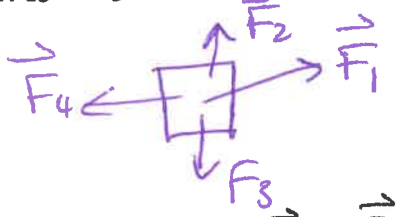


gravitational force
$F = 10^{-47}\ N$

Newton's Laws of Motion. (2)

1st Law

A body acted on by no Net Force moves with constant velocity (which maybe ZERO) and zero acceleration.



$$\text{if } \underbrace{\sum \vec{F}}_{\text{Net Force}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

Net Force = Total Force

$a = 0$; $v = \text{const}$

* $\sum \vec{F} = 0 \Rightarrow$ in equilibrium

$v = \text{const}; a = 0 \leftarrow$ (dengede)

INERTIA ≡ Eylemsizlik

the tendency of keep its state is inertia.

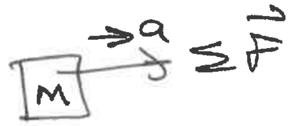
(durumunu koruma eğilimine eylemsizlik denir)

2nd Law $\sum \vec{F} = m\vec{a}$

if a net force acts on a body; the body accerelates. The direction of accelation is the save as net force mass of the body tries acceleration equals net force.

"concept mass"

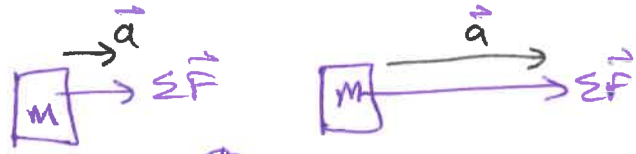
kütle nedir?



$$m = \frac{\sum \vec{F}}{\vec{a}}$$

measure

measure



mass ~~≠~~ weight

kütle ağırlık

⇓ ⇓

KG ≠ FORCE!

mass ≠ weight

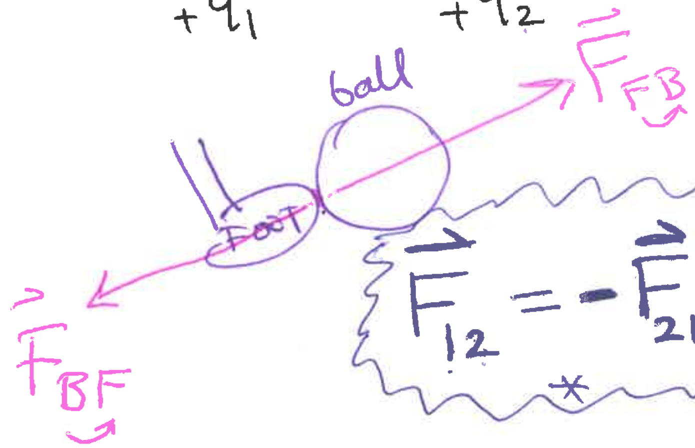
$1N = mg$

weight $= mg$

$m = 0.1$ kg

3rd LAW

if A exerts a force $\vec{F}$ on B (action) (etki) then B exerts a force $\vec{F}$ on A (reaction) (tepki)



$$\vec{F}_{12} = -\vec{F}_{21}$$ *

Phys I : 14.7.17 ①

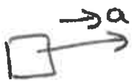
Newton's Law:

1st law: inertia ; tendency to keep its state (durum)

(eglim) (eylemsizlik)

$\sum \vec{F} = 0$

2nd Law: $\sum \vec{F} = m\vec{a}$



$\sum \vec{F} = m\vec{a}$

$m = \dfrac{\sum \vec{F}}{\vec{a}}$ ✓ ✓

↓ mass

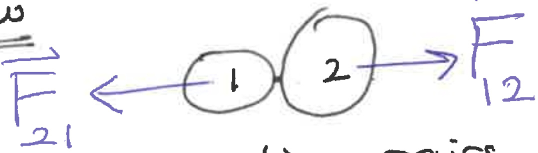
♀ 80kg — mass

weight $80 \times 9.8\,N$

mass ≠ weight

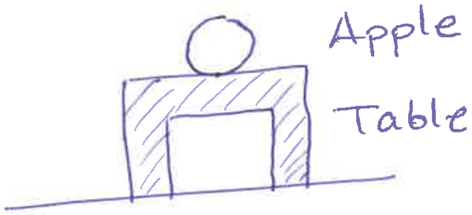
(kütle ≠ ağırlık)

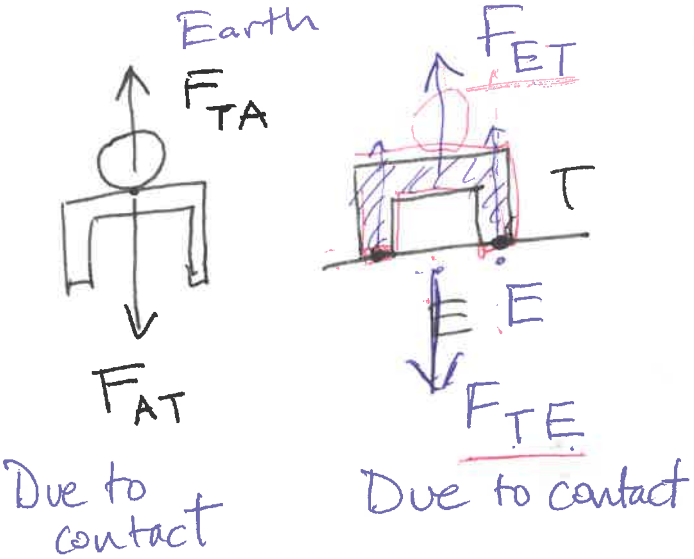
3rd Law



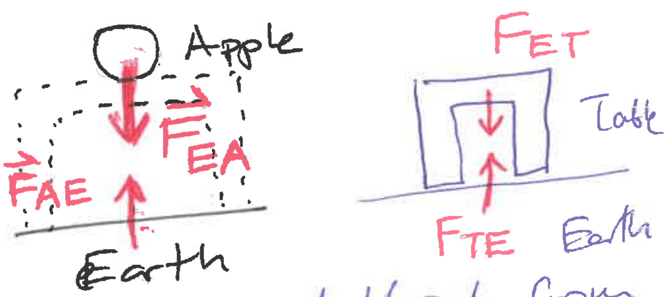
action-reaction pairs

$\vec{F}_{12} = -\vec{F}_{21}$





$F_G \Rightarrow$ gravitational force



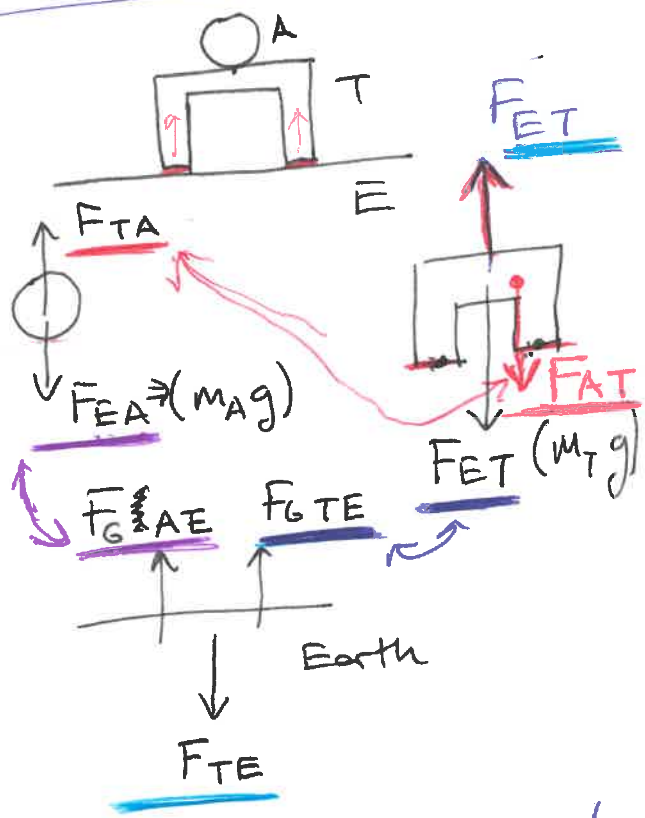
these forces are different from contact forces.

$\vec{F}_{EA} = m_A g(-\hat{j})$

$\vec{F}_{AE} = m_A g(\hat{j})$

$\vec{F}_{ET} = m_T g(-\hat{j})$

$\vec{F}_{TE} = m_T g(\hat{j})$



action-reaction pairs act on Different objects;

never on the SAME object

Free Body Diagram

FBD ; shows all the (2)
forces on the object
(Serbest Cisim Diyagramı)

$\Sigma \vec{F}$

show all the forces ; so that
the object "cut" out from
the scene.

Chapter 4 ends here
Chapter 5 ⇒ Application
of Newton's Laws

FBD

contact surfaces → ropes

Ropes always PULL
Surfaces " PUSH

Normal force (push)

F (pull)

surface

surface push force
"Normal Force"

object sits on surface

perpendicular to the surface

FBD

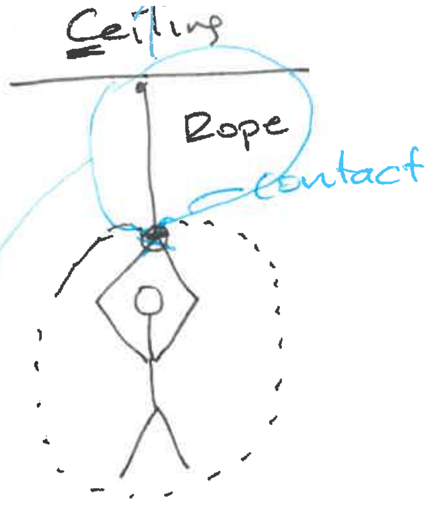
N ; F_{EO}

$F_G(EO)$

N

mg

FBD



FBD of the person

$T(F_{RP})$

P

$mg \cdot (F_{EP})$

FBD of the Rope

F_{CR}

Rope

$T(F_{PR})$

if the person is in equilibrium

y

T

mg

$a = 0$

$\Sigma \vec{F} = m\vec{a} = 0$

$T - mg = 0 \quad T = mg$

F_{CR}

$F_{PR}(T)$

$a = 0 \quad \Sigma \vec{F} = m_R a = 0$

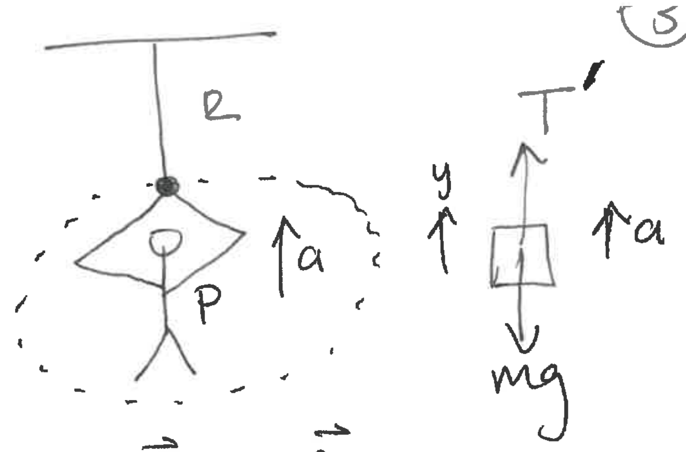
$F_{CR} - F_{PR} = 0$

$F_{CR} - T = 0$

$F_{CR} = T = mg$

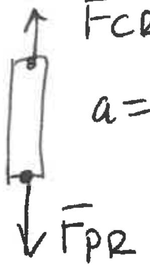
what happens if
the person pulls
himself up ?

(5)



$\sum \vec{F} = m\vec{a}$

$T' - mg = ma$



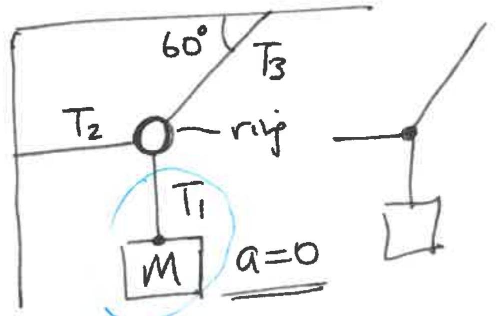
$\sum \vec{F} = m\vec{a}$

$F_{cR} - T' = 0$

$F_{cR} = T'$

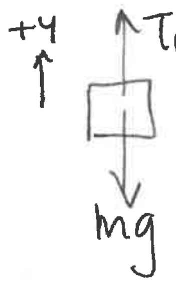
$F_{pR} \equiv T'$

Ex:



$T_1 = ?$ $T_2 = ?$ $T_3 = ?$

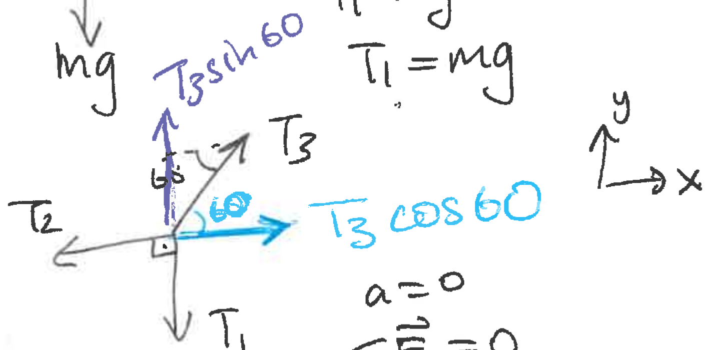
Tension of the ropes



$a=0$ $\sum \vec{F} = m\vec{a}$

$T_1 - mg = 0$

$T_1 = mg$



$a=0$

$\sum \vec{F} = 0$

$\hat{y}$ direction

$\sum F_y = 0$

$T_3 \sin 60 - T_1 = 0$

$\hat{x}$ direction

$\sum F_x = 0$

$T_3 \cos 60 - T_2 = 0$

$T_3 \sin 60 - T_1 = 0$

$T_1 = mg$

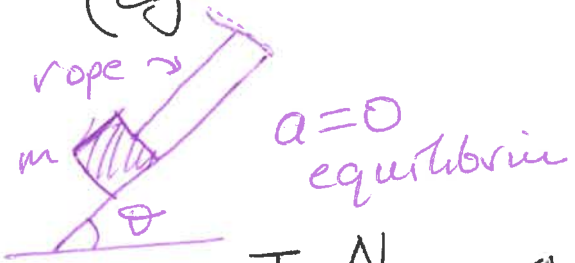
$T_3 = \frac{mg}{\sin 60}$

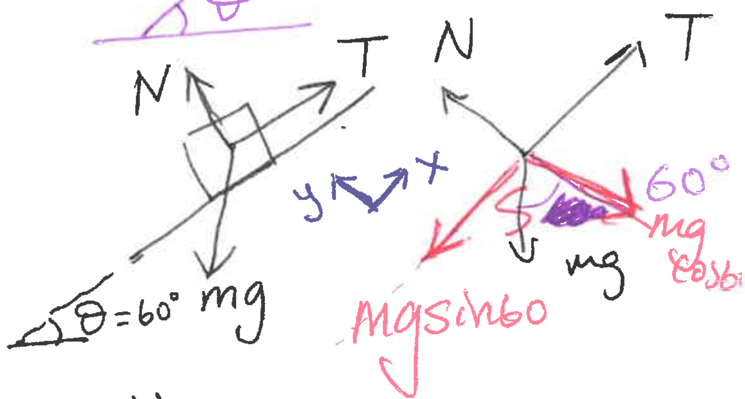
$T_3 \cos 60 - T_2 = 0$

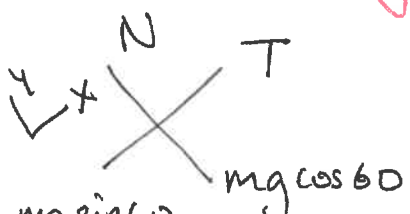
$T_2 = \frac{mg}{\sin 60} \cos 60$

$= \cancel{mg}$ $T_2 = \frac{mg}{\tan 60}$

Inclined plane (eğik duzlem)







$\sum \vec{F} = m\vec{a}$

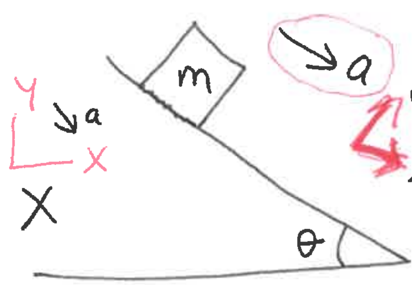
$a=0$

$\sum F_x = T - mg \sin 60 = 0$

$T = mg \sin 60°$

$\sum F_y = N - mg \cos 60 = 0$

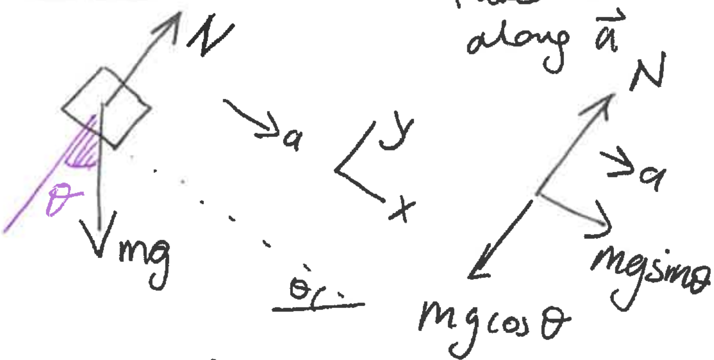
$N = mg \cos 60$



no friction

$a = ?$

take coordinate along $\vec{a}$



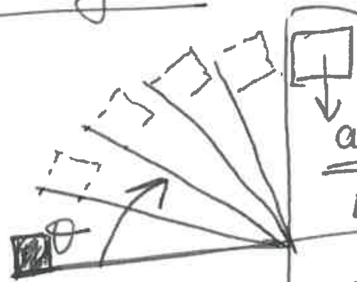
$\sum \vec{F} = m\vec{a}$

$\hat{y}$ $\quad a_y = 0$ $\qquad \hat{x}$ $\quad a_x = a$

$\sum F_y = m a_y$ $\qquad \sum F_x = m a_x$

$N - mg\cos\theta = 0$ $\qquad mg\sin\theta = ma$

$N = mg\cos\theta$ $\qquad a = g\sin\theta$



$\theta = 90°$

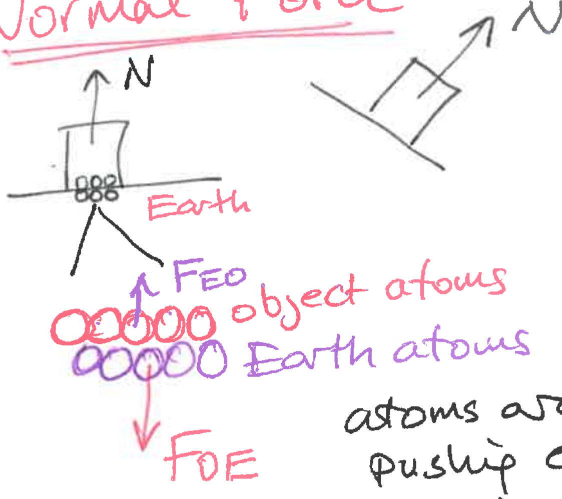
$a = g\sin 90 = g$

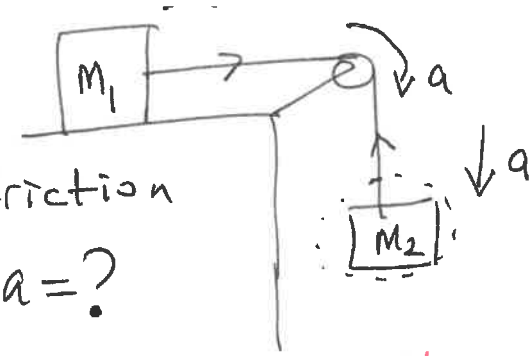
$N = mg\cos 90 = 0$

$\theta = 0 \Rightarrow a = g\sin\theta = 0$

flat surface $N = mg\cos 0 = mg$

Normal Force

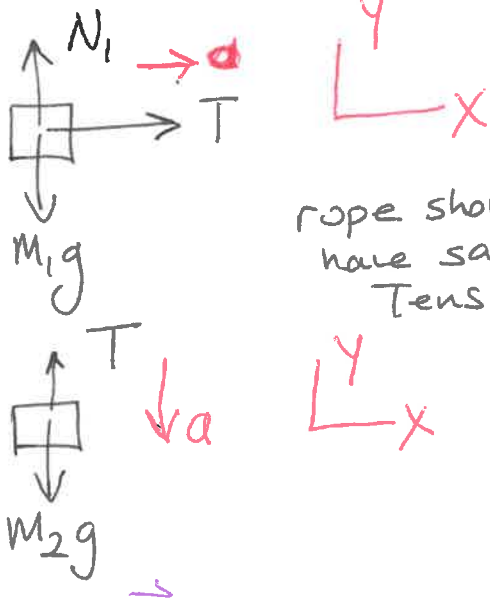




No friction

$a = ?$

FBD



rope should have same Tension

$\sum \vec{F} = m\vec{a}$

$\sum F_x = m_1 a_x$ $\qquad \sum F_y = m_1 a_y = 0$

$T = m_1 a$ ① $\qquad N_1 - m_1 g = 0$

$N_1 = m_1 g$

$\sum F_y = m_2 a_y$

$T - m_2 g = m_2(-a)$ ②

$m_2 g - T = m_2 a$ ②

$m_2 g - m_1 a = m_2 a$

$m_2 g = (m_1 + m_2) a$

$$a = \frac{m_2 g}{(m_1 + m_2)}$$

$$T = m_1 a = \frac{m_1 m_2 g}{(m_1 + m_2)}$$

How do you "feel" your weight? (5)

When do you "feel" weightless?

$N \equiv$ feel weight

$N \equiv 0$

weightless

jumping still in contact with surface

astronaut

F_G

$N=0$ weightless

earth

JUMPING

N' $\uparrow a$

$\Sigma F_y = ma_y$

$N' - mg = ma$

$N' > mg$

"feel heavier"

F_f mg F N

$F_f = \mu N$ → coefficient

μ

μ_s (static) when object STILL not MOVING

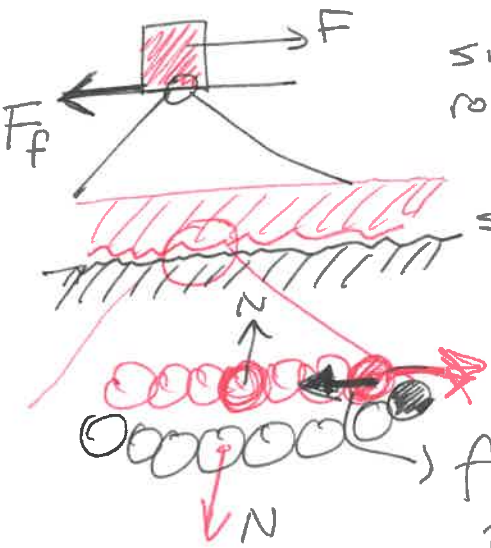
μ_k (kinetic) object moves

materials	μ_s >	μ_k
steel on steel	0.74	0.57
Al on Steel	0.61	0.47
Cu on Steel	0.53	0.36
Glass on Glass	0.94	0.40
Cu on Glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on Steel	0.04	0.04
Human joints eklemler	0.004	

	μ_s	μ_k
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet) rain	0.3	0.25

always be cautious when driving in rain!!

FRICTION FORCES

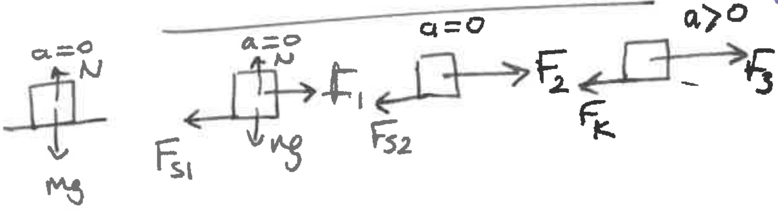


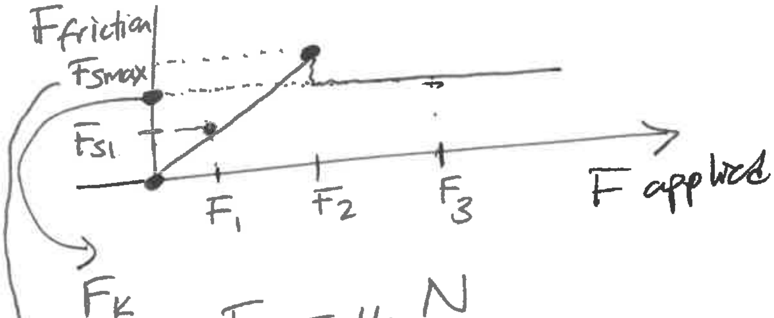
smooth → düz
rough → engebeli

surface rough (engebelilik)

frictional forces

Static vs Kinetic Friction (6)





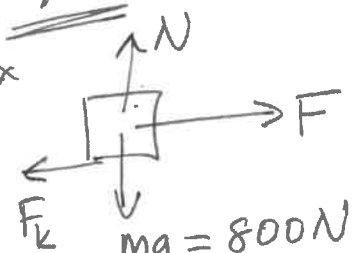
$F_k = \mu_k N$

$F_{s\,max} = \mu_s N$

static friction max value is $\mu_s N$

static friction is less than $\mu_s N$; when object is NOT MOVING

Ex



200 N (moving)

F (230 N) (stationary)

$mg = 800N$

$F_{s\,max} = 230N$

$F_k = 200\,N$

$\mu_s = ?\ \mu_k = ?$

$F_{s\,max} = \mu_s N = \mu_s\, 800$

$\mu_s = \dfrac{230}{800} = 0.29$

$F_k = \mu_k N \Rightarrow \mu_k = \dfrac{200}{800} = 0.25$

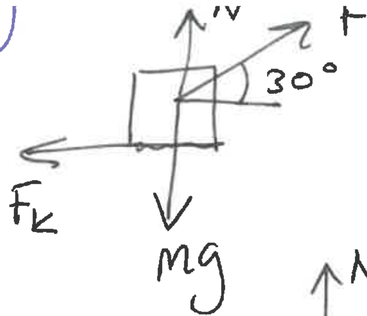
Ex:

$\mu_k = 0.4$

$mg = 500N$

F = ?

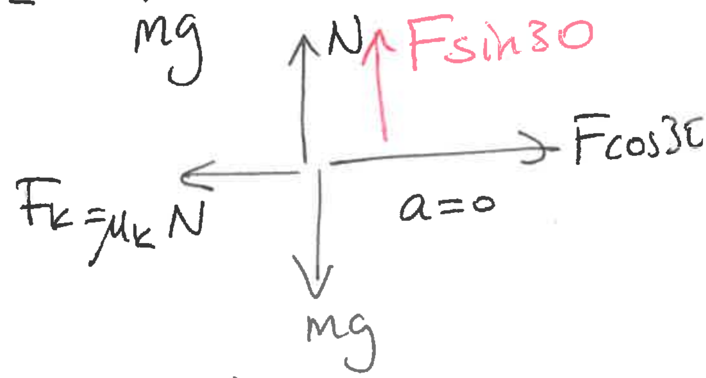
if it's moving const. velocity



a=0

v const.

$\mu_k = 0.4$



$\Sigma \vec{F} = m\vec{a}$ $\quad a=0$

$\hat{x}$: $F\cos\theta - \mu N = 0$

$\hat{y}$: $N + F\sin\theta - mg = 0$

$\cancel{F\ \frac{\mu N}{\cos\theta}}$

$N = \dfrac{F\cos\theta}{\mu}$

$\dfrac{F\cos\theta}{\mu} + F\sin\theta - mg = 0$

$F\left(\dfrac{\cos\theta}{\mu} + \sin\theta\right) = mg$

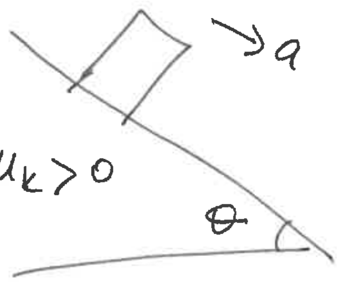
$F = \dfrac{mg}{\left(\dfrac{\cos\theta}{\mu} + \sin\theta\right)} = \dfrac{500}{\left(\dfrac{\sqrt{3}/2}{0.4} + \dfrac{1}{2}\right)}$

$F = 188N$

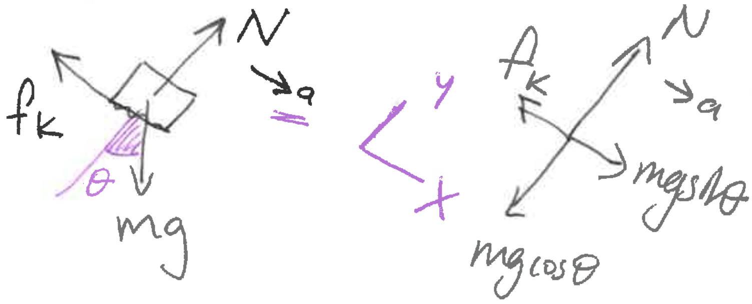
$N = \dfrac{F\cos\theta}{\mu} = 406\,N$

7

Friction forces on an inclined plane



$a = ?$



$\Sigma F_x = ma_x$; $\Sigma F_y = ma_y$

$mg\sin\theta - f_k = ma$; $N - mg\cos\theta = 0$

$N = mg\cos\theta$

$f_k = \mu_k N$

$f_k = \mu_k mg\cos\theta$

$mg\sin\theta - \mu_k mg\cos\theta = ma$

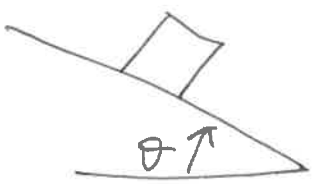
$a = g\sin\theta - \mu_k g\cos\theta$

rember when $\mu_k = 0$

$a = g\sin\theta$ ✓

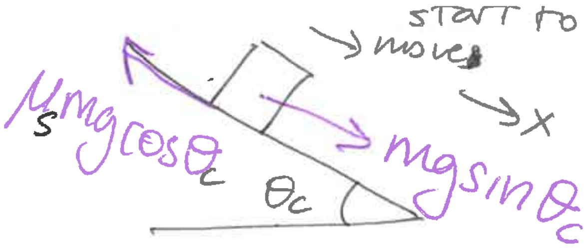
** measure μ_s easily ~~with~~ if you know the critic angle of the inclined plane.

have expt



θ from 0 increase it when object moves

stop: critic angle $\underline{\theta}$



$\Sigma F_x = m\cancel{a}^{\,0}$

$\cancel{mg}\sin\theta_c = \mu_s \cancel{mg}\cos\theta_c$

$\mu = \tan\theta_c$

max μ value ; μ_s (static)

Air resistance

(hava sürtünmesi)

friction ~~in~~ air

depends on the speed of the object

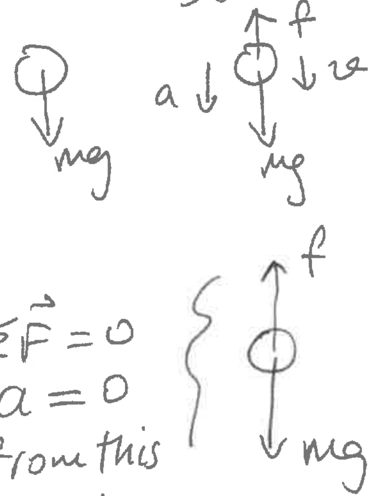
$f = (..)v^2$

jump: from a plane

$f = D\,v^2$ (D const)

$v = 0$

$f = 0$



as you drop

a ; $v\uparrow$

$f\uparrow$

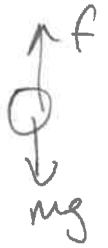
$\Sigma\vec{F} = 0$

$a = 0$

from this point

you don't speed up more

$v = \text{const} =$ terminal (velocity) speed at terminal speed



$mg - f = 0$

$mg = f = Dv^2$

$v_{\text{terminal}} = \sqrt{\dfrac{mg}{D}}$

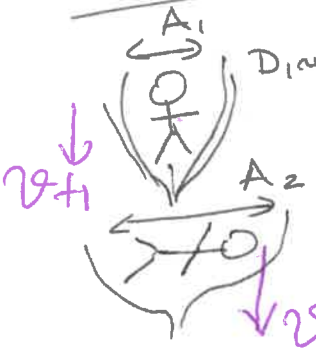
Ex: if $D = 0.25\ kg/m$

$m_p = 50kg$ $V_{\text{terminal}} = ?$

$v_{\text{term}} = \sqrt{\dfrac{50 \times 9.8}{0.25}} = 44\ m/s$

$V_{\text{term}} = 44\ \dfrac{m}{s} = \dfrac{44 \times 10^{-3}\ km}{\frac{1}{3600}\ hr}$

$V_{\text{term}} = 160\ km/hr$



$f = Dv^2$

$D_1 \sim A_1$

$D \sim \underline{Area}$

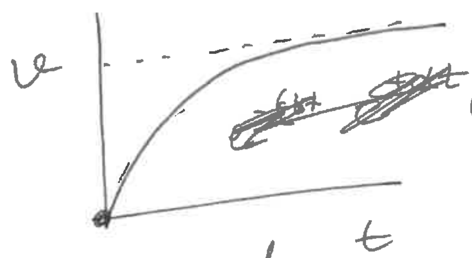
$D_2 \sim A_2$

$A_2 > A_1$

$D_2 > D_1$

$v_{t_2} < v_{t1}$

$v_t = \sqrt{\dfrac{mg}{D}}$



$(1 - e^{-\alpha t})$

~~for~~ higher speeds

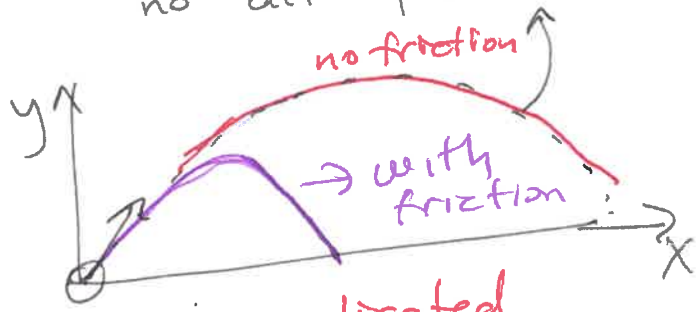
$f = Dv^2$

in liquids $f = kv$

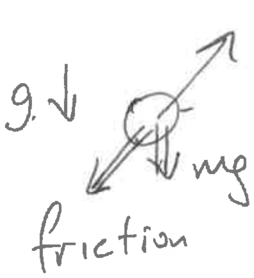
Projectile motion

we assumed

no air friction



more complicated with friction



$v(\theta)$

θ changes

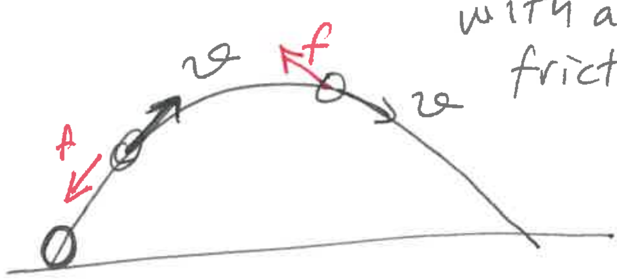
v changes

friction force direction and magnitude changes

a_x →

a_y ↓ ↓ g

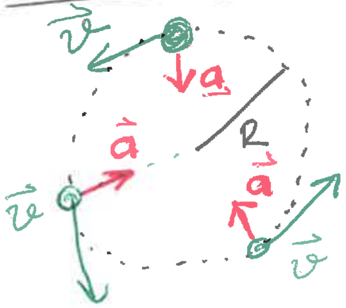
a_y ; a_x are changing all the time; it is a bit harder to solve.



with air friction.

Dynamics of Circular motion

1



$v = \text{const.}$

in uniform circular motion the acceleration is TOWARDS the center.

2nd Law: $\sum \vec{F} = m\vec{a}$

same direction

$\sum \vec{F}$ should be TOWARDS center

previously: $\vec{a}_{rad} = \frac{v^2}{R}(-\hat{r})$

radial

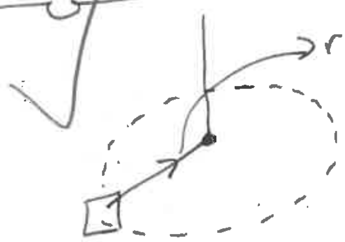
T: Period; time it takes a full revolution (circle.)

$v = \frac{2\pi R}{T}$

$$\boxed{a_r = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}}$$

$$\sum \vec{F} = m\vec{a} = m\frac{v^2}{R}(-\hat{r})$$

Ex:



rope

frictionLESS

uniform circ. motion

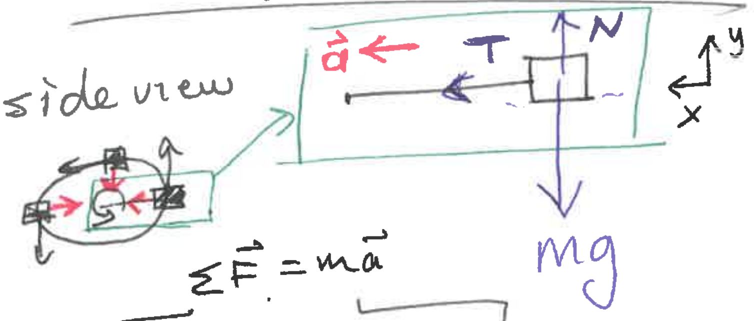
$m = 25\,kg$; length of rope $= 5\,m$

$F = ?$; draw FBD

5 revolutions in a minute.

Phys I 18.7.17

side view



$\sum \vec{F} = m\vec{a}$

$\sum F_x = ma_x$

$T = ma_r$

$a_r = v^2/R$ ✓

$\sum F_y = \cancel{ma_y}$ 0

$N - mg = 0$

$N = mg$

$T = m\frac{v^2}{R}$

$R = 5\,m$

5 revolution 1 minute

$v = \frac{\text{displacement}}{\text{time}} = \frac{5(2\pi R)}{60\,s}$

$= \frac{50 \times 3.14\,m}{60\,s} = 2.62\,m/s$

$T = m\frac{v^2}{R} = 25\frac{(2.62)^2}{5} = 34.$

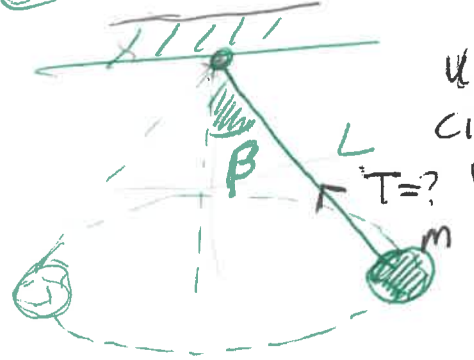
~~1.37~~ N

$a = \frac{v^2}{R}$

$a = 1.37\,m/s^2$; $T = 34.3\,N$

radial acceleration ; tension of rope

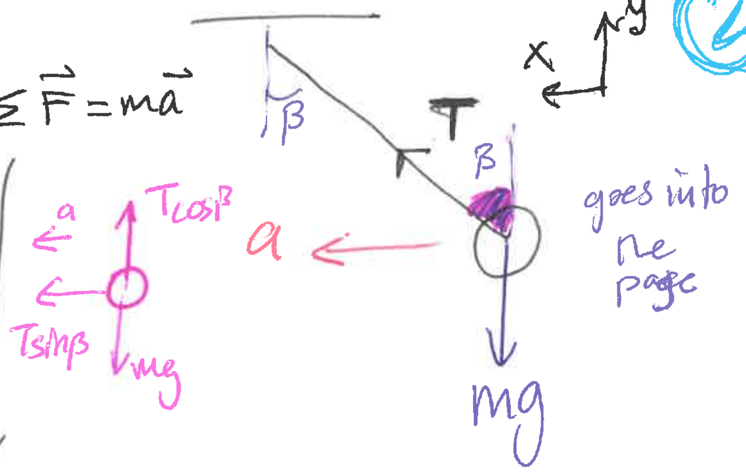
Ex: Conical Pendulum



uniform circ. motion

Tension in the wire?

FBD



$\sum \vec{F} = m\vec{a}$

$\hat{x}: \sum F_x = ma_x$

$T\sin\beta = m\dfrac{v^2}{R}$

$\hat{y}: \sum F_y = \cancel{ma_y} \to 0$

$T\cos\beta - mg = 0$

$T = \dfrac{mg}{\cos\beta}$

$\dfrac{mg}{\cos\beta}\sin\beta = m\dfrac{v^2}{R}$

$\cancel{m}g\tan\beta = \cancel{m}\dfrac{v^2}{R} = a$

$a = g\tan\beta$

tension in the wire?

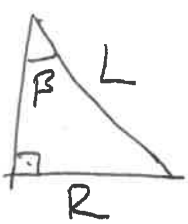
$T = mg/\cos\beta$

period of the motion?

$\dfrac{2\pi R}{T} = v \Rightarrow \dfrac{v^2}{R} = g\tan\beta$

$v = \sqrt{gR\tan\beta}$

$T = \dfrac{2\pi R}{v} = \dfrac{2\pi R}{\sqrt{gR\tan\beta}}$



$R = L\sin\beta$

$T = \dfrac{2\pi R}{\sqrt{R\tan\beta\, g}} = \dfrac{2\pi\sqrt{R}}{\sqrt{g\tan\beta}}$

$R = L\sin\beta$

$T = 2\pi\dfrac{\sqrt{L\cancel{\sin\beta}}}{\sqrt{g\dfrac{\cancel{\sin\beta}}{\cos\beta}}} = 2\pi\dfrac{\sqrt{L\cos\beta}}{\sqrt{g}}$

$\text{Force} = \dfrac{mg}{\cos\beta}$; $T = 2\pi\sqrt{\dfrac{L\cos\beta}{g}}$

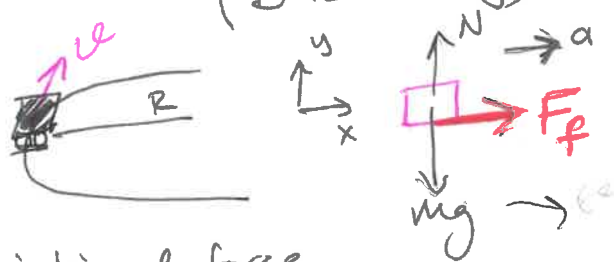
if $\beta\uparrow$; $\cos\beta\downarrow$

Force $\uparrow$

period $T\downarrow$; $v\uparrow$

Ex: Rounding a flat curve

(Düz Viraj)



Frictional force should be in $\vec{a}$ direction

$\sum \vec{F} = m\vec{a}$

$\sum F_y = \cancel{ma_y} \to 0$

$N - mg = 0$

$N = mg$

$\sum F_x = ma_x$

$F_f = ma_r$

$F_f = m\,v^2/R$

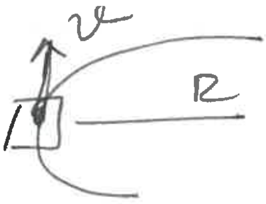
$F_f = \mu N = \mu mg$

$\mu\cancel{m}g = \cancel{m}v^2/R$

$v = \sqrt{\mu gR}$

3

flat curve



$v = \sqrt{\mu g R}$

dimensional (unit) analysis

$\left[\frac{m}{s}\right] = \sqrt{1\,\frac{m}{s^2}\,m}$ ✓

on a curve when you're driving

$\mu, g, R \Rightarrow$ const

$\sqrt{\mu g R} \Rightarrow$ fixed.

$v_{max} = \sqrt{\mu g R}$

$v > v_{max} \Rightarrow$ your car will skid

kayar!

$\mu N = F_f = m a_r$

$\mu = \mu_s$ = static frict. coefficient

If $\mu = 0.96$, $R = 230\,m$, $g = 9.8\,m/s^2$ } $V_{max} = \sqrt{\mu g R}$, $v_{max} = 47\,m/s$

$v_{max} = 47\,\frac{m}{s} = \dfrac{47 \times 10^{-3}\,km}{\frac{1}{3600}\,hr}$

$= 170\,km/hr$

Centripetal (merkezcil)

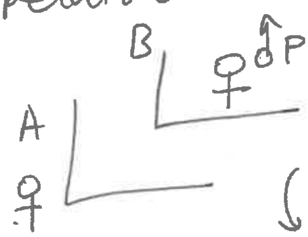
merkeze doğru ✓

Centrifugal (merkezkaç)

merkezden DIŞARI doğru

✗ YANLIŞ

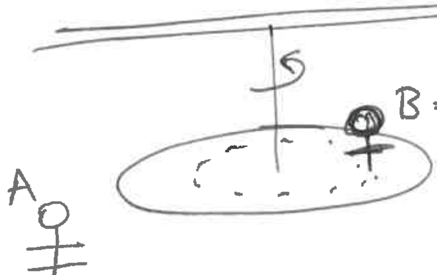
Relative motion



$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$

$\downarrow d/dt$

$\vec{a}_{P/A} = \vec{a}_{P/B} + \vec{a}_{B/A}$



ölçüm alındığı zaman ivmeli ortamda olmamamız lazım.

B ölçtüğü zaman yanlış olur.

çünkü ivmeli ortamda

(virajda B gibiyiz)

"Sanki dışarı doğru bir kuvvet hissediyoruz."

merkezkaç kuvveti

BU YANLIŞ!!

her zaman dışarıdan A'dan ölçmek lazım!!

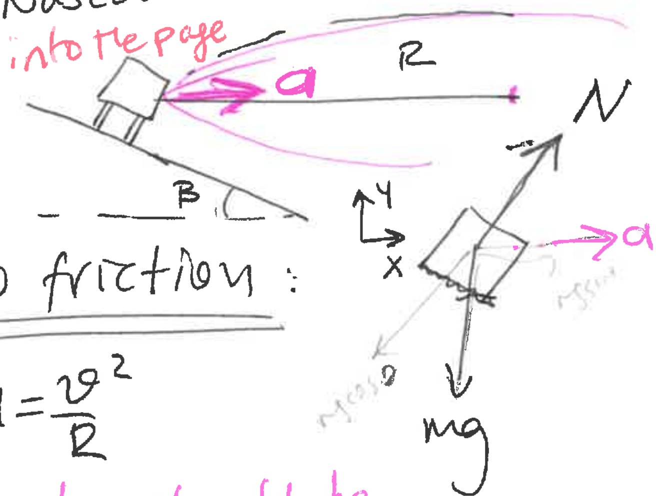
merkezcil doğru

4

Flat curve motion requires Friction

Ex: Rotation on a surface without friction
"Rounding on banked curve"

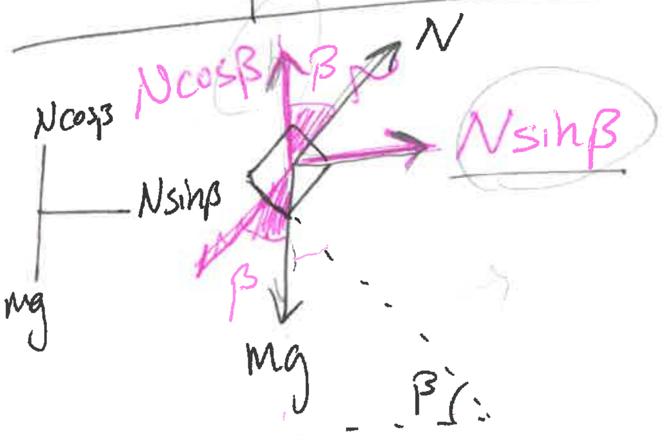
"Nascar races"



No friction:

$$a = \frac{v^2}{R}$$

coordinates should be chosen in $\vec{a}$ direction



$\Sigma F_y = m\cancel{a_y}^{0}$ $\quad \Sigma F_x = m a_x$

$N\cos\beta - mg = 0$, $\rightarrow N\sin\beta = m\frac{v^2}{R}$

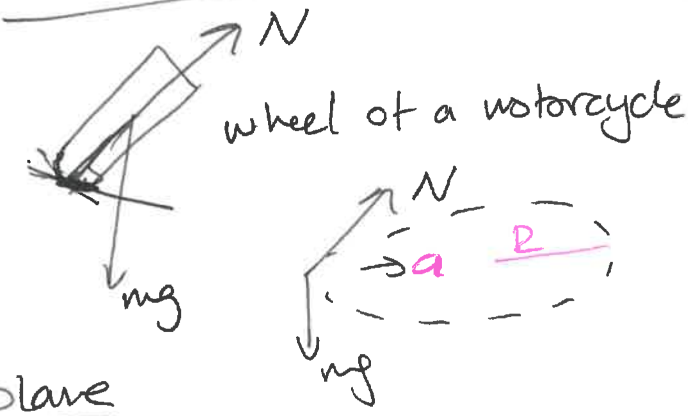
$N = \frac{mg}{\cos\beta}$ $\quad \cancel{m} g \tan\beta = \cancel{m}\frac{v^2}{R}$

$$v = \sqrt{gR\tan\beta}$$

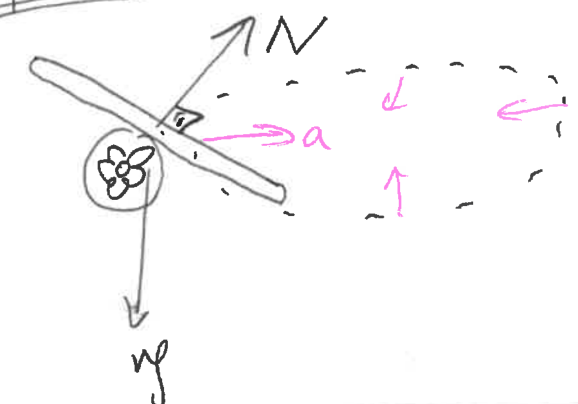
if $\beta = 15°$ $R = 230\,m$
$g = 9.8\,m/s$

$v = \sqrt{gR\tan\beta} = 25\,m/s$
max speed.

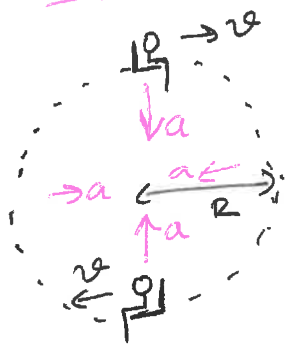
$v > 25\,m/s$ the car will skid
kayar!



plane



Ex: UCM in a vertical circle



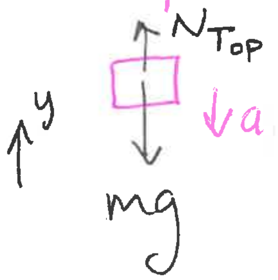
Ferris wheel
(dönme dolap)

$v = const$

Seat remains upright
Find force the seat exerts on the person at the TOP ?
at the BOTTOM?

FBD of person ⑤   End of Ch5

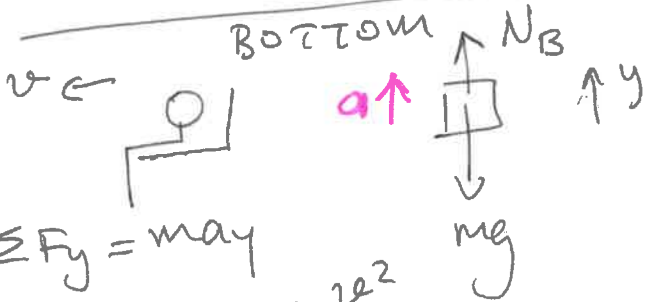
TOP



$\Sigma F_y = ma_y$

$N_T - mg = -m\frac{v^2}{R}$

$N_T = mg - m\frac{v^2}{R}$ **TOP**

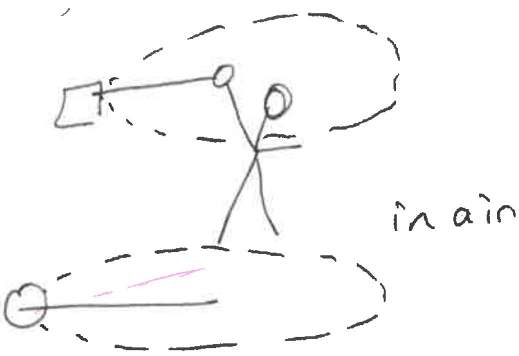


$\Sigma F_y = ma_y$

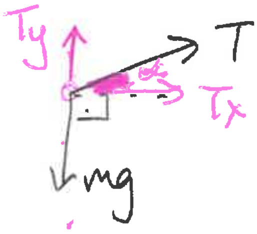
$N_B - mg = +m\frac{v^2}{R}$

$N_B = mg + m\frac{v^2}{R}$ Bottom

$N_B > N_T$ $N_B - N_T = 2m\frac{v^2}{R}$



can never be 90°



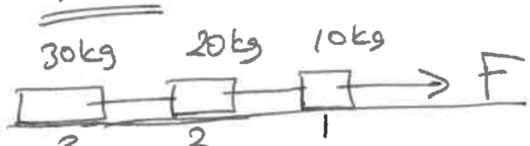
correct

- Drawing Free Body Diagram
  - * ropes PULL → T
  - * surfaces PUSH ⇒ N
  - * mg!
  - * friction forces $f = \mu N$
    - ↳ action reaction pairs between two bodies.
- Pick your coordinates, draw

- $\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$

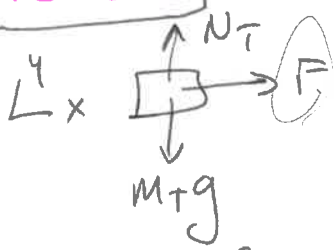
SOLVE!!

Ex: 5.14



no friction! $F = 132N$

a) $a = ?$ b) tension between 1 & 2 and 2 & 3



→a

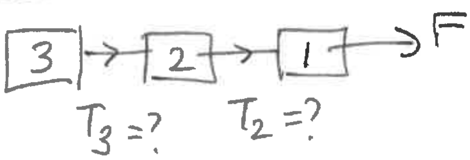
$\Sigma F_x = ma_x$

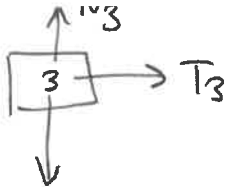
$132 = (10+20+30)a$

$a = 132/60 \ m/s^2$

$\Sigma F_y = ma_y^{\,0}$

$N_T - m_T g = 0$



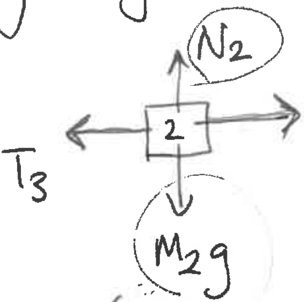


$\rightarrow a$

$\sum F_x = m_3 a_x$

$T_3 = m_3 a$

$T_3 = 30\left(\frac{132}{60}\right) N$ ✓



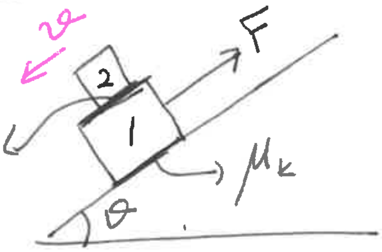
$\sum F_x = m_2 a_x$

$T_2 - T_3 = m_2 a$

$T_2 = 30\,\frac{132}{60} + 20\,\frac{132}{60}$ ✓

$T_2 = 50\left(\frac{132}{60}\right) N$

Ex: 5.33



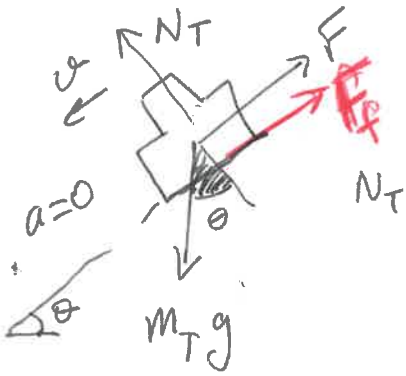
$\mu_s > 0$

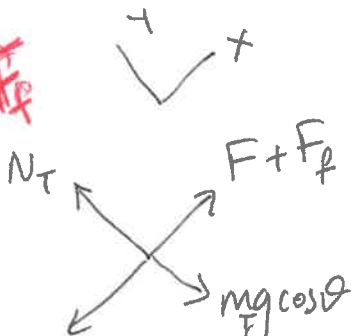
$\mu_k > 0$

Both 1 & 2 are going down at const. velocity; $a = 0$

$F = ?$ Draw FBD

1 and 2 are moving together.





$\sum F_y = m_T \cancel{a_y}^{0}$

$N_T - m_T g\cos\theta = 0$

$N_T = m_T g\cos\theta$

$\sum F_x = m_T \cancel{a_x}$

$F + F_f - m_T g\sin\theta = 0$

$F_f = \mu N_T$

(6) $F + F_f = m_T g\sin\theta$

$\mu N_T = \mu\, m_T g\cos\theta$

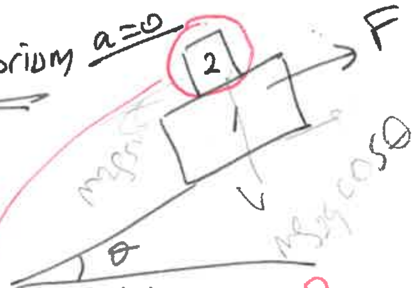
$F + \mu m_T g\cos\theta = m_T g\sin\theta$

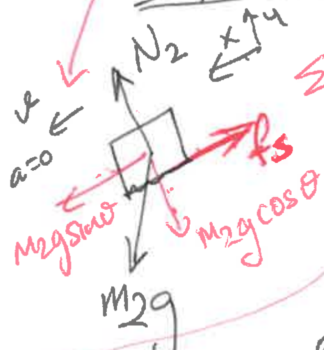
$$\boxed{F = m_T g(\sin\theta - \mu\cos\theta)}$$

FBD of m_1 ?

FBD of m_2

equilibrium $a=0$





$\sum F_x = 0$

$\sum F_x = m \cancel{a_x}^{0}$

$m_2 g\sin\theta - f_s = 0$

$f_s = m_2 g\sin\theta$

μN_2

$\sum F_y = m_2 \cancel{a_y}^{0}$

$N_2 - m_2 g\cos\theta = 0$

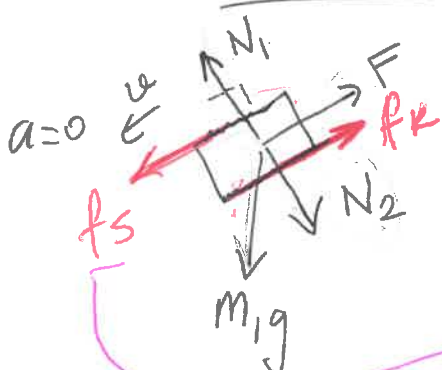
$N_2 = m_2 g\cos\theta$

$\mu N_2 = m_2 g \sin\theta$

$\mu \cancel{m_2 g}\cos\theta = \cancel{m_2 g}\sin\theta$

$$\boxed{\mu = \tan\theta}$$

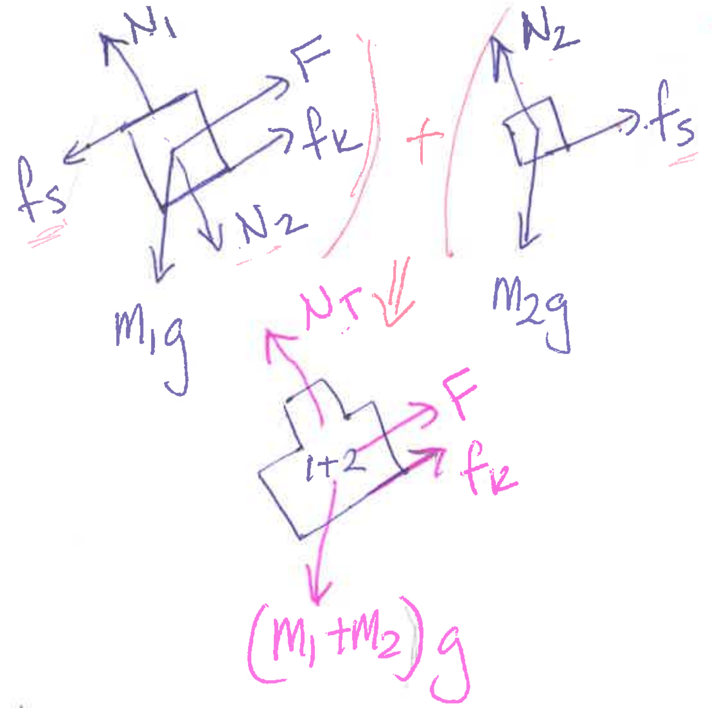
FBD 1



FBD 2

action reaction

(7) 1st object & 2nd object



① $T - m_1 g\sin\alpha - f_{k1} = m_1 a$

② $m_2 g\sin\beta - T - f_{k2} = m_2 a$

① + ②

$m_2 g\sin\beta - m_1 g\sin\alpha - f_{k1} - f_{k2} = (m_1+m_2)a$

$f_{k1} = \mu m_1 g \cos\alpha$

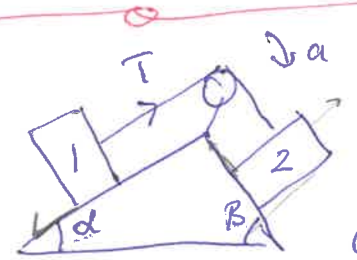
$f_{k2} = \mu m_2 g \cos\beta$

$$a = \frac{m_2 g\sin\beta - m_1 g\sin\alpha - f_{k1} - f_{k2}}{m_1+m_2}$$

find a

1st equ: $T = m_1 a + f_{k1} + m_1 g\sin\alpha$

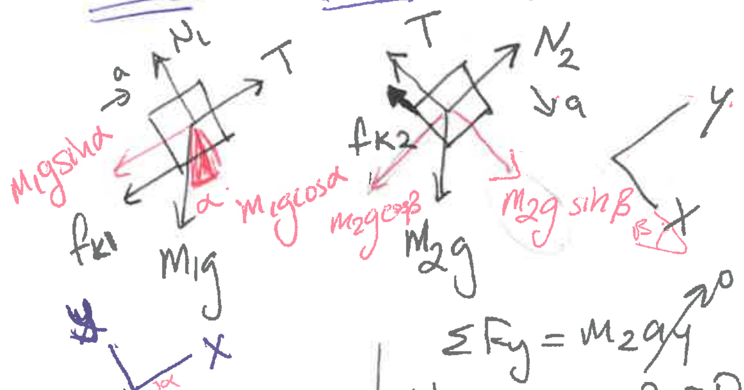
T-90



$\mu > 0$

$a = ?$

- FBD $a = ?$ $T = ?$



$\Sigma F_x = m a_x$

$T - m_1 g\sin\alpha - f_{k1} = m_1 a$

$\Sigma F_y = m_1 a_y^{\nearrow 0}$

$N_1 - m_1 g\cos\alpha = 0$

$N_1 = m_1 g\cos\alpha$

$f_{k1} = \mu N_1$

$f_{k1} = \mu m_1 g\cos\alpha$

$\Sigma F_y = m_2 a_y^{\nearrow 0}$

$N_2 - m_2 g\cos\beta = 0$

$N_2 = m_2 g\cos\beta$

$\Sigma F_x = m_2 a_x$

$m_2 g\sin\beta - T - f_{k2} = m_2 a$

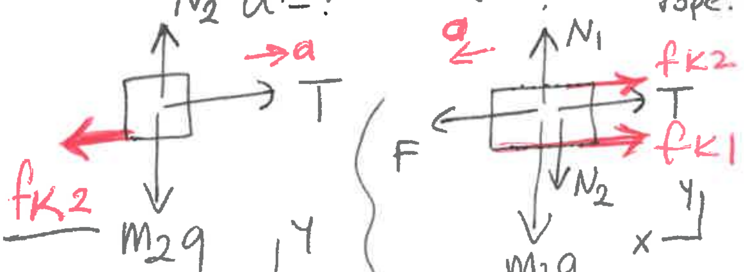
$f_{k2} = \mu N_2$

$f_{k2} = \mu m_2 g\cos\beta$

5-87 $\mu > 0$

FBD of ~~the~~ 1 & 2.:

N_2 $a = ?$ $T = ?$ in the rope.



$\Sigma F_y = m_2 a_y^{\nearrow 0}$

$N_2 - m_2 g = 0$ $N_2 = m_2 g$

$T - f_{k2} = m_2 a$

$T - \mu N_2 = m_2 a$

$\overset{?}{T} - \mu m_2 g = m_2 \overset{?}{a}$

$\Sigma F_y = m_2 a_y^{\nearrow 0}$

$N_1 - N_2 - m_1 g = 0$

$N_1 = m_1 g + m_2 g$

$f_{k2} = \mu N_2 = \mu m_2 g$

$f_{k1} = \mu N_1 = \mu(m_1+m_2)g$

(1) $T - \mu m_2 g = m_2 a$ (8)

(2) $\Sigma F_x = m_1 a$

(2) $F - T - f_{k1} - f_{k2} = m_1 a$

$f_{k1} = \mu (m_1 + m_2) g$

$f_{k2} = \mu m_2 g$

(1) + (2)

$$F - 2\mu m_2 g - \mu(m_1+m_2)g = (m_1+m_2)g$$

$$a = \frac{F - 2\mu m_2 g - \mu(m_1+m_2)g}{m_1+m_2}$$

(1) $T = m_2 a + \mu m_2 g$

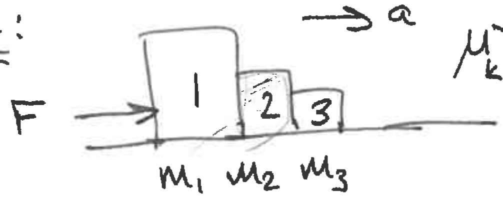
$\checkmark$

Phys I

20.7.17

① ①

Ex:



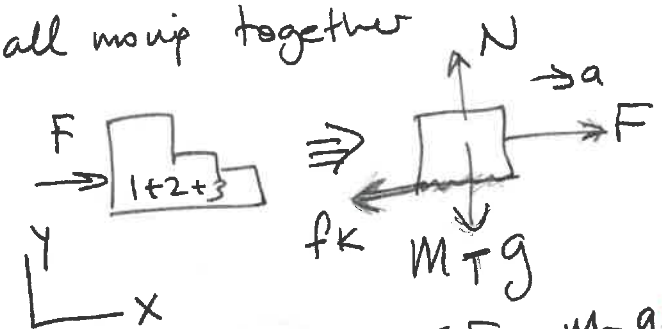
$\mu_k > 0$

→ FBD of each mass

→ $a = ?$ → Force between 1 & 2 on 2

→ Force between 2 & 3 on 3

all moving together



$\Sigma F_x = M_T a_x$

$\Sigma F_y = M_T \cancel{a_y}^{0}$

$F - f_k = M_T a$

$N - M_T g = 0$

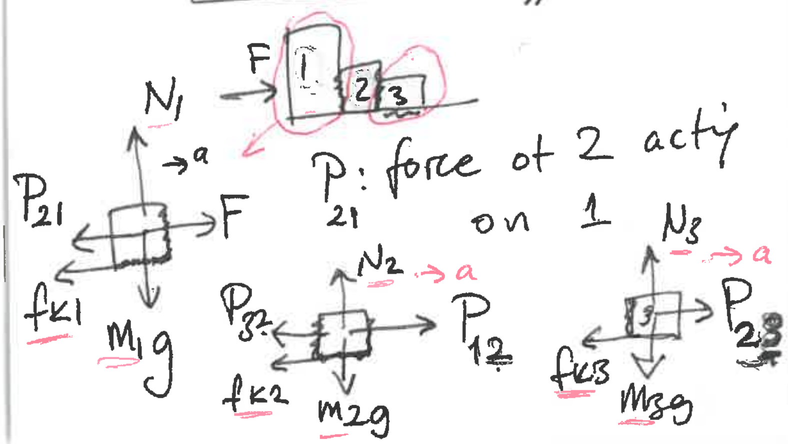
$N = M_T g$

$\mu N = \mu M_T g$

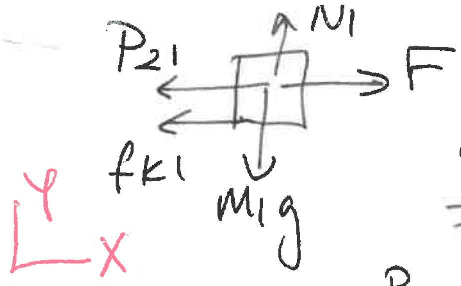
$F - \mu M_T g = M_T a$

$$a = \frac{F - \mu M_T g}{M_T}$$

$$\boxed{a = \frac{F}{M_T} - \mu g}$$



P_{21}: force of 2 acting on 1



→ a

$$a = \frac{F}{m_T} - \mu g$$

$P_{21} = ?$

$\Sigma F_y = m_1 \cancel{a_y}^{0}$

$N_1 - m_1 g = 0$ $\quad N_1 = m_1 g$

$f_{k1} = \mu N_1 = \mu m_1 g$

$\Sigma F_x = m_1 a_x$

$F - P_{21} - f_{k1} = m_1 a$

$P_{21} = F - f_{k1} - m_1 a$

$$P_{21} = F - \mu m_1 g - m_1\left(\frac{F}{M_T} - \mu g\right)$$

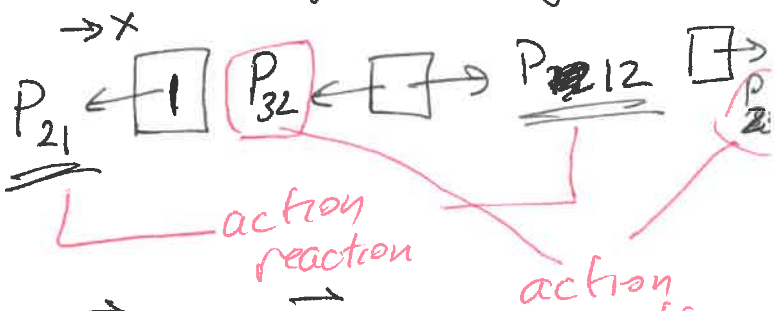
$M_T = M_1 + M_2 + M_3$

$$P_{21} = F - \cancel{\mu m_1 g} - \frac{m_1}{m_T} F + \cancel{m_1 \mu g}$$

$$= F\left(1 - \frac{M_1}{M_T}\right) = F\left(\frac{m_T - m_1}{m_T}\right)$$

$$P_{21} = \frac{M_2 + M_3}{M_1 + M_2 + M_3} F$$

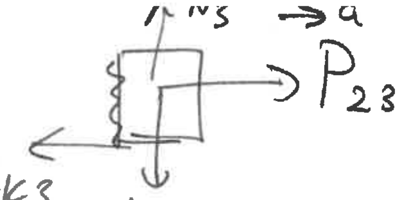
Force acting on 1 by 2nd obj



$$\vec{P}_{21} = -\vec{P}_{12}$$

$$\vec{P}_{21} = \frac{m_2 + m_3}{m_1 + m_2 + m_3} F(-\hat{\imath})$$

$P_{23} = ?$



2

$\sum F_y = m_3 \cancel{a_y}^{\,0}$

$\sum F_x = m_3 a_x$

$N_3 - m_3 g = 0$

$N_3 = m_3 g$

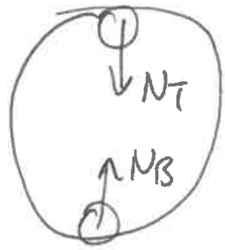
$f_{k3} = \mu N_3 = \mu m_3 g$

$P_{23} - f_{k3} = m_3 \underline{\underline{a}}$

$a = \dfrac{F}{m_T} - \mu g$

$P_{23} - \cancel{\mu m_3 g} = \dfrac{m_3}{m_T} F - \cancel{m_3 \mu g}$

$\vec{P}_{23} = \dfrac{m_3}{m_1 + m_2 + m_3} F \; (\uparrow)$



$N_T = \dfrac{m v^2}{R} - mg$

$N_B = \dfrac{m v^2}{R} + mg$

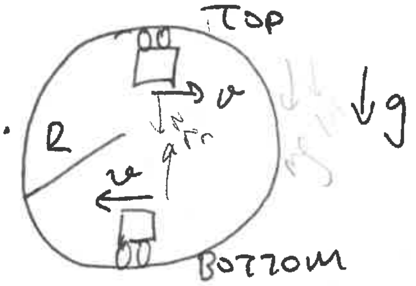
Normal force > 0; when there is contact

$\underline{N = 0} \Rightarrow$ no contact with the surface.

$N_T > 0$

$m\dfrac{v^2}{R} - mg > 0 \quad \underline{\underline{\dfrac{v^2}{R} > g}}$

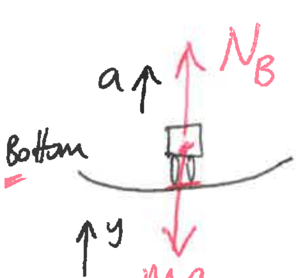
5.45



M
R } given
$v \to$ const.

Normal force exerted on the CAR by the wall of the circle

at BOTTOM? at TOP?



circular motion
a is toward the centre

uniform circ. mot.

$v = \text{const}$

$a_r = v^2/R$

$\sum F_y = m a_y$

$N_B - mg = m a_r$

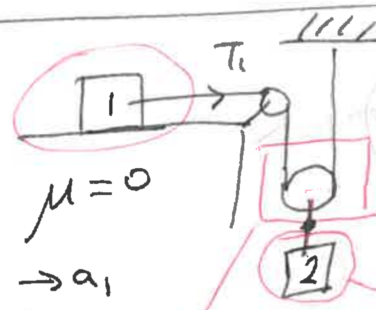
$N_B = mg + m\dfrac{v^2}{R}$

Bottom

$\sum F_y = m a_y$

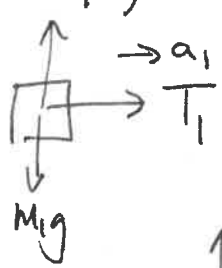
$-N_T - mg = -m a_r \quad \underline{N_T} = m\dfrac{v^2}{R} - mg$

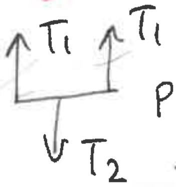
5.91



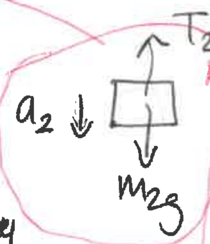
FBD? acceleration of each object?

$N_1 \quad \mu = 0$





pulley mass $= 0$



$\sum F_y = 0 \quad 2T_1 - T_2 = 0$

$2T_1 = T_2$

$\sum F_y = m_1 \cancel{a_y}^{\,0}$

$N_1 = m_1 g$

$\sum F_x = m_1 a_x$

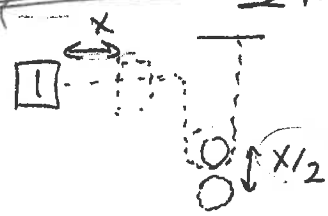
$T_1 = m_1 a_1$

$\sum F_y = m_2 a_y$

$T_2 - m_2 g = m_2(-a_2)$

$2T_1 - m_2 g = -m_2 a_2$

$2 m_1 \underline{\underline{a_1}} - m_2 g = -m_2 a_2$



$\dfrac{d^2 x}{dt^2} = a_1 \Rightarrow 2a_2$

$X_1 = 2X_2$

X_1 = distance of 1st object
X_2 " 2nd object

3

$X_1 = X$; $X_2 = X/2$

$a_1 = a$ $\quad a_2 = a/2$

$$2m_1a_1 - m_2g = -m_2a_2$$

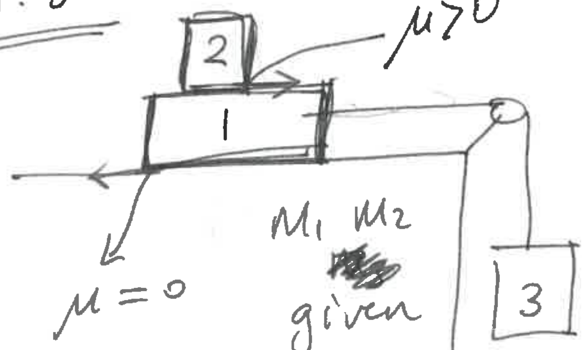
$a/2$

$$2m_1a + m_2\frac{a}{2} = m_2g$$

$$a\left(2m_1 + \frac{m_2}{2}\right) = m_2g$$

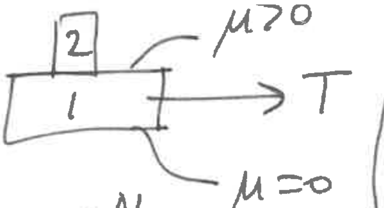
$$a = \frac{m_2g}{\left(2m_1 + \frac{m_2}{2}\right)}$$ of 1st object

5.92



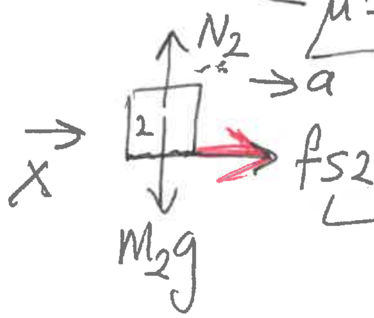
what would be the mass of 3rd object if 2nd obj does not slide on 1st one when moving?

FBD



$\Sigma F_y = m_2 g$

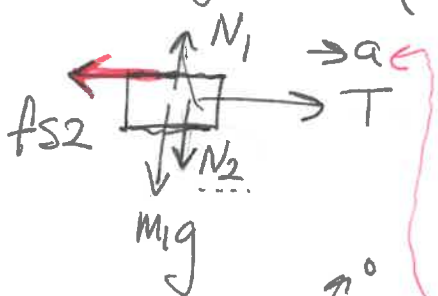
$N_2 = m_2g$



$fs2 = \mu N_2$

$fs2 = \mu m_2 g$

static (not sliding)



$\Sigma F_x = m_2a_x$

$fs2 = m_2a$

$\mu m_2 g = m_2 a$

$\mu g = a$

$\Sigma F_y = m_1 a_y^{\,0}$

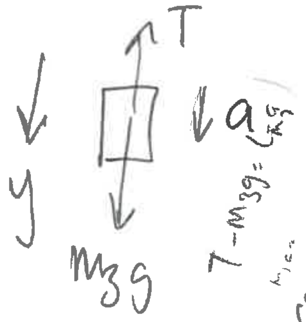
m_2g

$N_1 - N_2 - m_1g = 0$

$N_1 = m_1g + m_2g$

$\Sigma F_x = m_1a_x$

$T - fs2 = m_1a$ (2)



$\Sigma F_y = m_3a_y$

$m_3g - T = m_3a$ (3)

(2) + (3)

$$m_3g - fs2 = m_1a + m_3a$$

$$m_3g = \mu m_2g + (m_1 + m_3)a$$

μg

$\boxed{\cancel{m_3g = \mu g(m_1 + m_2 + m_3)}}$

$$m_3\cancel{g} - \mu m_3\cancel{g} = \mu\cancel{g}(m_1 + m_2)$$

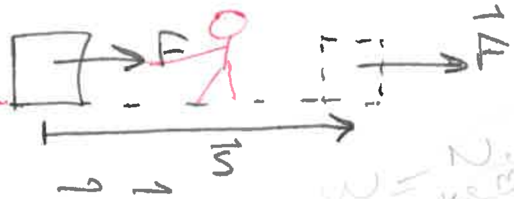
$$m_3 = \frac{\mu(m_1 + m_2)}{(1 - \mu)}$$

Chapter 6
work & Kinetic Energy

Work = $(\vec{Force}) \cdot (\vec{distance})$

$W = \vec{F} \cdot \vec{s}$ ↳ distance.

Force, distance } constant.



$W = \vec{F} \cdot \vec{s}$

units?

the work done by Force on the object

Joule = Newton. meter

$J = Nm$

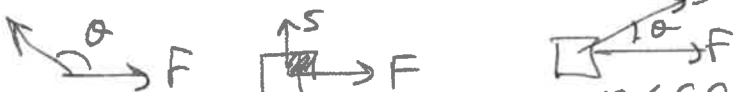
$= \left(kg \frac{m}{s^2}\right) \cdot m$

$\left[J = kg\, m^2/s^2 \right]$

$W = \vec{F} \cdot \vec{s}$ ⇒ scalar, dot product

$W = Fs\cos\theta$

$W = Fs\cos\theta$

$W<0$	$W=0$	$W>0$
		
$\theta > 90$	$\theta = 90$	$\theta < 90$
$W<0$	$W=0$	$W>0$

Ex: $\vec{F} = (160\hat{i} ~~+~~ 40\hat{j})\, N$

$\vec{s} = (14\hat{i} + 11\hat{j})\, m$

$\hat{i} \cdot \hat{j} = 0$

$W = \vec{F} \cdot \vec{s} = (160\hat{i} - 40\hat{j}) \cdot (14\hat{i} + 11\hat{j})$

1

$W = 160(14) - 40(11)$

$W = 1800\, J$

④ what is the angle between $\vec{F}$, $\vec{s}$

$\underline{W} = \vec{F} \cdot \vec{s}$

$1800 = Fs\cos\theta$

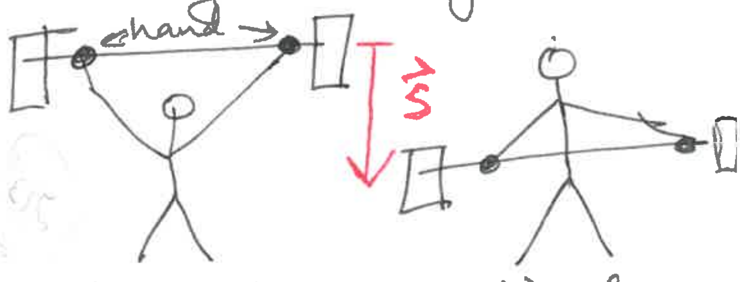
$|\vec{F}| = F = \sqrt{160^2 + 40^2} = 210\, N$

$s = \sqrt{14^2 + 11^2} = 18\, m$

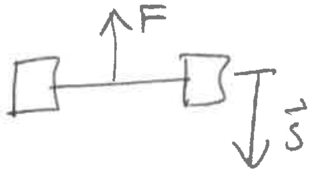
$1800 = 210(18)\cos\theta$

$30° = \theta = \cos^{-1}\left(\frac{1800}{210(18)}\right)$

Lowering the dumbell by the weightlifter.



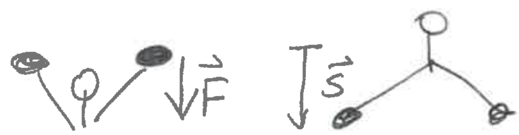
⇒ What is the work done by the hands on the dumbell?



$W_{on\ dumbell} = Fs\cos 180 = -Fs$

$W<0$

⇒ what is the work done by the dumbell on the hands?



$W = \vec{F} \cdot \vec{s} = Fs\cos 0 = Fs$

$W>0$

$$\Sigma W = \frac{m v_2^2}{2} - \frac{m v_1^2}{2}$$

$$K \equiv \frac{m v^2}{2}$$

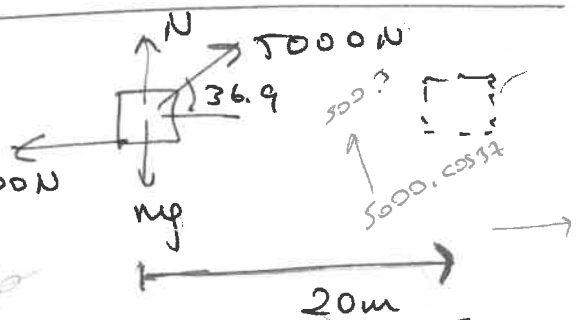
$$\boxed{\Sigma W = K_2 - K_1 = \Delta K} \; !$$

→ if $\Sigma W > 0$ $K_2 > K_1$ speeds up!

→ if $\Sigma W < 0$ $K_2 < K_1$ slows down!

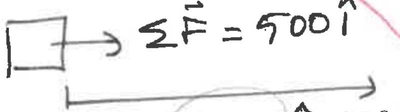
→ if $\Sigma W = 0$ $K_2 = K_1$ speed constant

Ex:



$v_1 = 2\,m/s$ $v_2 = ?$

$m = 1500\,kg$



$$\Sigma W = \Sigma \vec{F} \cdot \vec{s} = 10000\,J$$

$$\Sigma W = \Delta K = K_2 - K_1$$

$$10000 = \frac{m v_2^2}{2} - m\frac{v_1^2}{2}$$

$$10000 = \frac{1500\, v_2^2}{2} - \frac{1500\; 2^2}{2}$$

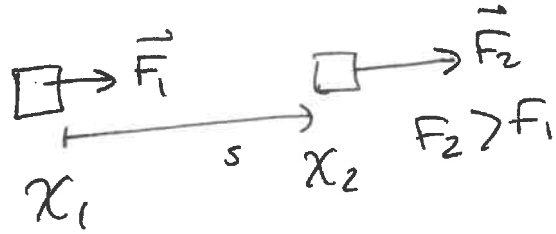
$$v_2 = 4.2\,m/s$$

$$\frac{m v_2^2}{2} - \frac{m v_1^2}{2}$$

K_2:

5) When the force is varying (not constant) along the path:

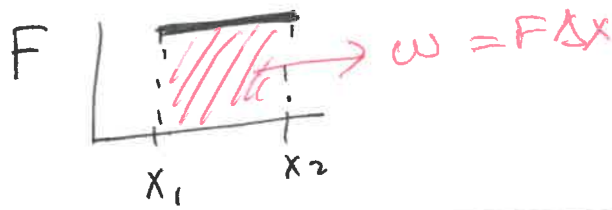
$$W = \vec{F} \cdot \vec{s}$$ const.

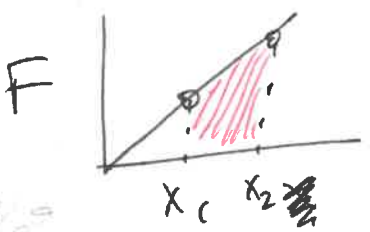


$$W_{1\to 2} = \int_{x_1}^{x_2} F\,dx$$

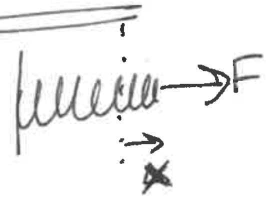
when F const

$$W = F \int dx = F \Delta x$$





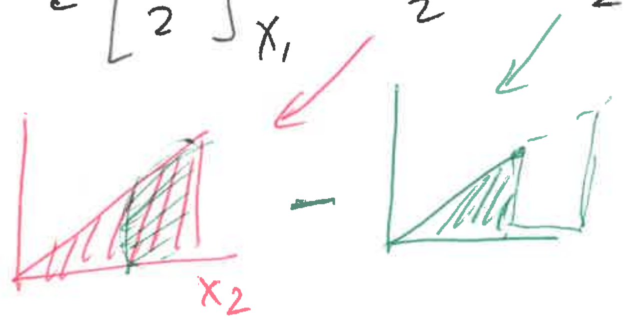
$F = kx$



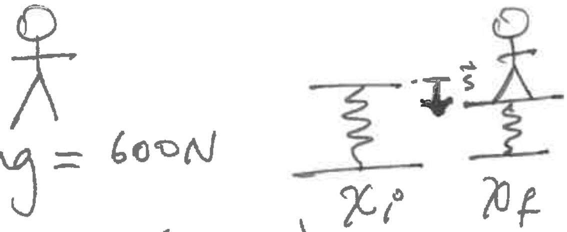
spring (yay)

$$\int_{x_1}^{x_2} F\,dx = \int kx\,dx = k\int x\,dx$$

$$W = k\left[\frac{x^2}{2}\right]_{x_1}^{x_2} = \frac{k x_2^2}{2} - \frac{k x_1^2}{2}$$



Ex



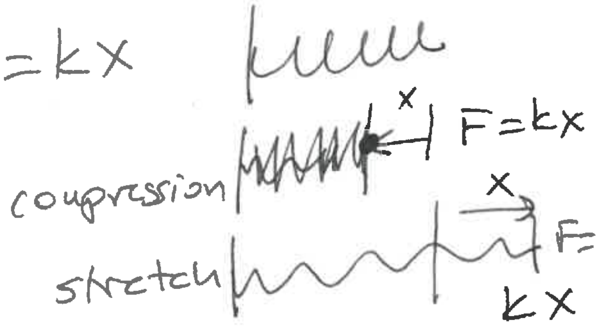
$mg = 600N$

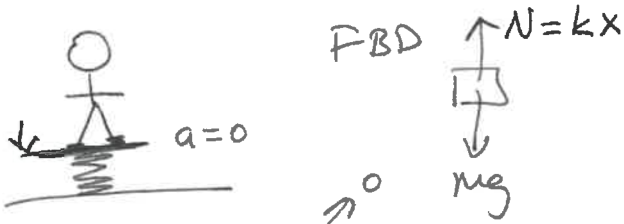
compression (sikisma) is $1cm = 0.01m$

→ Force constant of spring ?

→ ΣW on the spring during compression

$F = kx$



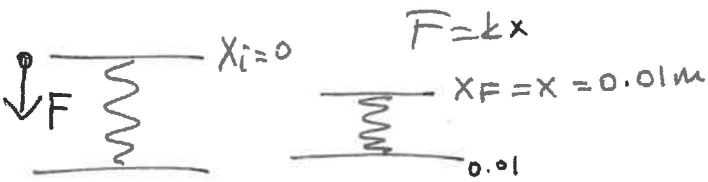


$\Sigma F = m\cancel{a}^{\,0}$

$kx - mg = 0$

$k = \frac{mg}{x}$

$k = \frac{600N}{0.01m} = 6\times10^4 N/m$



$\Sigma W = \int F dx = \int_0^{0.01} kx\,dx$

$= k\left[\frac{x^2}{2}\right]_0^{0.01}$

$= 6\times10^4\left[\frac{0.01^2}{2} - 0\right]$

$W = 3J > 0$

(6) $W_{tot} = \Sigma W = \int F_x\,dx$

$= \int m a_x\,dx$

$= \int m \frac{dv_x}{dt}\,dx$

$= \int m \frac{dv_x}{\cancel{dx}} \frac{dx}{dt} \cancel{dx}$ v_x

$= \int m\, v_x\, dv_x$

$\Sigma W = m\int v\,dv = \left[m\frac{v^2}{2}\right]_{v_i}^{v_f}$

$\Sigma W = \Delta K = K_2 - K_1$

POWER

$P_{av} = \frac{\Delta W}{\Delta t}$ $\frac{\text{work}}{\text{time}}$

$\left[Watt = \frac{J}{s} = W\right]$

$P = \frac{dW}{dt} = \frac{d}{dt}(\vec{F}\cdot\vec{s})$

$= \vec{F}\cdot\frac{d\vec{s}}{dt}$ velocity

$P = \vec{F}\cdot\vec{v}$

1 horsepower = 1hp = 746 Watt

7

Ex. plane engine A380

$F = 322\,000\ N$

$v = 900\ km/hr = 250\ m/s$

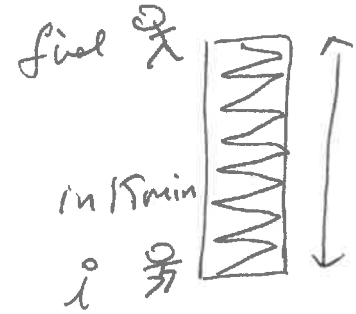
$P = ? = (322 \times 10^3)\ 250$

$= 8.05 \times 10^7$ Watt

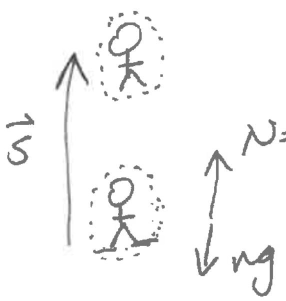
$= 108\,000$ hp

Ex: a runner runs up stairs of 443 m tall building. she finishes in 15 minutes.

$P_{av} = ?$ of the runner.



$P_{av} = \dfrac{\Delta W}{\Delta t}$ → done against mg



$N = \quad N \geq mg$

$\Delta W = \vec{N} \cdot \vec{s} > 0$

$= Ns$

$= mgs$

$P_{av} = \dfrac{\Delta W}{\Delta t} = \dfrac{mgs}{\Delta t}$

$= \dfrac{50 \times 9.8 \times 443}{15 \times 60} \dfrac{J}{s}$

$P_{av} = 241\ W = 0.32$ hp

Midterm 2

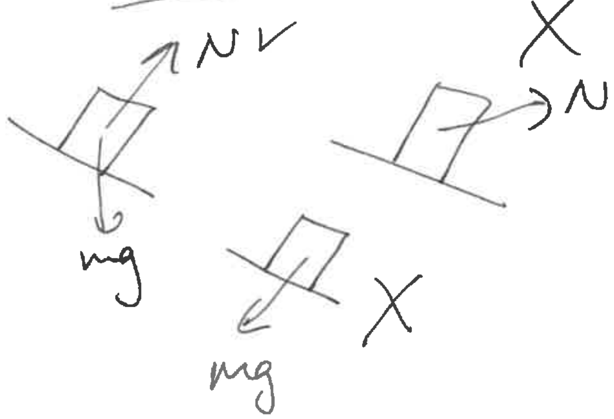
60–70 %
- ch4 + ch5
- newton's law ↳ FBD → $\Sigma \vec{F} = m\vec{a}$ solve for $\vec{F}, \vec{a}, \ldots ?$

at least 2 question

ch6

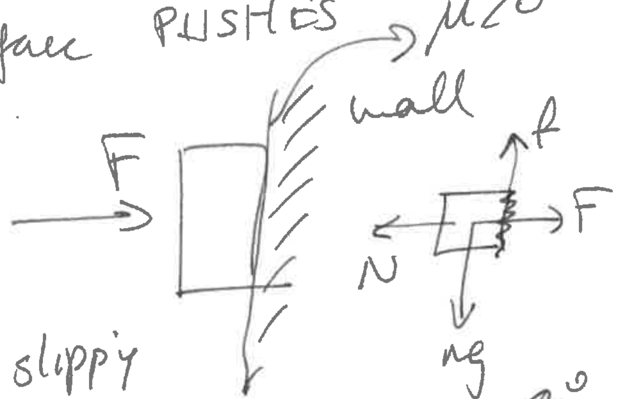
30% { Work & Kinetic energy

FBD important



rope PULLS

surface PUSHES → $\mu \geq 0$



no slipping

$\mu = ?$

$\Sigma F_x = m a_x^{\,0}$

$N - F = 0$

$N = F$

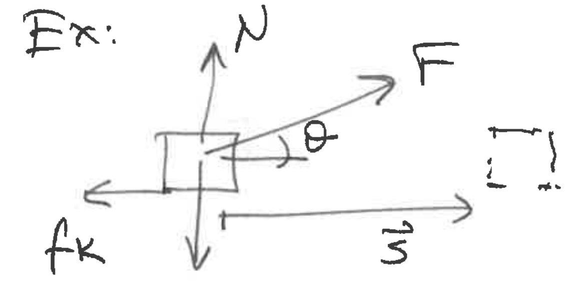
$\Sigma F_y = m a_y^{\,0}$

$mg - f = 0$

$\mu N = \mu F$

$mg = \mu F \quad \mu = \dfrac{mg}{F}$

Ex:



$\theta = 36.9°$

$s = 20m$ $\quad f_k = 3500N$

$F = 5000N$

$mg = 14700N$

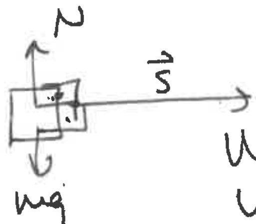
a) Find the work done by each Force on the object

b) Find the total work done on the object

$W_F = \vec{F} \cdot \vec{s} = Fs\cos 36.9°$

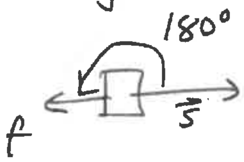
$= 5000(20)(0.8)$

$= 80000 J$



$W_N = Ns \cos 90$

$W_N = 0$



$W_{mg} = mgs \cos 90$

$W_{mg} = 0$

$W_{f_k} = (3500)(20)\cos 180$

$= -70000 J$

$\Sigma W = W_F + W_{mg} + W_N + W_{fk}$

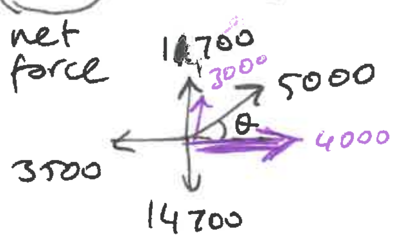
$= 80000 + 0 + 0 - 70000$

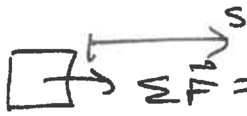
$\Sigma W = 10000 J$

total work

$\Sigma W = (\Sigma \vec{F}) \cdot \vec{s} = (\vec{F} + m\vec{g} + \vec{N} + \vec{f_k}) \cdot \vec{s}$

net force





$\Sigma \vec{F} = 500\hat{i}$

$\Sigma W = (\Sigma \vec{F}) \cdot \vec{s}$

$= 500\hat{i} \cdot 20\hat{i}$

$\Sigma W = 10000 J$

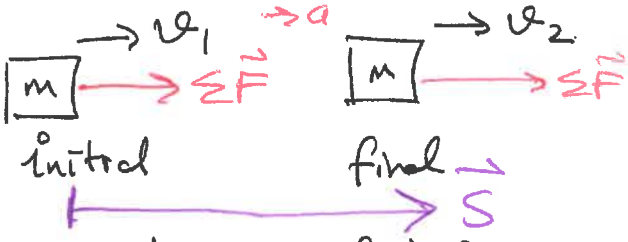
$\Sigma W = ?$

1st way: $W_{F1} + W_{F2} + W_{F3}$ ✓

equal

2nd: $(\Sigma \vec{F}) \cdot \vec{s}$ ✓

WORK – ENERGY theorem and KINETIC ENERGY



speeding up $v_2 > v_1$

$v_2^2 = v_1^2 + 2a\underbrace{(X_2 - X_1)}_{s}$

$a = \frac{v_2^2 - v_1^2}{2s}$

$\Sigma \vec{F} = m\vec{a}$

$\Sigma F_x = ma_x$

$\Sigma F = m \frac{v_2^2 - v_1^2}{2s}$

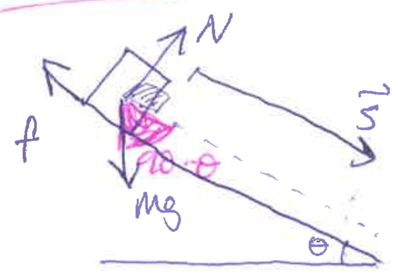
$(\Sigma \vec{F} \cdot \vec{s})$; $\cos\theta = 1$ $\quad \theta = 0$

$\Sigma F s = m \frac{v_2^2 - v_1^2}{2\cancel{s}} \cancel{s}$

$\Sigma W = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$

$= \Delta K$ (kinetic energy difference

PHYS I 21.7.17 (1)

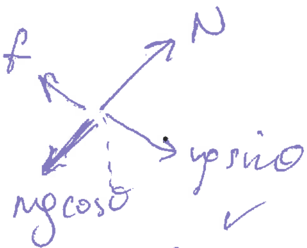


$W_N = \vec{N}\cdot\vec{s}$
$= Ns\cos 90$
$= \underline{\underline{0}}$

$W_{mg} = m\vec{g}\cdot\vec{s}$
$= mgs\cos(90-\theta)$
$= mgs\sin\theta$

$W_f = \vec{f}\cdot\vec{s}$
$= fs\cos 180 = -fs$
$= -\mu Ns$

$\Sigma W = W_{TOT} = (mg\sin\theta)s - \mu Ns$
$= (mg\sin\theta - \mu mg\cos\theta)s$

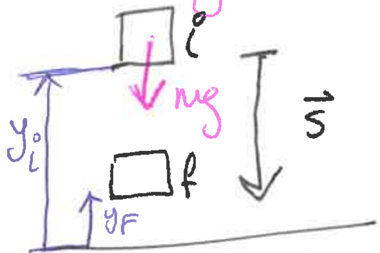


$\Sigma W = K_2 - K_1$

$P = \Sigma\vec{F}\cdot\vec{v} \rightarrow \frac{v_1+v_2}{2}$

$= \frac{\Delta W}{\Delta t} \rightarrow v_2 = v_1 + at$

Chapter 7
Potential Energy and Energy Conservation



$W_{mg} = m\vec{g}\cdot\vec{s} > 0$
$= mgs \rightarrow (y_i - y_F)$

$y_i = y_1 \quad y_F = y_2$

$W_{mg} = mgs = mgy_1 - mgy_2$

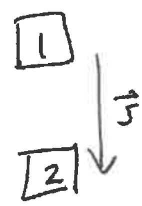
$U_g \equiv mgy$ gravitational potential energy

$mgs = U_1 - U_2 = mgy_1 - mgy_2$

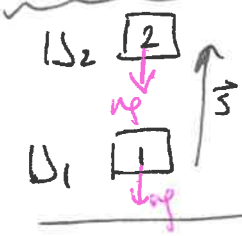
$W_{mg} = U_1 - U_2 = -(U_2 - U_1)$ ← final − init

Δ:

$\boxed{W_{mg} = -\Delta U}$



$W_{mg} = mgs > 0 = -\Delta U$
$U_2 < U_1$!



$W_{mg} < 0 = -mgs$
$-mgs = -\Delta U$
$mgs = U_2 - U_1$
$U_2 > U_1$!

$\boxed{W_{mg} = -\Delta U}$ ✓

work & KE theorem ②

$W_F = \Delta K$

$W_{mg} = \Delta K$ } equal

$W_{mg} = -\Delta U$

$\Delta K = -\Delta U$

$\boxed{\Delta K + \Delta U = 0}$

change in KE + change in Pot. E = ZERO

$K_F - K_i + U_F - U_i = 0$

$\boxed{K_i + U_i = K_F + U_F}$

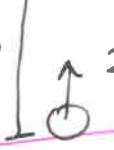
ENERGY conservation.

$E = K + U$

↳ mechanical energy

when only mg force does the work

Ex:



$v_F = 0$ } final

$h = ?$

20 m/s, $m = 0.145\,kg$ } initial

↳ $y_i = 0$

$E_i = E_F$

$U_i + K_i = U_F + K_F$

$mg\cancel{y_i} + \frac{1}{2}m v_i^2 = mg y_F + \frac{1}{2}m \cancel{v_F^2}$

$\frac{1}{2}(\cancel{0.145})\,20^2 = (\cancel{0.145})(9.8)\,y_F$

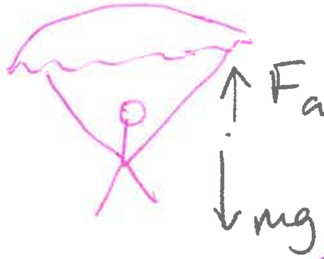
$y_F = \dfrac{20^2}{2(9.8)} = 20.4\,m$

mg is the only force!!

When Forces other than Gravity does work

$W_{other} + W_{mg} = \Delta K$

$\Sigma W = W_{TOT} = \Delta K$



} other force

$W_{other} + \underbrace{(-\Delta U)}_{W_{mg}} = \Delta K$

$W_{other} - (U_F - U_i) = K_F - K_i$

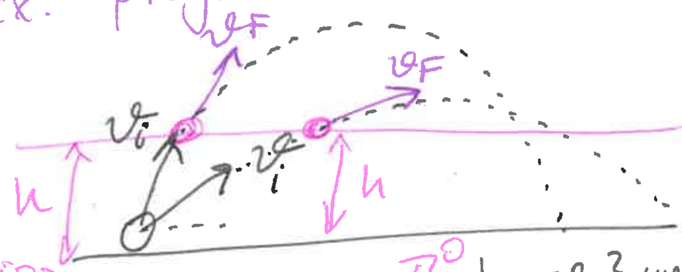
$\boxed{W_{other} + U_i + K_i = U_F + K_F}$

the most general form of energy conservation

$W_{other} + mg y_i + \frac{1}{2}m v_i^2 = mg y_F + \frac{1}{2}m v_F^2$

W_{other}: mostly frictional forces

Ex: projectile motion

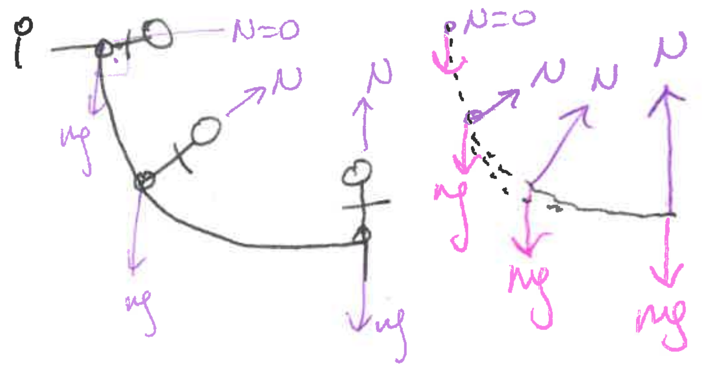


$\frac{1}{2}m v_i^2 + mg\cancel{y_i}^{\,0} = \frac{1}{2}m v_F^2 + mg y_F$

at same height h:

both have the same v_F

motion along a curved path (3)

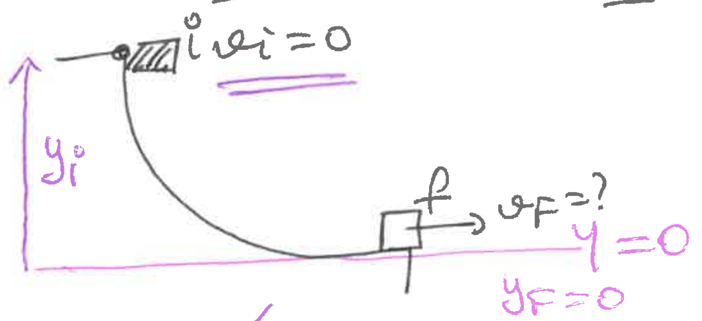


no friction!

$U_i + K_i = U_F + K_F$

Normal Force is always $\perp$ perpendicular to the path

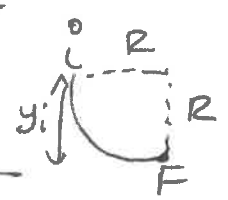
$W_N = \vec{N}\cdot\vec{s} = Ns\cos 90 = 0$



$\frac{1}{2}m\cancel{v_i^2} + mgy_i = \frac{1}{2}mv_F^2 + \cancel{mgy_F}$

$\cancel{m}gy_i = \frac{1}{2}\cancel{m}v_F^2$

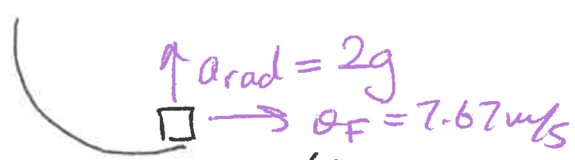
$v_F = \sqrt{2gy_i}$



$v_F = \sqrt{2gR}$

(b) what's the a_{rad} at final point?

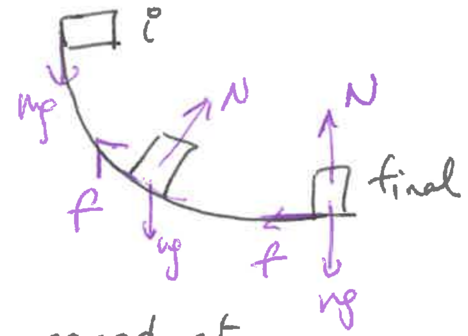
$a_{rad} = \frac{v_F^2}{R} = \frac{2gR}{R} = 2g$



$R = 3m \quad g = 9.8\ m/s^2$

$v_F = \sqrt{2gR} = 7.67\ m/s$

when there's friction.

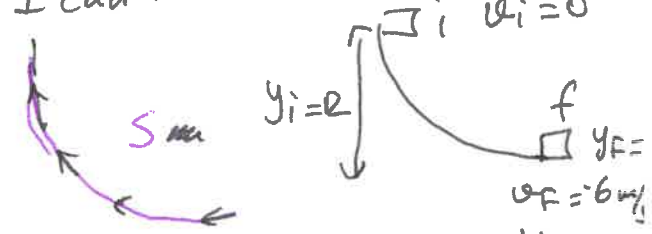


if the speed at bottom (final) is 6 m/s. what's the work done by the frictional force?

$(m = 25kg) \quad (R = 3m)$

$W_f = \vec{f}\cdot\vec{s} = fs\cos\theta$ not const.

I can't use it.



$W_{other} + K_i + U_i = K_F + U_F$

$W_f + \frac{1}{2}m\cancel{v_i^2} + mgy_i = mg\cancel{y_F} + \frac{1}{2}mv_F^2$

$W_f + mgR = \frac{1}{2}mv_F^2$

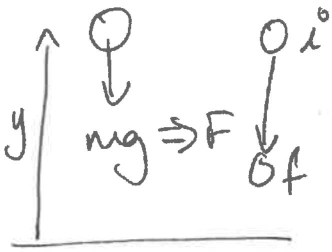
friction $W_F = \frac{1}{2}mv_F^2 - mgR$

$= \frac{1}{2}25(6^2) - 25(9.8)3$

$W_F = -285\ J$

friction always DOES negative work

Phys I 25.7.17 (1)



$E_i = E_f$

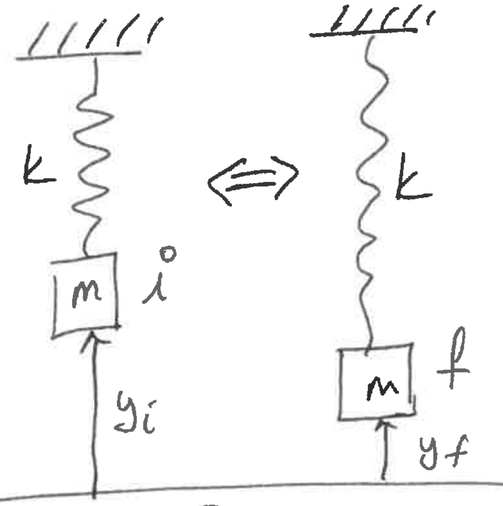
$K_i + U_i = K_f + U_f$

$K = \frac{1}{2}mv^2;\ U = mgy$

** $\downarrow g$

$F_g = mg$

$F_{el} = -ky$



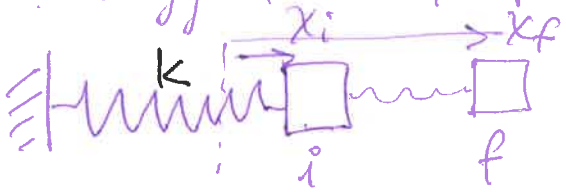
$E_i = E_f$

$K_i + U_{g,i} + U_{el,i} = K_f + U_{g,f} + U_{el,f}$

Displacement is in $\pm\hat{y}, (\hat{j})$ direction

Elastic Potential Energy

Energy of a Spring



$\underline{\underline{U_{el}}} = \frac{1}{2}kx^2$

elastic

*** if there is friction

$\underset{\text{other}}{W} + E_i = E_f$

$\llcorner$ friction

$W_{friction} < 0$

$W_{other} + K_i + U_{g,i} + U_{el,i}$

$= \{K_f + U_{g,f} + U_{el,f}\}$

$W_{el} \Rightarrow$ work done by elastic force

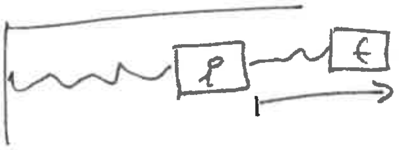
elastic force $\equiv F_{el} = -kx$

$W_{el} = -\Delta U_{el}$

$W_{el} = U_{el,i} - U_{el,f}$

* if there is no displacement in $\hat{j}\ (\hat{y})$ direction; $\underline{\underline{U_g}}$ gravitational pot. energy

$\Delta U_g = 0$;



only $F = -kx$!

$K_i + U_{el,i} = K_f + U_{el,f}$

energy conservation

solving problems

⇒ is there a displacement in y direction

→ YES → U_g

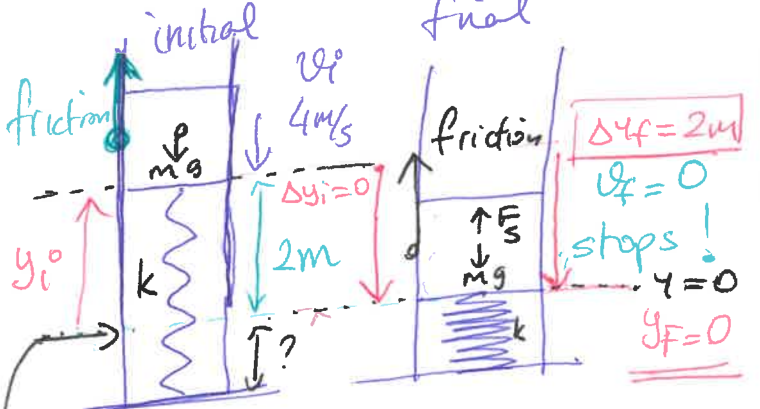
⇒ is there a spring;

→ YES → U_{el}

⇒ is there friction

→ YES → $W_{friction}$

Ex: A 2000 kg elevator (2) with broken cables is falling at 4 m/s when it contacts a spring at the bottom. The spring is compressed 2m; ALSO during the motion a safety brake applies 17000N friction force. What is the $\underline{k}$ spring constant?



$W_{friction} + E_i = E_f$

$y=0$ establish (choose) $y=0$!!

$W_f + K_i + U_{gi} + U_{eli} = K_f + U_{gf} + U_{elf}$

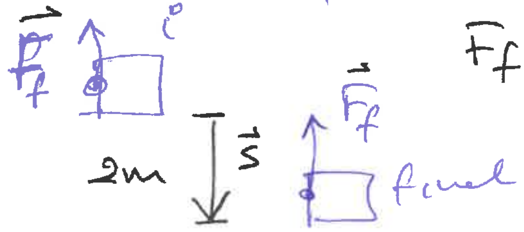
$W_f + \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}k\cancel{\Delta y_i^2} = \frac{1}{2}m\cancel{v_f^2} + mg\cancel{y_f} + \frac{1}{2}k\Delta y_f^2$

$\Delta y_i = 0$ because there is no compression / strech at initial position!

$W_f + \frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}k(\Delta y_f)^2$

$W_f + \frac{1}{2}(2000)4^2 + 2000(9.8)2 = \frac{1}{2}\overset{?}{k}2^2$

What is $W_f = ?$



$F_f = 17000N$

$W_f = \vec{F_f} \cdot \vec{s} = F_f s \underbrace{\cos 180}_{-1}$

$= - F_f s$

$= -(17000)2$

$W_f = -34000 J$

$W_f + \frac{1}{2}2000\, 4^2 + 2000(9.8)2$

$= \frac{1}{2}\underline{\underline{k}}\, 2^2$

$\boxed{F = kx} \Rightarrow k\left[\frac{N}{m}\right]$

$-34000 + 16000 + 39200 = 2k$

$\underline{\underline{k}} = 1.06 \times 10^4 = \underline{\underline{10600}}$

$= 10600\ N/m$

Conservative

nonconservative forces

(korunumlu / korunumsuz kuvvetler)

conservative forces:

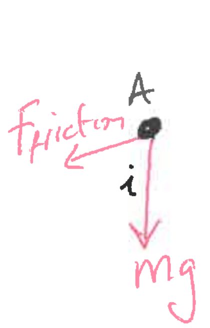
- it is reversible (tersinir)
- it is independent of the path, depends only on starting and ending points
- when starting, ending points are SAME, $W_{tot} = 0$!

→ it is conserving energy!

— it can be written as the difference between initial and final values of a potential energy.

conservative forces $\Rightarrow$ $\underline{\underline{mg}}$; $\underline{\underline{kx}}$ ✓

nonconservative forces $\Rightarrow$ frictional forces



$\underline{\underline{A \to B}}$

$B \to A$

$W_{mg} = 0$!

conservative !

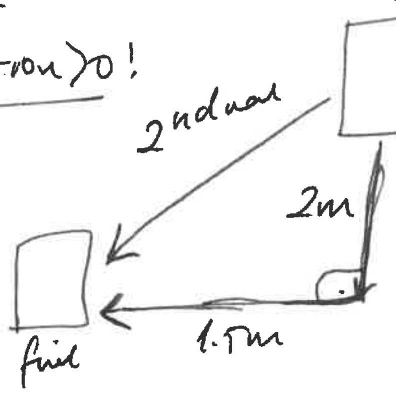
$W_{Ff} \neq 0$

$A \to B$

nonconservative !

Ex: coffee table, initial

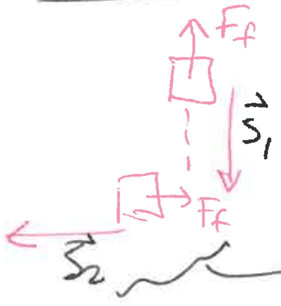
friction > 0 !



$\mu_k = 0.2$

$m = 40 kg$

$W_{f\ i \to f}$ 1st way =



$\Big\}\ -\mu N (2m)$

$\underline{\underline{N = mg}}$

$-\mu N (1.5m)$

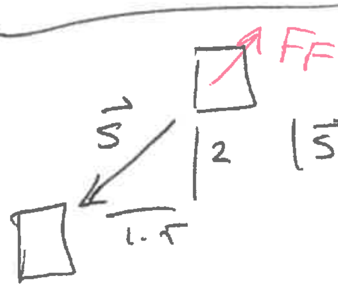
③

$W_{Ff}^{1st\ way} = -\mu mg\, 2 - \mu mg\, 1.5$

$= -\mu mg (3.5)$

$= -(0.2)(40)(9.8)(3.5)$

$W_{Ff}^{1st} = -274$ Joule



$|\vec{S}| = \sqrt{2^2 + 1.5^2}$
$= 2.5 m$

$W_{Ff}^{2nd\ way} = -\mu N (2.5)$
$= -196$ Joules

Force & Potential Energy

$W = -\Delta U$ $\Big\}$ conservative forces !

$\Downarrow$

$F \Delta x = -\Delta U$

$F = -\dfrac{\Delta U}{\Delta x}$; $F_x = -\dfrac{dU}{dx}$

If you know $\underline{\underline{U}} \Rightarrow$ can find $\underline{\underline{F}}$

3 dimension $U \to (x, y, z)$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\vec{F} = \left(-\frac{\partial U}{\partial x}\hat{i}\right) + \left(-\frac{\partial U}{\partial y}\hat{j}\right) + \left(-\frac{\partial U}{\partial z}\hat{k}\right)$$

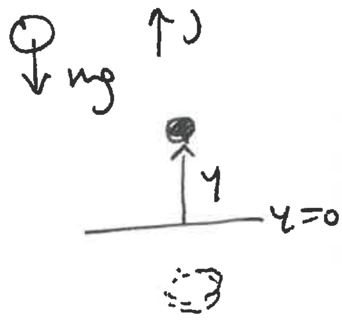
$\vec{F} = -mg\hat{j}$ ✓

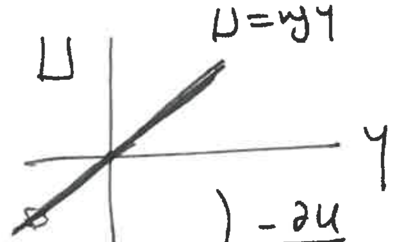
$U = mgy = U(y)$

$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$

$= -\frac{\partial}{\partial x}(mgy)\hat{i} - \frac{\partial}{\partial y}(mgy)\hat{j} - \frac{\partial}{\partial z}(mgz)\hat{k}$

$\vec{F} = -mg\hat{j}$ ✓





$-\frac{\partial U}{\partial y}$

$\vec{F} = -mg\hat{j}$

Spring

$U = \frac{1}{2}kx^2$ ✓

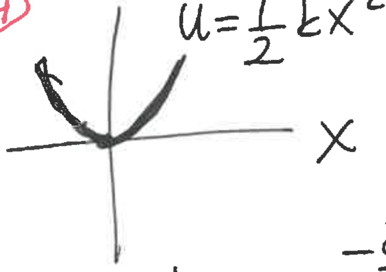
$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$

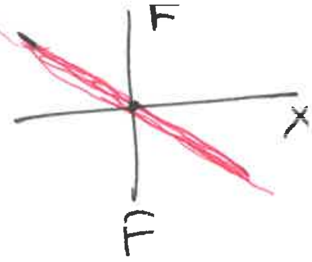
$\vec{F} = -\frac{\partial}{\partial x}\left(\frac{kx^2}{2}\right)\hat{i} - \frac{\partial}{\partial y}\left(\frac{kx^2}{2}\right)\hat{j} - \frac{\partial}{\partial z}\left(\frac{kx^2}{2}\right)\hat{k}$

$= -\frac{k}{2}\frac{\partial}{\partial x}(x^2)\hat{i}$

$\vec{F} = -kx\hat{i}$ ✓

(4)





$U = \frac{1}{2}kx^2 \quad -\frac{\partial U}{\partial x} \quad F = -kx$

Ex: $U(x,y) = \frac{1}{2}k(x^2+y^2)$

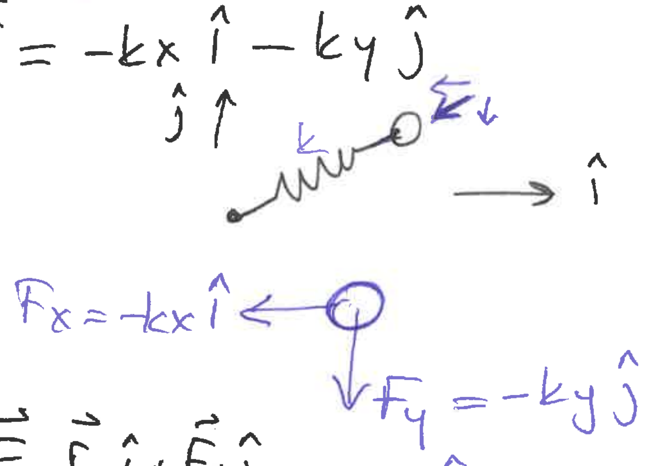
$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$

$= -\frac{\partial}{\partial x}\left(\frac{kx^2}{2} + \frac{ky^2}{2}\right)\hat{i} \Rightarrow -kx$

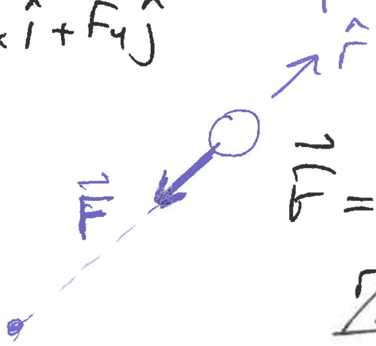
$-\frac{\partial}{\partial y}\left(\frac{kx^2}{2} + \frac{ky^2}{2}\right)\hat{j} \Rightarrow -ky$

$-\frac{\partial}{\partial z}\left(\frac{kx^2}{2} + \frac{ky^2}{2}\right)\hat{k} = 0$

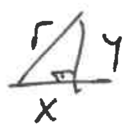
$\vec{F} = -kx\hat{i} - ky\hat{j}$



$\vec{F} = \vec{F}_x\hat{i} + \vec{F}_y\hat{j}$



$\vec{F} = F(-\hat{r})$



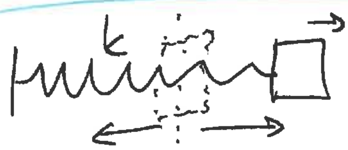
$U = \frac{1}{2}k(x^2+y^2) = \frac{kr^2}{2}$

$-\frac{\partial U}{\partial r}$

$r = \sqrt{x^2+y^2}$

$\vec{F} = -kr\hat{r}$

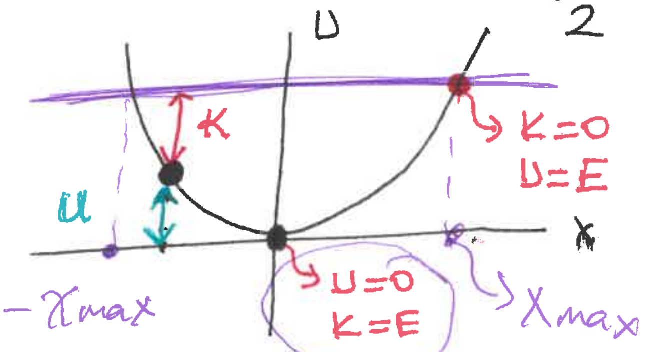
ENERGY DIAGRAMS (1) U



$E = E_i = E_F$

$K_i + U_i = K_F + U_F$

$X = 0$ $\qquad U = \frac{kx^2}{2}$



$K + U = E$

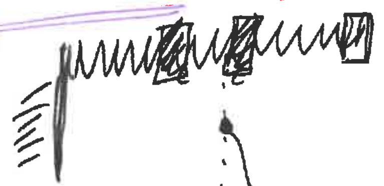
equilibrium point

$X = 0$

$U \to F$

$-\frac{\partial U}{\partial x}$

$0 = \sum \vec{F} = m\cancel{\vec{a}}^{\,0}$



U	F
$\frac{1}{2}kx^2$	$-kx$

$X = 0$

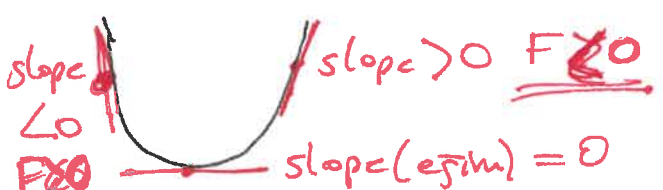
$F = 0$

$F = 0$

$F = k(0)$

$X = 0$

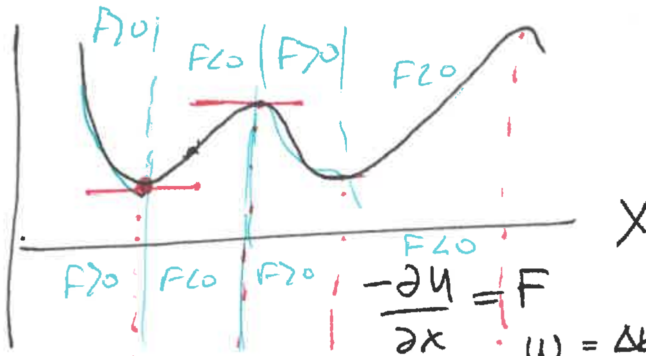
U



$-\frac{\partial U}{\partial x} = F = 0!!$

$\frac{\partial U}{\partial x}$ = slope $\to +$

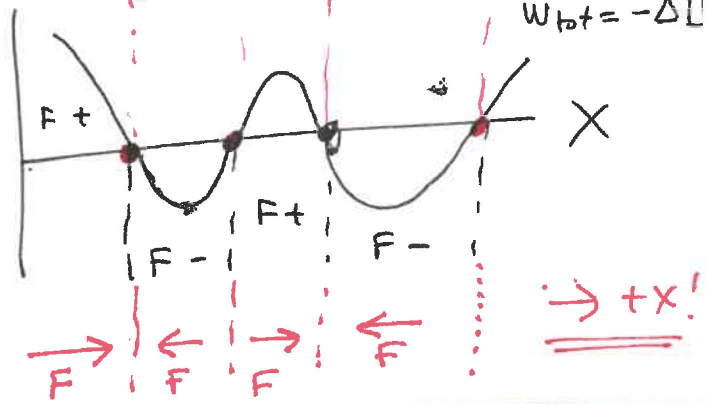
$-\frac{\partial U}{\partial x}$ = force $\to -$



$-\frac{\partial U}{\partial x} = F$

$W_{tot} = \Delta K$

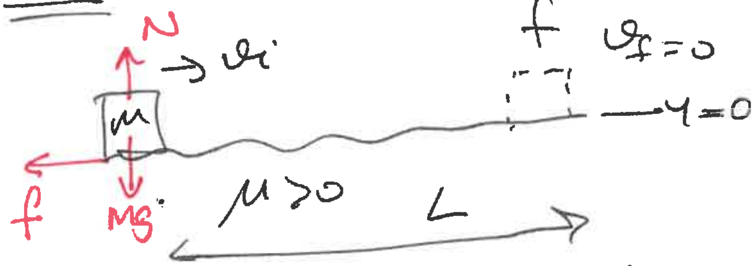
$W_{tot} = -\Delta U$



$\to +x!$

end of the chapter!

Ex:



find L in terms of other variables!

$W_f + E_i = E_f$

$W_f + K_i + \cancel{U_i} = K_F + \cancel{U_F}$

$K_F = \frac{1}{2}m\cancel{v_F^2} = 0$

$W_f + K_i = 0$

$-fL + \frac{1}{2}mv_i^2 = 0$ $\qquad f = \mu N = \mu mg$

$L = \frac{mv_i^2}{2f} = \frac{\cancel{m}v_i^2}{2\mu \cancel{m}g}$

$$\boxed{L = \frac{v_i^2}{2\mu g}}$$

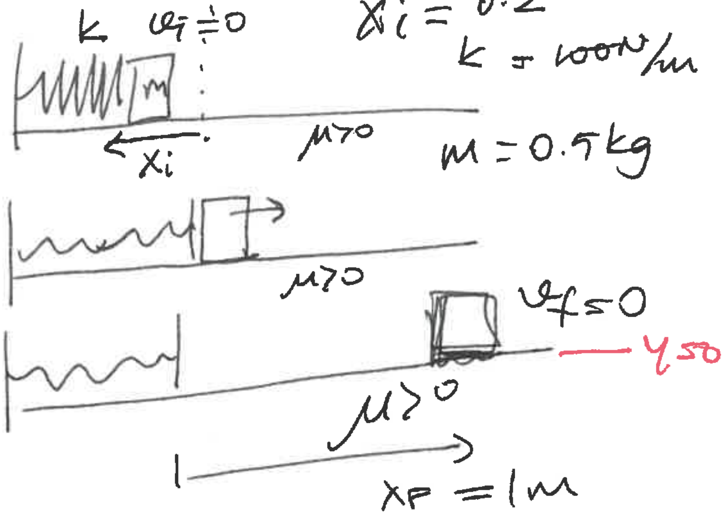
7.39 ⑥

$x=0$

$k \quad v_i \doteq 0 \qquad x_i = 0.2$

$k = 100\,N/m$

$m = 0.5\,kg$



$\mu = ?$

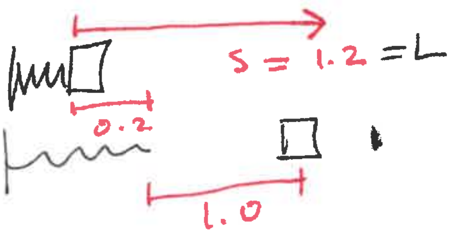
$W_f + E_i = E_f \quad \cancel{v_F} = 0$

$W_f + U_{eei} + \cancel{K_i}^{\,0} = \cancel{K_F}^{\,0} + \cancel{U_F}^{\,0}$

$+ \cancel{U_{gi}}^{\,0} \qquad v_i = 0$

$W_f + U_{eei} = 0$

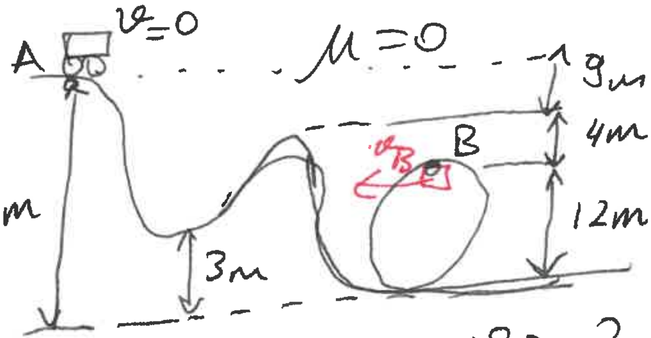
$-fL + \frac{1}{2} k x_i^2 = 0 \qquad f = \mu m g$



$-\mu m g L + \frac{1}{2} k x_i^2 = 0$

$\mu = \frac{k x_i^2}{m g L} = \frac{100\,(0.2)^2}{0.5\,(9.8)(1.2)}$

7.41 Roller coaster

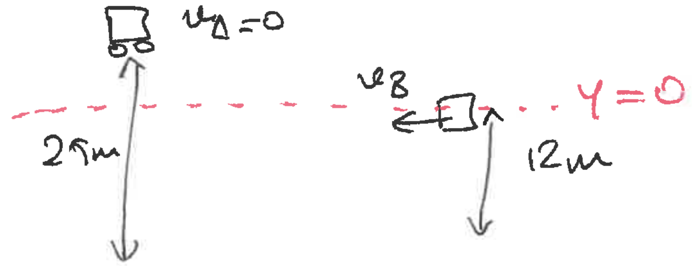


$m = 350\,kg \qquad v_B = ?$

$N_B = ?$

$E_A = E_B$

$\frac{1}{2} m v_A^2 + m g y_A = \frac{1}{2} m v_B^2 + m g y_B$

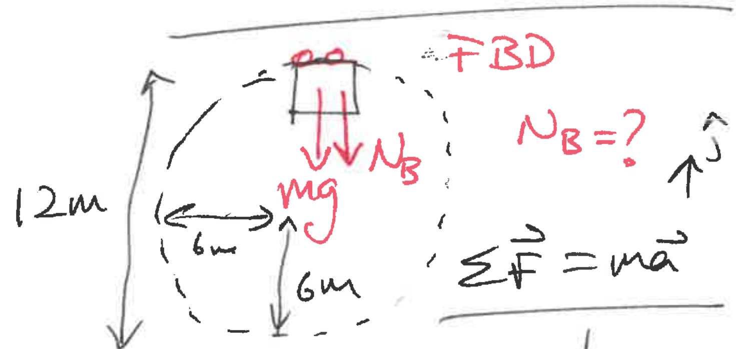


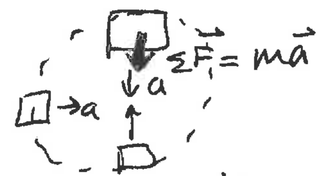
$y_A = 13\,m \qquad v_A = 0$

$y_B = 0 \qquad v_B = ?$

$0 + \cancel{m} g (13) = \frac{1}{2} \cancel{m} v_B^2 + 0$

$\checkmark \; v_B = \sqrt{2(9.8)\,13} \; m/s$





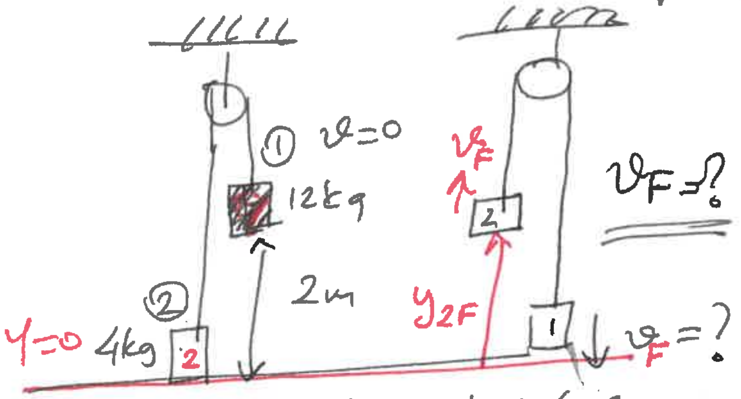
$-N_B - mg = m\left(\frac{-v^2}{R}\right)$

$N_B + mg = m \frac{v_B^2}{R}$

$N_B = m\left(\frac{v_B^2}{R} - g\right)$

$N_B = 350 \left(\frac{2(9.8)13}{6} - 9.8 \right) N$

7.57 Atwood machine



find speed just before it hits the ground?

$$E_i = E_f$$

$$\cancel{K_{1i}} + U_{1i} + \cancel{K_{2i}} + \cancel{U_{2i}} = K_{1F} + \cancel{U_{1F}} + K_{2F} + U_{2F}$$

$v_{1i}=0$

$v_{2i}=0$

$y_{2i}=0$

$y_{1i}=2m$

$y_{1F}=0$

$y_{2F}=y=2m$

$v_{1F}=v_{2F}=v_F$

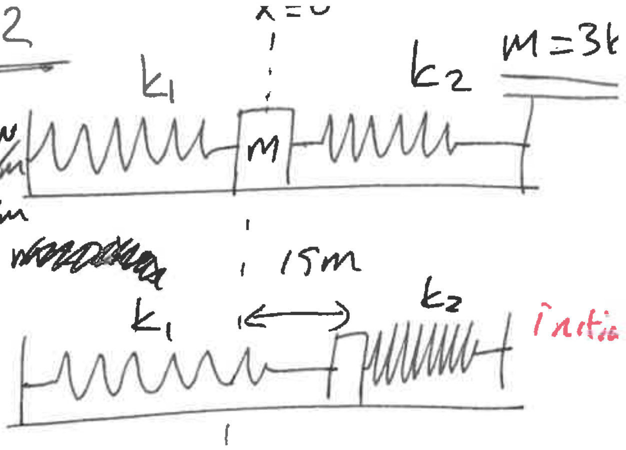
$$m_1 g y_{1i} = \frac{1}{2}m_1 v_F^2 + \frac{1}{2}m_2 v_F^2 + m_2 g y_{2F}$$

$$12(9.8).2 = \frac{1}{2}(12+4)v_F^2 + 4(9.8)2$$

$$v_F = \sqrt{\frac{16(9.8)}{8}} = \sqrt{2(9.8)}$$

$$v_F = \sqrt{19.6}\ m/s$$

find 7-62



$k_1 = 25 N/m$

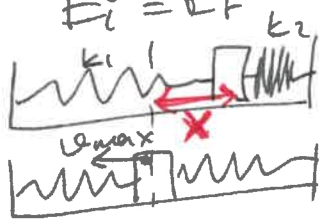
$k_2 = 20 N/m$

$M=0$

m is pushed 15m to the right and released from rest ($v=0$)

a) What is the max speed of m?

$$E_i = E_F$$



$U=\frac{1}{2}kx^2$

$$K_i + U_{1i} + U_{2i} = K_F + U_{1F} + U_{2F}$$

$$\frac{1}{2}k_1 x^2 + \frac{1}{2}k_2 x^2 = \frac{1}{2}m v_{max}^2$$

$$v_{max} = \sqrt{\frac{(k_1+k_2)x^2}{m}}$$

$$= \sqrt{\frac{(25+20)15^2}{3}}$$

$$v_{max} = 15\sqrt{15}\ m/s$$

Phys I 27.7.17

(1)

Momentum – Impulse and collisions Ch 8

$\sum \vec{F} = m\vec{a}$

$\sum \vec{F} = m\dfrac{d\vec{v}}{dt} = \dfrac{d(m\vec{v})}{dt}$

Momentum $\boxed{\vec{p} = m\vec{v}}$ **

$\boxed{\sum \vec{F} = \dfrac{d\vec{p}}{dt}}$ *

net force is change in momentum over tie.

$\vec{P} = m\vec{v} \Rightarrow P = \left[kg\,\frac{m}{s}\right]$

vektor

impulse (itme)

$\vec{J} \equiv \sum \vec{F}\,\Delta t \qquad J = [N\,s]$

$J = \left[kg\,\frac{m}{s^2}\,s\right]$

$\vec{J} = \sum \vec{F}\,\Delta t$

$\sum \vec{F} = \dfrac{\Delta \vec{P}}{\Delta t} = \dfrac{\vec{P}_f - \vec{P}_i}{t_f - t_i}$

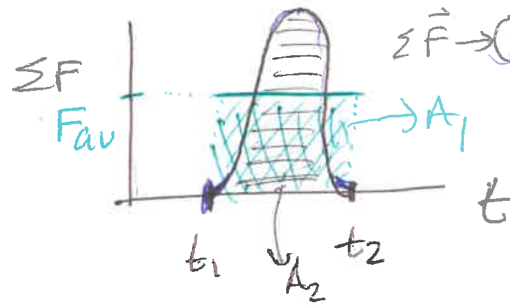
$\vec{J} = \sum \vec{F}\,\Delta t = \dfrac{\Delta \vec{P}}{\Delta t}\,\Delta t$

$\boxed{\vec{J} = \Delta \vec{P}}$ * impulse is the change in momentum

$\vec{J} = \sum \vec{F}\,\Delta t = \Delta \vec{P}$

impulse — change in mom.

$\vec{J} = \sum \vec{F}\,\Delta t \qquad \sum \vec{F} \to$ O t_1, $\sum \vec{F} \to$ O t_2



tie that force is applied

$J = (F_{av})(t_2 - t_1) = A_1$

$= \int \sum F\,dt = A_2$

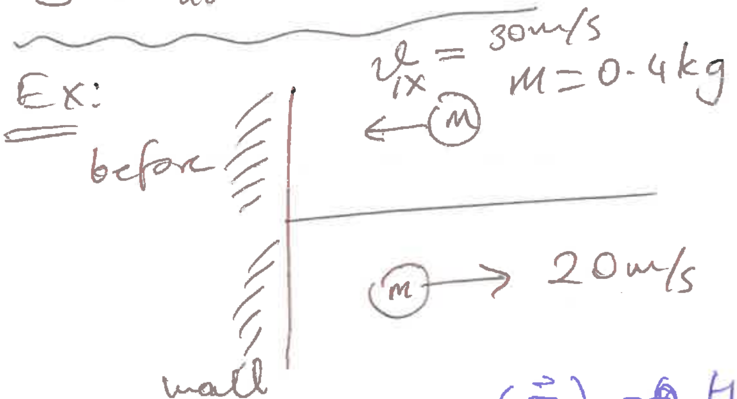
$A_1 = A_2$ (area)

$t_2 - t_1 = \Delta t \equiv$ contact tie

F_{av} = averge force applied

$\vec{J} = \vec{F}_{av}\,\Delta t = \Delta \vec{P}$

Ex:



a) Find the impulse ($\vec{J}$) of the ball during its collision.

$\longrightarrow \hat{x}, \hat{i}(+$

$\vec{J} = \Delta \vec{P} = \vec{P}_f - \vec{P}_i$

$= m\vec{v}_f - m\vec{v}_i$

$= m(\vec{v}_f - \vec{v}_i)$

$= 0.4\,[20\hat{i} - (-30\hat{i})]$

$\vec{J} = 20\ \frac{kg\,m}{s}\,\hat{i}$

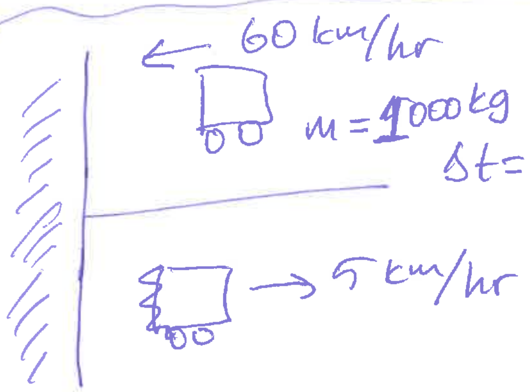
(6) if the collision time is 0.01 s ; whats the avarge force applied to ball.

$\vec{J} = \underbrace{\Sigma\vec{F}}_{?} \Delta t = \Delta\vec{P}$

$\vec{F}_{av} = \frac{\Delta\vec{P}}{\Delta t} = \frac{20\frac{kgm}{s}\hat{i}}{0.01 s}$

$\vec{F}_{av} = 2000\hat{i}$ $\frac{kg\,m}{s^2}$ Natn

Ex:



Fav = ?

$\vec{F}_{av} = \frac{\Delta\vec{P}}{\Delta t} = \frac{\vec{P}_f - \vec{P}_i}{\Delta t}$

$60 \frac{km}{hr} \rightarrow 60 \frac{1000 m}{3600 s}$

$\frac{60}{3.6} m/s = 16.7 m/s$

$5 km/hr \rightarrow \frac{5}{3.6} m/s = 1.4 m/s$

i ← 16.7 m/s

f → 1.4 m/s +x

$\vec{F}_{av} = \frac{\Delta\vec{P}}{\Delta t}$

$\frac{1000(1.4 - (-16.7))}{0.1} =$

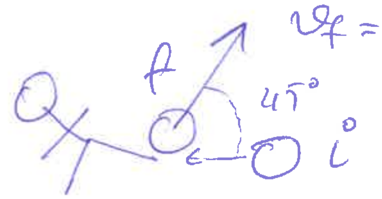
$18.1 \times 10^4 N = F_{av}$

181000 N

1000kg person ⇒ 1000N

(2) Always buckle up"! herzaman kemeri tak!

Ex:



$v_f = 30 m/s$

$v_i = 20 m/s$

$m = 0.4 kg$

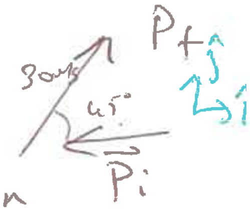
$\Delta t = 0.01 s$

$\vec{J} = ?$

$\vec{F}_{av} = ?$

$\vec{J} = \Delta\vec{P} = \vec{F}_{av}\Delta t$

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i \Rightarrow$



$\vec{P}_f = m v_{fx}\hat{i} + m v_{fy}\hat{j}$

$= 0.4(30\cos 45)\hat{i} + 0.4(30\sin 45)\hat{j}$

$= 12\frac{\sqrt{2}}{2}\hat{i} + 12\frac{\sqrt{2}}{2}\hat{j}$

$\vec{P}_f = 6\sqrt{2}(\hat{i} + \hat{j})$

$\vec{P}_i = 0.4(-20\hat{i}) = -8\hat{i}$

$\vec{P}_f - \vec{P}_i = 6\sqrt{2}\hat{i} + 6\sqrt{2}\hat{j} - (-8\hat{i})$

$\vec{J} = \Delta\vec{P} = (6\sqrt{2} + 8)\hat{i} + 6\sqrt{2}\hat{j}$

$|\vec{J}| = \sqrt{(6\sqrt{2}+8)^2 + (6\sqrt{2})^2}$

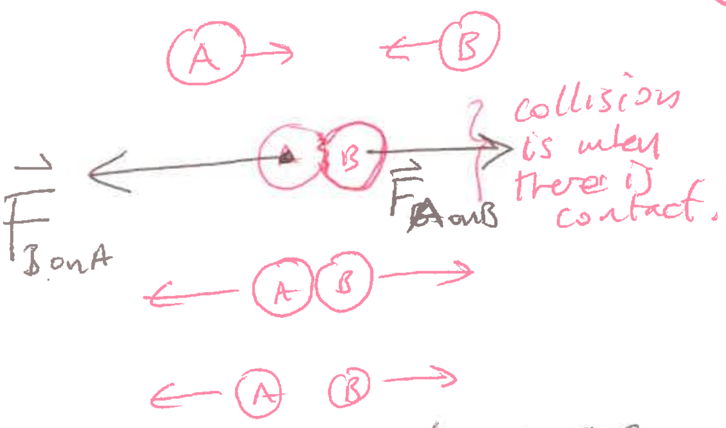
$= 18.5 \frac{kgm}{s}$

$\vec{F}_{av} = \frac{\vec{J}}{\Delta t}$; $|\vec{F}_{av}| = \frac{|\vec{J}|}{\Delta t}$

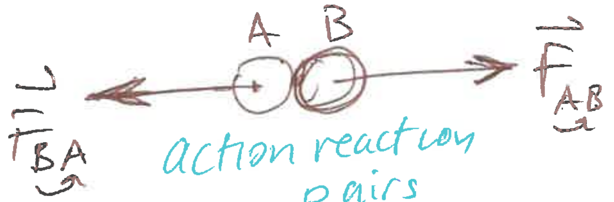
$F_{av} = \frac{18.5}{0.01} = 1850 N$

Conservation of Momentum

③



during collision forces are action reaction pairs; these are ~~coming~~ due to contact collision



$$\vec{F}_{BA} = -\vec{F}_{AB}$$

$$\vec{F}_{BA} + \vec{F}_{AB} = 0 \qquad \vec{F}_{av}\Delta t = \Delta \vec{P}$$

$$\frac{\Delta \vec{P}_A}{\Delta t} + \frac{\Delta \vec{P}_B}{\Delta t} = 0$$

$$\Delta \vec{P}_A + \Delta \vec{P}_B = 0$$

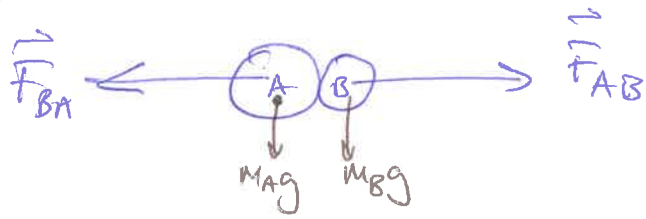
$$\left(\vec{P}_{AF} - \vec{P}_{Ai}\right) + \left(\vec{P}_{BF} - \vec{P}_{Bi}\right) = 0$$

$$\vec{P}_{Ai} + \vec{P}_{Bi} = \vec{P}_{AF} + \vec{P}_{BF}$$

total momentum before collision = total momentum after the collision

Conservation of momentum.

when in air



$F_{BA}; F_{AB} \gg m_Ag, m_Bg$

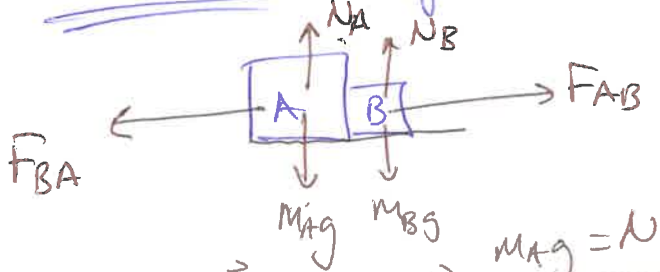
ignore mg forces !!

↳ momentum is conserved

Ex: $F_{BA} \sim 1800N$

$mg \sim 5N \Rightarrow$ ignore !!

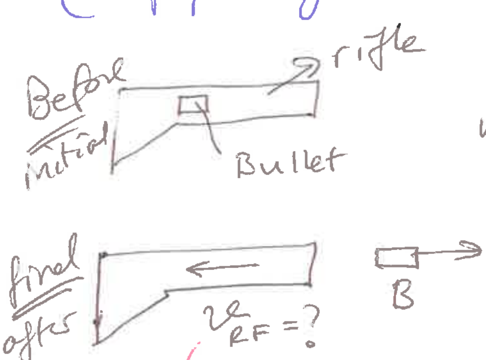
on road traffic accidents



$\Sigma \vec{F} = 0 \Rightarrow m_Ag = N_A$

momentum is conserved !!

Ex: Recoil of a rifle (tüfeğin geri tepmesi)



$M_{Bullet} = 5$ gr

$M_{rifle} = 3$ kg

$v_{BF} = 300 m/s$

recoil velocity?

$\sum P_i = \sum P_f$

initial mom = final mom

$\vec{P}_{Ri} + \vec{P}_{Bi} = \vec{P}_{RF} + \vec{P}_{BF}$

$\square \rightarrow v_{BF}$

$\rightarrow +x$

since

$v_{Bi} = 0; P_{Bi} = 0$

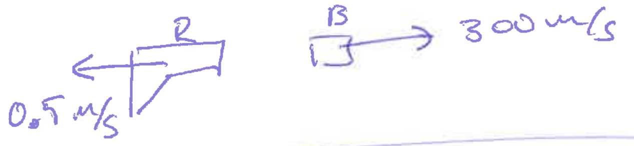
$v_{Ri} = 0; P_{Ri} = 0$

$0 + 0 = m_R \vec{v}_{RF} + m_B \vec{v}_{BF}$

$0 = 3 \vec{v}_{RF} + \underbrace{(0.005)(300\hat{i})}$

$0 = 3\vec{v}_{RF} + 1.5\hat{i}$

$\vec{v}_{RF} = -\frac{1.5}{3}\hat{i} = -0.5\hat{i}$ m/s



(b) what's KE of Bullet?

" " Rifle?

$KE_B = \frac{1}{2} m_B v_{BF}^2 = \frac{0.005}{2}(300)^2$

$KE_R = \frac{1}{2} m_R v_{RF}^2 = \frac{3}{2}(0.5)^2$

$KE_B = 225$ Joules

$KE_R = 0.375$ Joules

Ex: i $\boxed{A} \rightarrow 2$ m/s $\quad \leftarrow 2$ m/s $\boxed{B}$

f $v_{AF} = ?$ $\leftarrow \boxed{A}$ $\quad \boxed{B} \rightarrow 2$ m/s

$m_A = 0.5$ kg $\quad \rightarrow +\hat{i}$

$m_B = 0.3$ kg

$\sum \vec{P}_i = \sum \vec{P}_f$

$m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{AF} + m_B \vec{v}_{BF}$

$(0.5)2\hat{i} + (0.3)(-2\hat{i}) = 0.5\vec{v}_{AF} + 0.3(2\hat{i})$

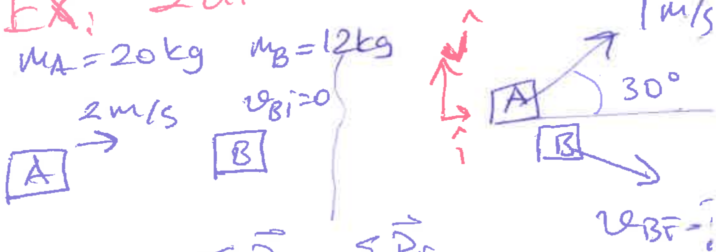
(4) $1\hat{i} - 0.6\hat{i} = 0.5\vec{v}_{AF} + 0.6\hat{i}$

$-0.2\hat{i} = 0.5\vec{v}_{AF}$

$\vec{v}_{AF} = -0.4\hat{i}$ m/s

Ex: 2 dimension collision

$m_A = 20$ kg $\quad m_B = 12$ kg



$\sum \vec{P}_i = \sum \vec{P}_f$

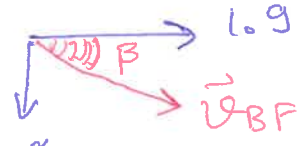
$m_A \vec{v}_{Ai} + m_B \cancel{\vec{v}_{Bi}} = m_A \vec{v}_{AF} + m_B \vec{v}_{BF}$

$\underbrace{20(2\hat{i})}_{40\hat{i}} + 0 = 20(1\cos 30\,\hat{i} + 1\sin 30\,\hat{j}) + 12\vec{v}_{BF}$

$40\hat{i} = \underbrace{20\frac{\sqrt{3}}{2}\hat{i}}_{17.3\hat{i}} + 20\frac{1}{2}\hat{j} + 12\vec{v}_{BF}$

$\vec{v}_{BF} = \frac{22.7\hat{i} - 10\hat{j}}{12}$

$\vec{v}_{BF} = (1.9\hat{i} - 0.8\hat{j})$ m/s



$|\vec{v}_{BF}| = \sqrt{1.9^2 + 0.8^2} = 2.1$ m/s

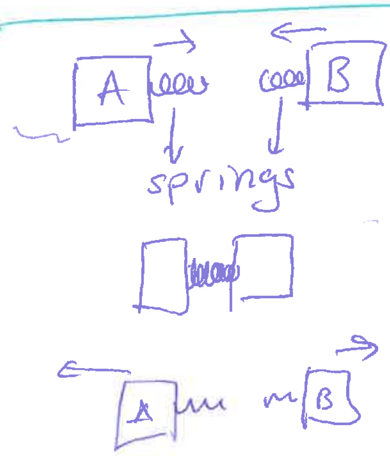
$\tan\beta = \frac{-0.8}{1.9} = -24°$

Elastic & Inelastic Collisions

KE is conserved

KE is NOT conserved.

Elastic Collisions



During collision the KE of the objects are STORED as potential energy of springs!!

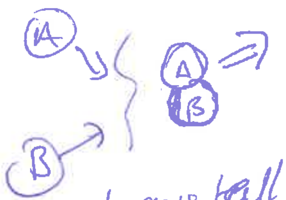
→ at each instant momentum is conserved!

→ KE is conserved

you can not know whether a collision is elastic unless it's told so!!

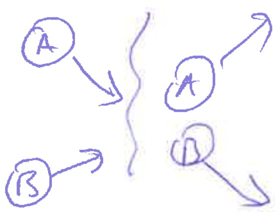
Inelastic collisions

Completely inelastic: objects sticks to each other



can always tell this is inelastic.

inelastic



can be inelastic. they should tell whether it's inelastic.

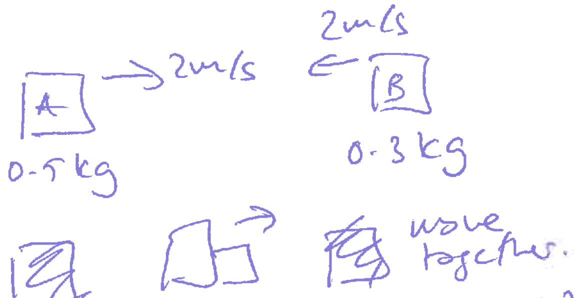
5. Inelastic collisions

we can calculate the % of energy loss.

$K_i > K_F$

$$\frac{\Delta K}{K_i} = \% \text{ of lost energy.}$$

Ex:



$V_F = ?$ % of energy lost?

$$\Sigma \vec{P}_i = \Sigma \vec{P}_F$$

$$m_A \vec{V}_{Ai} + m_B \vec{V}_{Bi} = m_A \vec{V}_{AF} + m_B \vec{V}_{BF}$$

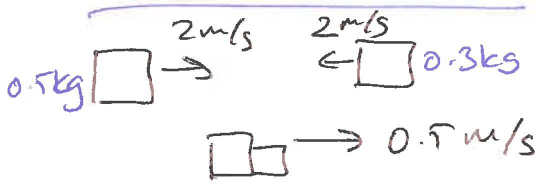
$V_{AF} = V_{BF} = V_F$

stick together

$$0.5(2\hat{i}) + 0.3(-2\hat{i}) = \underbrace{(m_A + m_B)}_{0.8} \vec{V}_F$$

$$0.4\,\hat{i} = 0.8\,\vec{V}_F$$

$$\vec{V}_F = 0.5\,\hat{i} \text{ m/s}$$



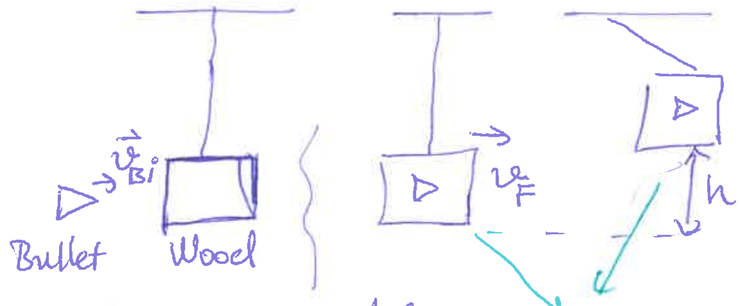
$$K_i = \frac{0.5}{2} 2^2 + \frac{0.3}{2} 2^2 = 1.6\ J$$

$$K_F = \frac{0.8}{2}(0.5)^2 = 0.1\ J$$

$$\% \text{ lost energy} = \frac{1.6 - 0.1}{1.6} = \frac{1.5}{1.6} \Rightarrow 0.94$$

94 % LOST

Ballistic Pendulum (Balistik Sarkaç)



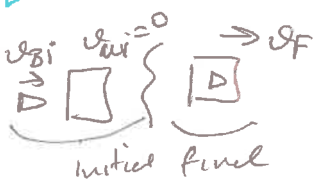
collision

momentum conserved

$E_i = E_F$ mechanical energy conserved.

$\underline{M_B}, \underline{M_W}$ know this ✓

$v_{Bi} = ?$

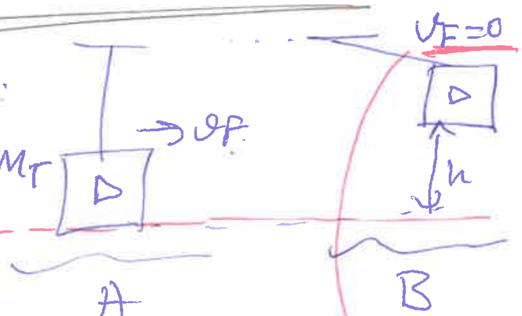


1st $\quad \sum \vec{P}_i = \sum \vec{P}_F$

$$M_B \vec{v}_{Bi} + M_W \cancel{\vec{v}_{wi}}^{\,0} = (M_B + M_W)\, \vec{v}_F$$

$$\vec{V}_F = \frac{M_B}{M_B + M_W} \vec{v}_{Bi}$$

2nd step:



$M_T = M_B + M_W$

$E_A = E_B$

$$K_A + U_A = K_B + U_B$$

$$\frac{1}{2} M_T v_F^2 + M_T g \cancel{y_A}^{\,0} = \frac{1}{2} M_T \cancel{v_B^2}^{\,0} + M_T g \underbrace{y_B}_{h}$$

$$\frac{1}{2} \cancel{M_T} v_F^2 = \cancel{M_T} g h$$

$$v_F = \sqrt{2gh}$$

put it in the ~~fir~~ 1st eqn

$$v_F = \frac{M_B\, v_{Bi}}{(M_B + M_W)}$$

$$v_F = \sqrt{2gh}$$

$$\left(\sqrt{2gh}\right)(M_B + M_W) = M_B\, v_{Bi}$$

$$\boxed{v_{Bi} = \frac{M_B + M_W}{M_B} \sqrt{2gh}}$$

by measuring $\underline{h}$ you can find out the speed of the bullet

Ex:

$M_B = 5\,g \quad M_W = 2\,kg \quad h = 3\,cm$

$$v_{Bi} = \frac{(0.005 + 2)}{(0.005)} \sqrt{2(9.8)(0.03)}$$

$$v_{Bi} = 307\ m/s$$

v_F (speed when bullet/wood move together)

$$V_F = \sqrt{2gh} = 0.76\ m/s$$

% energy lost:

$$K_i > K_F$$

$$K_i = \frac{1}{2} M_B v_{Bi}^2 \;; \quad K_F = \frac{1}{2}(M_B + M_W) v_F^2 = (M_B + M_W) g h$$

$$K_i = 236\,J \qquad K_F = 0.59\,J$$

$$\frac{236 - 0.6}{236} = 0.997$$

99.7% is LOST

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Momentum Conservation – Contd

Elastic Collisions

$\Sigma \vec{P}$ conserved. (always)

$\Sigma K_i = \Sigma K_F$ conserved. (elastic)

Ⓐ $\vec{v}_{Ai}$ Ⓑ $\vec{v}_{Bi}$ } Ⓐ $\vec{v}_{AF}$ Ⓑ $\vec{v}_{BF}$

$\Sigma \vec{P}_i = \Sigma \vec{P}_f$

* $m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{AF} + m_B \vec{v}_{BF}$

$\Sigma K_i = \Sigma K_F$

* $\frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2} m_A v_{AF}^2 + \frac{1}{2} m_B v_{BF}^2$

you don't have to memorize any other equation.

1 dimensional collision

$v_{Bi} = 0$

$v_{Bi} = 0$ $\rightarrow \hat{x}$

Ⓐ → v_{Ai} Ⓑ

Ⓐ → v_{AF} Ⓑ → v_{BF}

$\Sigma \vec{P}_i = \Sigma \vec{P}_F$

$m_A v_{Ai} = m_A v_{AF} + m_B v_{BF}$

$\frac{m_A v_{Ai}^2}{\cancel{2}} = \frac{m_A v_{AF}^2}{\cancel{2}} + \frac{m_B v_{BF}^2}{\cancel{2}}$

$\Sigma K_i = \Sigma K_F$

$m_B v_{BF}^2 = m_A (v_{Ai}^2 - v_{AF}^2)$

$v_{BF} = m_A (v_{Ai} - v_{AF})$

Dividing the eqn's

(1) $v_{BF} = v_{Ai} + v_{AF}$

I know m_A m_B v_{Ai}

I don't know v_{AF}; v_{BF}

$m_B v_{BF} = m_A (v_{Ai} - v_{AF})$ (2)

$m_B (v_{Ai} + v_{AF}) = m_A (v_{Ai} - v_{AF})$

$m_B v_{Ai} + m_A v_{AF} = m_A v_{Ai} - m_B v_{Ai}$

$v_{AF}(m_B + m_A) = (m_A - m_B) v_{Ai}$

$$\boxed{v_{AF} = \frac{m_A - m_B}{m_A + m_B} v_{Ai}}$$

$v_{BF} = v_{Ai} + v_{AF}$

$= \left(1 + \frac{m_A - m_B}{m_A + m_B}\right) v_{Ai}$

$= \frac{m_A + \cancel{m_B} + m_A - \cancel{m_B}}{m_A + m_B} v_{Ai}$

$$\boxed{v_{BF} = \frac{2 m_A}{m_A + m_B} v_{Ai}}$$

$v_{Bi} = 0$

Ⓐ → Ⓑ } v_{AF} v_{BF}

m_A m_B

elastic collision in 1 dimension.

Don't memorize the eqn's in rectangles above!!

1d elastic collision

$m_A >> m_B$

bowling ball   pingpong ball

$$v_{AF} = \frac{m_A - m_B}{m_A + m_B} v_{Ai}$$

if $m_A >> m_B$ 1000g   1g

$$= \frac{999}{1001} v_{Ai}, \quad \frac{999}{1001} \approx 1$$

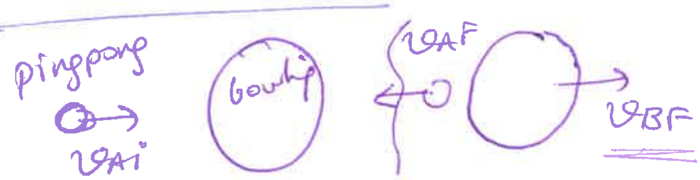
$\boxed{v_{AF} \simeq v_{Ai}}$ (bowling ball velocity is still same)

$$v_{BF} = \frac{2 m_A}{m_A + m_B} v_{Ai}$$

$$v_{BF} \simeq 2 \frac{1000}{1001} v_{Ai}, \quad \frac{1000}{1001} \approx 1$$

$$v_{BF} \simeq 2 v_{Ai}$$

other scenario



$m_A << m_B$

$$v_{AF} = \frac{m_A - m_B}{m_A + m_B} v_{Ai}$$

$$= \frac{1 - 1000}{1001} \cdot v_{Ai}, \quad \frac{1-1000}{1001} \simeq -1$$

$$v_{AF} \simeq -v_{Ai}$$

$$v_{BF} = \frac{2 m_A}{m_A + m_B} v_{Ai}$$

$$v_{BF} = \frac{2(1)}{1001} v_{Ai} \Rightarrow v_{BF} \simeq 0, \quad \frac{2(1)}{1001} \simeq 0$$

② if $m_A = m_B$

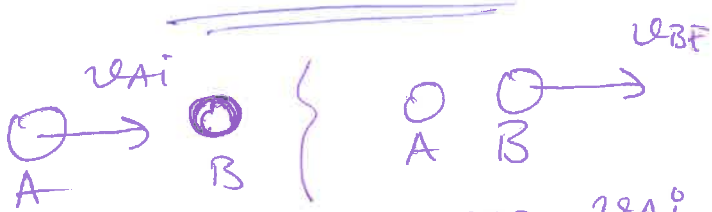
$$v_{AF} = \frac{(m_A - m_B)}{m_A + m_B} v_{Ai}, \quad (m_A - m_B) \to 0$$

$$v_{AF} = 0$$

$$v_{BF} = 2 \frac{m_A}{m_A + m_B} v_{Ai}$$

$$v_{BF} = \frac{2m}{2m} v_{Ai}$$

$$v_{BF} = v_{Ai}$$



$v_{BF} = v_{Ai}$
$v_{AF} = 0$

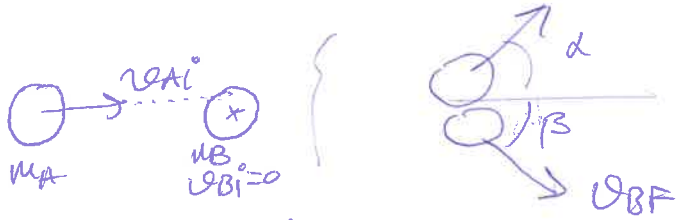
Don't memorize any of these

$$v_{AF} = \frac{m_A - m_B}{m_A + m_B} v_{Ai}$$

$$v_{BF} = \frac{2 m_A}{m_A + m_B} v_{Ai}$$

TRUE when $v_{Bi} = 0$ and Elastic 1 dimensional collision

2 dimensional collision
elastic



m_A, m_B, v_{Ai} given + v_{AF} given

$m_A = 0.5\,kg;\ m_B = 0.3\,kg$

$v_{Ai} = 4\,m/s \qquad v_{AF} = 4\,m/s$

$$\sum \vec{P_i} = \sum \vec{P_F}$$

$$m_A \vec{v_{Ai}} = m_A \vec{v_{AF}} + m_B \vec{v_{BF}}$$

$$(0.5)\,4\hat{i} = 0.5\,(4\cos\alpha)\,\hat{i} + (0.5)\,4\sin\alpha\,\hat{j} + 0.3\,v_{BF}\cos\beta\,\hat{i} + 0.3\,v_{BF}\sin\beta\,(-\hat{j})$$

$(\hat{i})$ ① $2\hat{i} = 2\cos\alpha\,\hat{i} + 0.3\,v_{BF}\cos\beta\,\hat{i}$

$(\hat{j})$ ② $0 = 2\sin\alpha\,\hat{j} - 0.3\,v_{BF}\sin\beta\,\hat{j}$

① $2 = 2\cos\alpha + 0.3\,v_{BF}\cos\beta$

② $2\sin\alpha = 0.3\,v_{BF}\sin\beta$

$$\sum K_i = \sum K_F$$

$$\cancel{\tfrac{1}{2}}\,m_A v_{Ai}^2 = \cancel{\tfrac{1}{2}}\,m_A v_{AF}^2 + \cancel{\tfrac{1}{2}}\,m_B v_{BF}^2$$

$$(0.5)(4^2) = (0.5)(2^2) + 0.3\,v_{BF}^2$$

$$\sqrt{\frac{6}{0.3}} = v_{BF} = \sqrt{20} = 4.47\,m/s$$

③ $2 = 2\cos\alpha + 0.3\sqrt{20}\cos\beta$

$2\sin\alpha = 0.3\sqrt{20}\sin\beta$

$$\cos\beta = \frac{2 - 2\cos\alpha}{0.3\sqrt{20}}$$

$$\sin\beta = \frac{2\sin\alpha}{0.3\sqrt{20}}$$

$$\cos^2\beta + \sin^2\beta = 1$$

$$\frac{(2-2\cos\alpha)^2}{0.3^2\,20} + \frac{4\sin^2\alpha}{0.3^2\,20} = 1$$

$$\frac{4 + 4\cos^2\alpha - 8\cos\alpha + 4\sin^2\alpha}{(0.3)^2\,20} = 1$$

4

$$4 + 4 - 8\cos\alpha = \underbrace{(0.3)^2\,20}_{0.09\times 20 = 1.8}$$

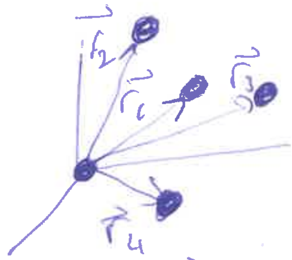
$$\cos\alpha = \frac{8 - 1.8}{8}$$

$\alpha = 39°$ $\qquad \beta = 70°$

$\alpha = 39.19°$ $\qquad \beta = 70.40°$

Center of Mass

④



4 point mass
$\vec{r}_i$; 3d vector

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

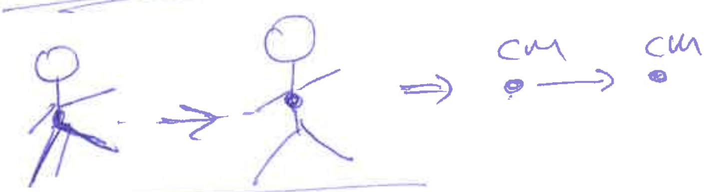
$$\vec{r}_{cm} = \frac{\vec{r}_1 m_1 + \vec{r}_2 m_2 + \vec{r}_3 m_3 + \vec{r}_4 m_4}{m_1 + m_2 + m_3 + m_4}$$

$$\vec{r}_{cm} = X_{cm}\hat{i} + Y_{cm}\hat{j} + Z_{cm}\hat{k}$$

$$X_{cm} = \frac{m_1 X_1 + m_2 X_2 + m_3 X_3 + m_4 X_4}{m_1 + m_2 + m_3 + m_4}$$

$$X_{cm} = \frac{\sum m_i x_i}{\sum m_i} \; ; \; Y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$

motion of center of mass



$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$\frac{d}{dt} \downarrow$

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i} \Rightarrow \text{** momentum conservation}$$

$\frac{d}{dt} \downarrow$

$$a_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$$

$$\sum m_i = M$$

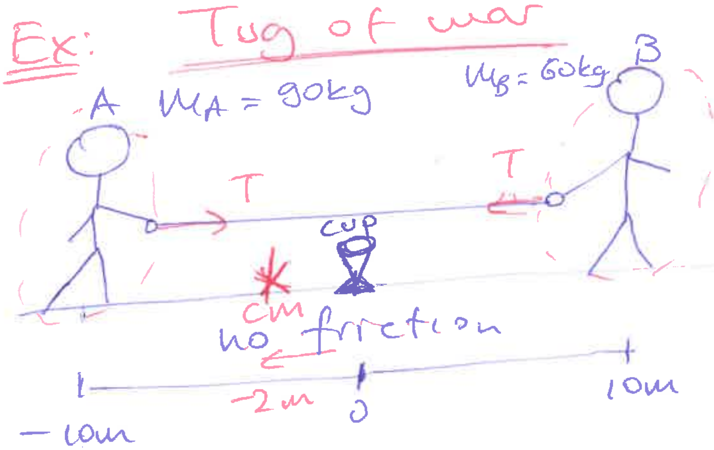
$$M \vec{v}_{cm} = \underbrace{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots}_{\text{momenta of individual partic}}$$

total mass ↓ M; CM velocity ↓ $\vec{v}_{cm}$

2 body system

$$\underset{(M_s + M_p)}{M} \vec{v}_{cm} = M_s \vec{v}_s + M_p \vec{v}_p$$

Ex: Tug of war



who reaches to the cup first ?

$$X_{cm} = \frac{M_A X_A + M_B X_B}{M_A + M_B} = \frac{90(-10) + 60(10)}{90 + 60}$$

$$X_{cm} = \frac{-300}{150} = -2m$$

When A moves 6m to the cup; how much B moves?

Ex: The rocket ejects (6) fuel in the 1st second it ejects $\frac{1}{120}$ of its initial mass. at a speed of 2400 m/s. What's rocket's initial acceleration?

$$M\,dv = -v_{ex}\,dM$$

$$M\frac{dv}{dt} = -v_{ex}\frac{dM}{dt}$$

$\vec{a}$

$$a = -\frac{v_{ex}}{M}\frac{dM}{dt}$$

$$\frac{dM}{dt} = \frac{M_0/120}{1s}\ ; \left[\frac{kg}{s}\right]$$

$$a = \frac{v_{ex}\ M_0/120}{M_0}$$

$$a = \frac{2400}{120} = 20\,m/s^2$$

(b) $\frac{3}{4}$ of initial mass of the rocket is fuel. it's consumed at a constant rate in 90s.

Final mass = $\frac{M_0}{4}$

if; $v_i = 0 = v_0$ $\quad v_F = ?$

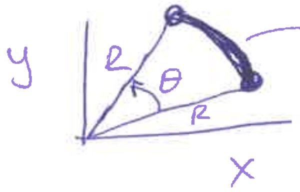
$$\boxed{v_F - v_i = v_{ex} \ln\frac{M_0}{M}}\ \ M \to M_0/4$$

$$v_F - 0 = 2400 \ln\frac{M_0}{M_0/4}$$

$$v_F = 2400 \ln 4 = 3327\,m/s$$

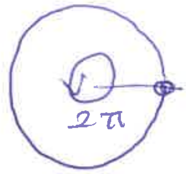
Chapter 9

Rotation of Rigid Bodies



→ length of the arc (path)

$$S = R\theta$$



$$\underbrace{2\pi}_{\theta}\ \underbrace{R}_{R} = S$$

Radians	⟺ Degree
0	0
$\pi/2$	90
π	180
2π	360

$\pi = 3.14$

$90° \to \frac{\pi}{2} \to \frac{3.14}{2}$ rad

$180 \to \pi \to 3.14$ rad

$?° \quad \frac{180°}{\pi} = \quad 1$ rad

$\frac{180}{3.14} = 57.3° \to 1$ rad

$\theta \to$ rad (angular displacement) rotation

previously

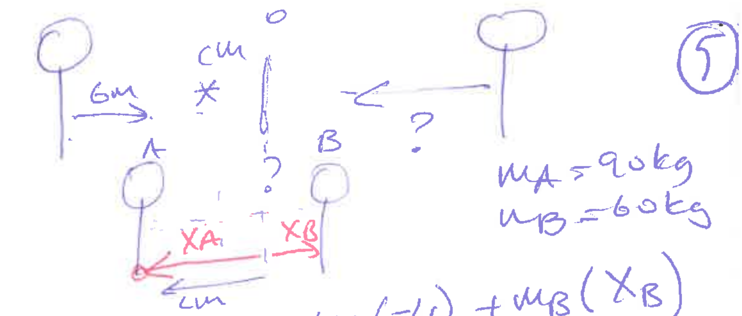
$X \to$ displacement $\leftarrow \theta$

$\frac{dx}{dt}$ $v \to$ velocity $\leftarrow \omega$ $\quad d/dt$

d/dt

$a \to$ acceleration. $\leftarrow \alpha$ $\quad d/dt$

(5)



$m_A = 90kg$
$m_B = 60kg$

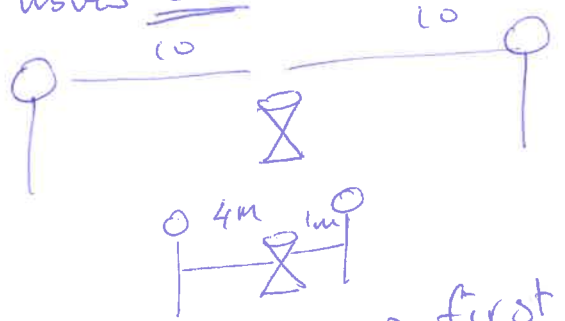
$$X_{cm} = -2m = \frac{m_A(-4) + m_B(X_B)}{m_A + m_B}$$

$$(-2)(150) = 90(-4) + 60X_B$$

$$-300 + 360 = 60 X_B$$

$$1m = \frac{60}{60} = X_B \quad (1m \text{ from origin})$$

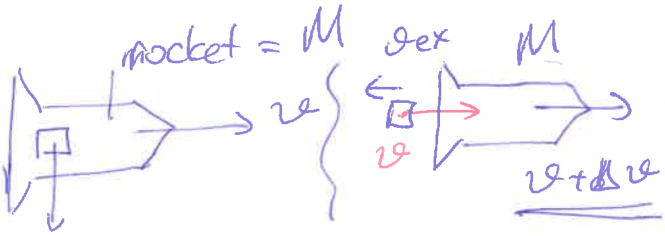
A moves 6m
B moves 9m



B gets the cup first

Rocket Propulsion

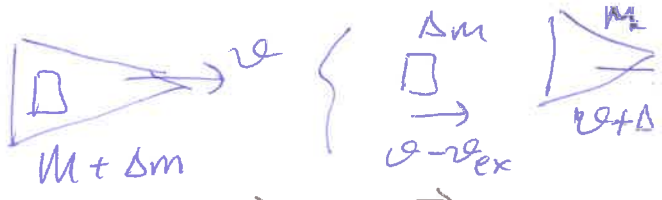
to move in space ; rocket has to exhaust fuel.



v_{ex}: exhaust velocity with respect to the rocket.

fuel

v_{ex} ← □ → v velocity of fuel $(v - v_{ex})$



$$\sum \vec{P}_i = \sum \vec{P}_F$$

$$(M + \Delta m) v = M(v + \Delta v) + \Delta m (v - v_{ex})$$

$$\cancel{Mv} + \Delta m v = \cancel{Mv} + M\Delta v + \Delta m v - \Delta m v_{ex}$$

$$\Delta m v_{ex} = M \Delta v$$

* important relation between $\underline{M}$ and $\underline{\Delta m}$
rocket fuel

$$\text{Total mass} = (M + \Delta m) = \text{const}$$

$$\Delta m = -\Delta M$$
fuel rocket

$$-\Delta M \leftarrow \Delta m \; v_{ex} = + M\Delta v$$

$$-\Delta M v_{ex} = M \Delta v$$

$$-dM v_{ex} = M dv$$

$$\int dv = \int -\frac{dM}{M} v_{ex}$$

$$[\Delta v]_{v_i = v_0}^{v_F} = -v_{ex} [\ln M]_{M_i}^{M_F}$$

M_i = total mass = M_0 at the beginning.

$$v_F - v_0 = -v_{ex} \ln \frac{M_F}{M_0}$$

$$\boxed{v_F - v_0 = v_{ex} \ln \frac{M_0}{M}}$$

Angular Velocity (ω) omega

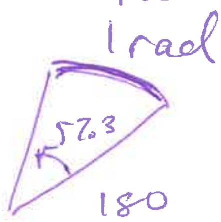
$$\omega = \frac{d\theta}{dt} \quad \text{(instantaneous ang. velocity)}$$

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \quad \text{(average ang. velocity)}$$

$$\underline{\underline{\omega = \left[\frac{rad}{sec}\right]}} = \left[\frac{revolution}{second}\right] \neq \left[\frac{rev}{min}\right]$$

revolution = rotation $\Rightarrow$ $\underline{\underline{2\pi}}$ rad

rpm = $\frac{rotation}{minute}$



$\frac{180}{\pi}$

$$1\ \frac{rad}{s} = \frac{180^\circ}{\pi}\frac{1}{s}$$

$$= \frac{360^\circ}{2\pi}\frac{1}{s}$$

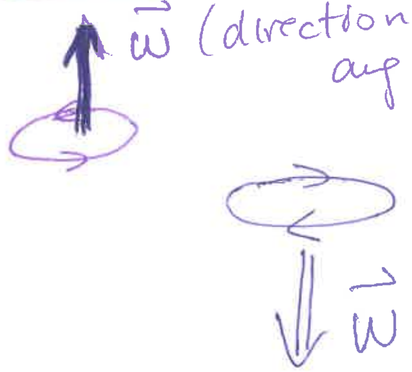
$$1\ \frac{rad}{s} = \frac{1\ rotation}{2\pi}\frac{1}{s}$$

$$= \frac{1\ rotation}{2\pi}\frac{1}{\frac{1}{60}min}$$

$$= \frac{60}{2\pi}\frac{rot}{min}$$

$$1\ rad/s = \frac{60}{2\times 3.14} \simeq 10rpm$$

$\vec{\omega}$ (direction of ang. velocity)



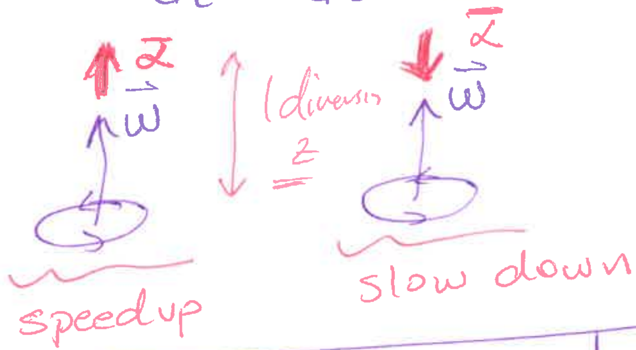
7 Angular Acceleration

α (alpha)

$$\alpha = \frac{d\omega}{dt} \quad \text{(instantaneous ang. accelerat.)}$$

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} \quad \text{(average angular acceleratn.)}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad [rad/s^2]$$



Equations α = const.

previously

$$v_F = v_i + at$$

$$\boxed{\omega_F = \omega_i + \alpha t} \rightarrow \text{new}$$

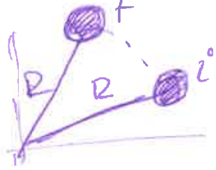
$$x_F = x_i + v_i t + \frac{1}{2}at^2$$

$$\boxed{\theta_F = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2} \rightarrow \text{new}$$

$$v_F^2 = v_i^2 + 2a(x_F - x_i)$$

$$\boxed{\omega_f^2 = \omega_i^2 + 2\alpha(\theta_F - \theta_i)} \rightarrow$$

Relation between linear motion and angular motion



$\vec{v}_i$ related to $\underline{\omega, R}$

$\underbrace{S}_{\text{linear path}} = R\,\underbrace{\theta}_{\text{ang path}}$ $\quad \underline{R = \text{const.}}$ 8

$$\frac{ds}{dt} = \frac{d}{dt}(R\theta)$$

$$\underbrace{\frac{ds}{dt}}_{v} = R\,\underbrace{\frac{d\theta}{dt}}_{\omega}$$

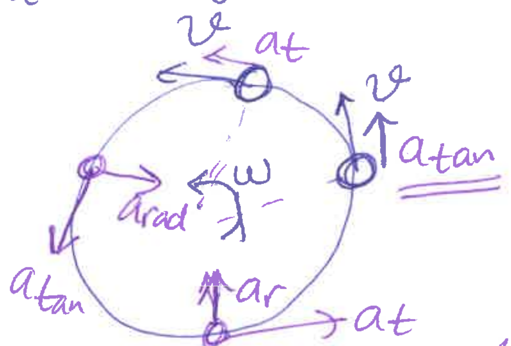
$$\boxed{v = R\omega}$$

d/dt

$$\boxed{a_t = R\alpha}$$

linear acceleration
it's not radial acceleration

$a_t = a_{tangential}.$



$$\underline{a_t} = R\alpha \neq \underline{a_{rad}} = \frac{v^2}{R}$$

Phys I : 1.8.17

rotational motion

(1)

θ : rotational displacement

$\boxed{\theta} \xrightarrow{d/dt} \boxed{\omega} \xrightarrow{d/dt} \boxed{\alpha}$

displacement — rot. velocity — rot. acceleration

$X \longrightarrow v \longrightarrow a$

linear motion

$X = S = R\theta$

$v = R\omega$

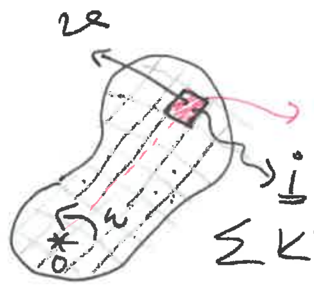
$a_t = R\alpha$

$\to$ tangential acceleration

$$\boxed{\vec{a} = \vec{a}_t + \vec{a}_r}$$

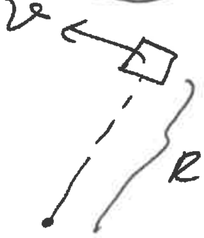
$a_t = \alpha R$; $a_r = \dfrac{v^2}{R}$

ENERGY in ROTATIONAL MOTION



$KE = \frac{1}{2} m v^2$

$\sum KE = \sum \frac{1}{2} m_i v_i^2$



$v = \omega R$

$v_i = \omega R_i$ — const. for the body.

$$KE = \sum \frac{1}{2} m_i \omega^2 R_i^2$$

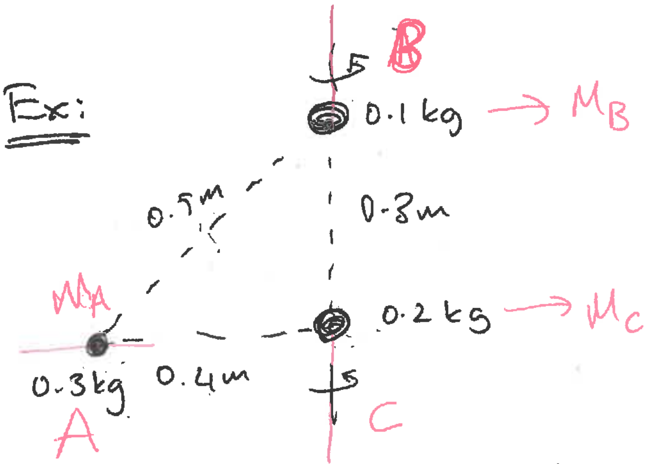
$$= \omega^2 \frac{1}{2} \boxed{\sum m_i R_i^2}$$

I ← moment of inertia ⇓ (inertia of rotating object)

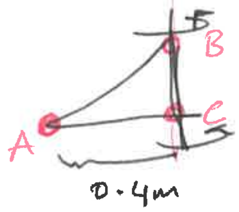
$$\boxed{I = \sum m_i R_i^2}$$

$$KE = \frac{1}{2} \underline{\underline{I}} \omega^2 \simeq \frac{1}{2} m \underset{R^2\omega^2}{v^2}$$

Ex:



(a) I ; if it rotates along $\underline{\underline{BC}}$



$I = \sum m_i R_i^2$

R : distance from obj to rotating axis.

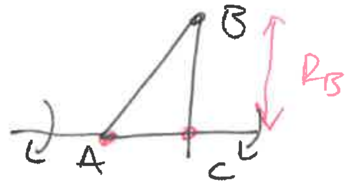
$I = m_A R_A^2 + m_B R_B^2 + m_C R_C^2$

$R_B = R_C = 0$ along rot. $\underline{\underline{BC}}$

$I_{BC} = m_A R_A^2 = 0.3\,(0.4)^2\ kgm^2$

$= 0.048\ kgm^2$

(b) I if it rotates along $\underline{\underline{AC}}$ axis

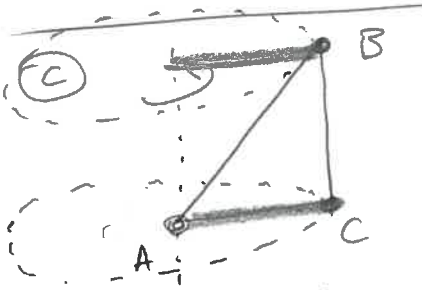


$$I_{AC} = m_A R_A^2 + m_B R_B^2 + m_C R_C^2$$

$R_A = 0$ $\qquad R_C = 0$

$$I_{AC} = m_B R_B^2 = 0.1(0.3)^2$$
$$= 0.009\ kgm^2$$

(c)



$I_A = ?$

$R_C = R_B = 0.4m$

$$\underline{\underline{I_A}} = \cancel{m_A R_A^2}^{\,0} + m_B R_B^2 + m_C R_C^2$$
$$= 0.1\ (0.4)^2 + 0.2\ (0.4)^2$$
$$I_A = 0.3\ (0.4)^2 = 0.048\ kgm^2$$

(d) $I_A = 0.048\ kgm^2$

$w = 4\ rad/s$

$KE = ? \quad = \frac{1}{2} I w^2 = K$

$$\frac{1}{2}\ (0.048)\ 4^2 = 0.46\ J$$

$$kgm^2 \frac{rad^2}{s^2}$$ rad^2 → birimsiz unitless

---

the meaniŋ of $\underline{\underline{I}}$

inertia ~ mass

$I_1 = 0.009\ kgm^2$

$I_2 = 0.048\ kgm^2$

it is EASIER to rotate $\underline{\underline{I_1}}$

I depends on the $\underline{\underline{AXIS}}$

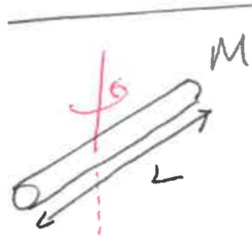
$$I = \sum \frac{1}{2} m_i \underline{\underline{R_i}}^2$$
↓
depends on axis

3d geometric objects

$$I = \sum m_i R_i^2$$
↓ continuous
$$I = \int dm\, R^2$$

---

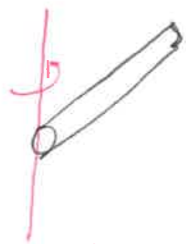
from this relation ROD, sphere, DISK CYLINDER, PLATE can be calculated.



ROD

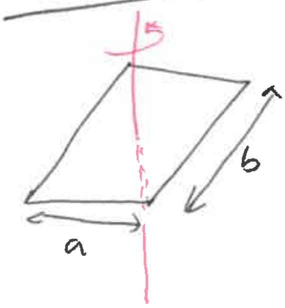
axis through center

$$I = \frac{ML^2}{12}$$



axis through one end

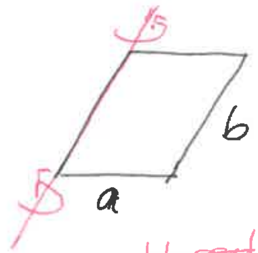
$$I = \frac{1}{3} ML^2$$



PLATE

axis through center

$$I = \frac{1}{12} M(a^2+b^2)$$



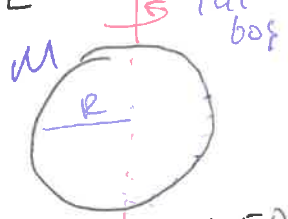
axis ~~through~~ along edge

$$I = \frac{1}{3} Ma^2$$

SPHERE

FULL SPHERE

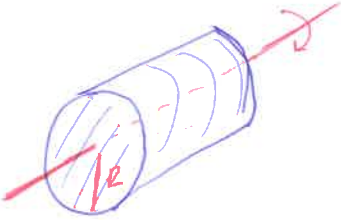
$$I = \frac{2}{5} MR^2$$



THIN WALLED HOLLOW (SHELL) SPHERE

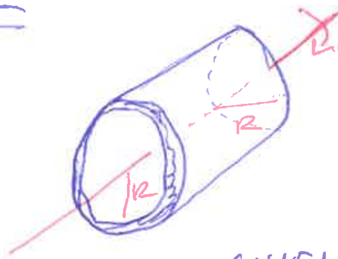
$$I = \frac{2}{3} MR^2$$

CYLINDER



FULL (dolu)

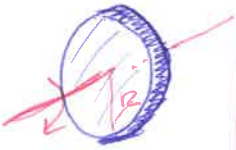
$I = \frac{1}{2} MR^2$



HOLLOW (SHELL) içi boş

$I = MR^2$

↓ DISC



$I = \frac{1}{2} MR^2$

* makara dönen

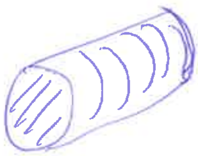
HOLLOW CYLINDER kısmi boş/dolu

R_2

R_1

$I = \frac{1}{2} M (R_1^2 + R_2^2)$

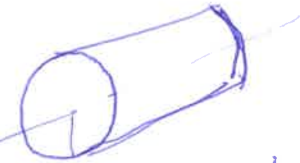
*



$R_1 = 0$

$R_2 = R$

$I = \frac{1}{2} M (0 + R^2) = \frac{MR^2}{2}$

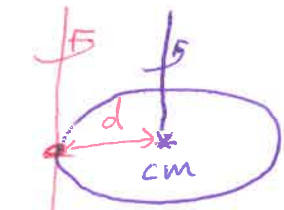


$R_1 = R_2 = R$

(empty) cylinder

$\frac{1}{2} M (R^2 + R^2) = \underline{MR^2}$ shell cylinder

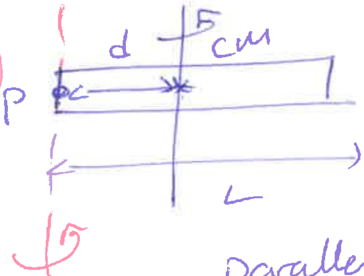
PARALLEL AXIS THEOREM



P — Center of mass

I_{cm} axis thru cm

$I_p = I_{cm} + Md^2$



$I_{cm} = \frac{ML^2}{12}$

$I_p = ?$

parallel axis theorem

$I_p = I_{cm} + Md^2$

d: distance between P and CM

$d = \frac{L}{2}$ $\quad I_p = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$

$I_p = ML^2\left(\frac{1}{12} + \frac{1}{4}\right) = ML^2\left(\frac{1+3}{12}\right)$

$I_p = \frac{ML^2}{3}$ $\quad I_{cm} = \frac{ML^2}{12}$

Ch 9 yol θ hız ω ivme α $\longleftrightarrow$ $x \to$ yol, $v \to$ hız, a_t ivme

* $I = \sum m_i R_i^2$

$v = \omega R$

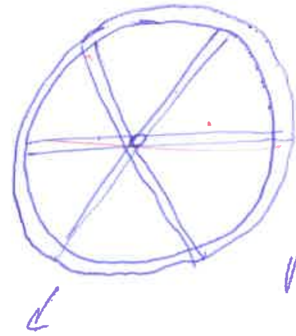
$x = \theta R$

$a_t = \alpha R$

objects: rod, sphere, cylinder, plate — I will be given

9-33

wagon wheel



at arabası tekerleği

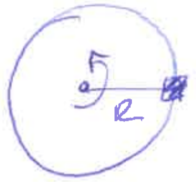
$I = ?$

M

$I_{rod} = \frac{ML^2}{12}$

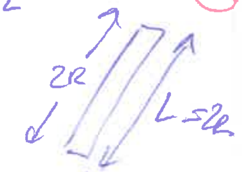
$M_1 + M_2 + M_2 + M_2$

(4)



$$I = M_1 R^2$$

$$I_{ROD} = \frac{M L^2}{12}$$

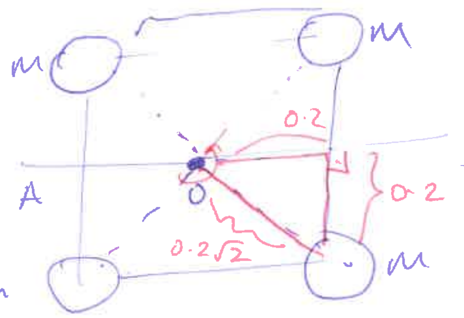


$$I_{wheel} = M_1 R^2 + 3\left(\frac{M_2 (2R)^2}{12}\right)$$

$$I_{total} = M_1 R^2 + M_2 R^2$$

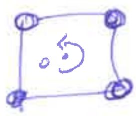
SQUARE

9.28



$m = 0.2\,kg$

$I_0 = ?$



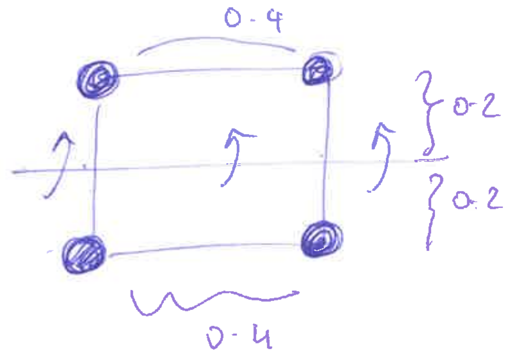
$$I_0 = \sum m_i R_i^2$$

$$= 4m(0.2\sqrt{2})^2$$

$$4m(0.04)2$$

$$I_0 = 0.32m = 0.064\,kgm^2$$

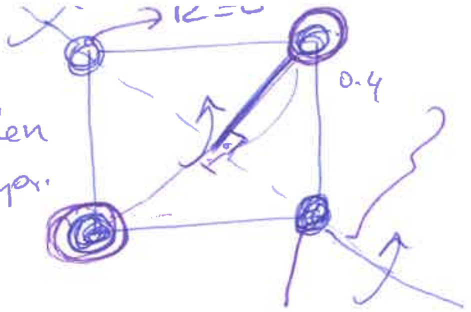
(b) around axis AB



$$I_{AB} = 4m(0.2)^2$$

$$= 0.16m = 0.032\,kgm^2$$

(c) köşegenden dönüyor.

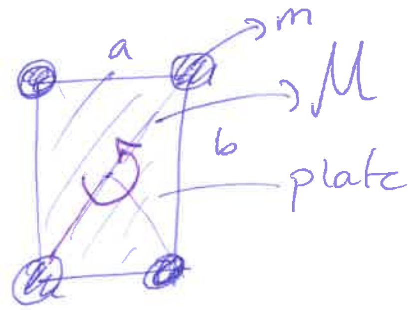


$$2m(0.2\sqrt{2})^2$$

$$(0.04)4m$$

$$0.16m = 0.032\,kgm^2$$

Ex:



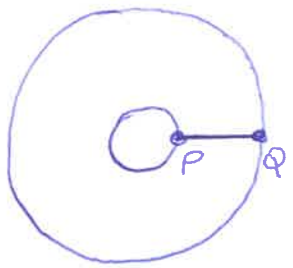
$$I = I_{plate} + I_{4\ balls}$$

$$I_{plate} = \frac{1}{12} M(a^2+b^2)$$

$$R = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$I = \frac{M}{12}(a^2+b^2) + 4\left[m\left(\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2\right)\right]$$

Ex: DISC CD is slowing down to stop.



at $t = 0s$

$\omega = 27.5\,rad/s$

$\alpha = -10\,rad/s^2$

PQ lies along X axis at $t = 0s$.

a) $w_f = ?$ at $t = 0.3s$

$$w_f = w_i + \alpha t$$

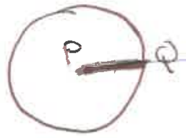
(like $v_f = v_i + at$)

$$= 27.5 - 10(0.3)$$

$$w_f = 24.5 \text{ rad/s}$$

final angular velocity

b) what's the angle of PQ lie at $t = 0.3$ s ? θ_f



$\theta_i = 0$

like $x_F = x_i + v_i t + \frac{1}{2} a t^2$

$t = 0s$

$w_i = 27.5$ rad/s

$\alpha = -10$ rad/s^2

$$\theta_F = \theta_i + w_i t + \frac{1}{2} \alpha t^2$$

$$= 0 + 27.5(0.3) + \frac{1}{2}(-10)\, 0.3^2$$

$$\theta_F = 7.8 \text{ rad}$$

π rad $= 180°$

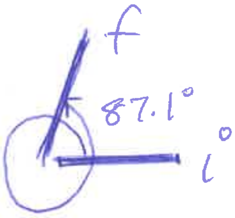
(3.14) rad $= 180°$

$1 \text{ rad} = \frac{180°}{3.14}$

$1 \text{ rad} = 57.3°$

$7.8 \text{ rad} = 447.1°$

$f \quad = 360 + 87.1°$



1 revolution rotation $= 2\pi$ rad

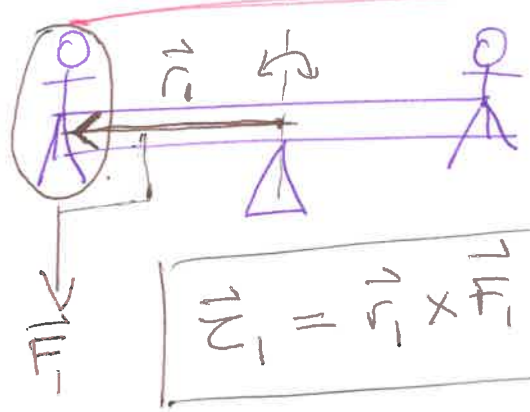
1 rev $= 6.28$ rad

1 rev/6.28 $=$ 1 rad

Chapter 10

Dynamics of Rotational Motion

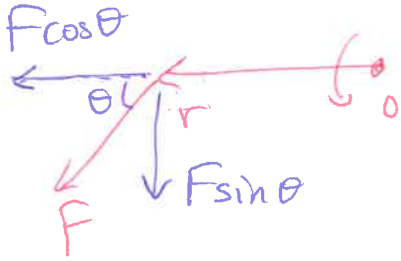
TORQUE



$$\boxed{\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1}$$

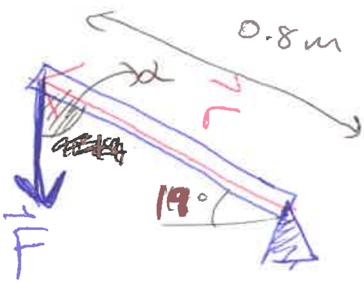
$$|\vec{\tau}| = F_1 r_1 \sin\theta$$

Tork: bir kuvvetin cismi dondurebilme kapasitesi



$$|\vec{r} \times \vec{F}| = Fr\sin\theta = (F\sin\theta)\, r$$

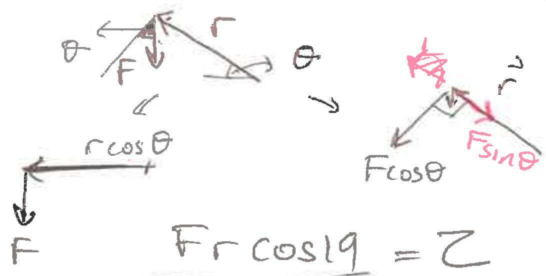
Ex:



$\vec{\tau} = \vec{r} \times \vec{F}$
$= Fr\sin\alpha$

$|\vec{F}| = 900N$

$|\vec{\tau}| = ?$ direction of $\tau = ?$



$$Fr\cos 19 = \tau$$

$\tau = Fr \cos 19$

$= 900(0.8)\cos 19$

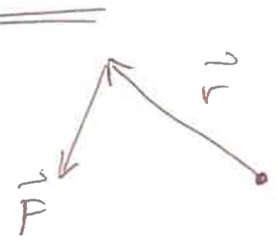
$= 680 \ N\cdot m$

$=$

$\tau = [N.m]$ unit of torque

not joule !

direction ?



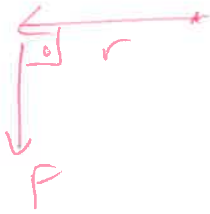
$\vec{r} \times \vec{F}$

$\vec{\tau}$ is out of page !

$\theta = 90^\circ$

$|\vec{\tau}| = Fr = 1$

R



$|\vec{\tau}| = \tau = FR$

$\downarrow$

ma

$\downarrow$

αR

$\tau = m\alpha RR$

$\tau = mR^2 \alpha$

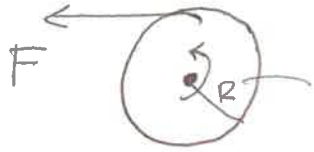
I = moment of inertia

$\boxed{\vec{\tau} = I\vec{\alpha}}$

like

$\boxed{\vec{F} = m\vec{a}}$

ⓑ Ex:



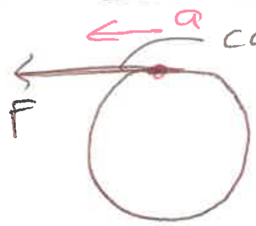
$I; M, R, F$ given

$\alpha = ?$

$\vec{\tau} = \vec{R} \times \vec{F} =$

$\tau = FR = I\alpha$

$\alpha = \frac{FR}{I}$



cable ; what's cable's acceleration

$a_t = ?$

$a_t = \alpha R$

$= \cancel{\alpha}$ $a_t = \frac{FR}{I} R$

$\boxed{a_t = \frac{FR^2}{I}}$

$F = 900 N$ $R = 0.06 \ m$ $M = 50 kg$

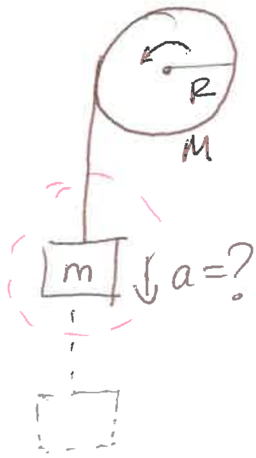
$I = \frac{MR^2}{2} = \frac{50(0.06)^2}{2}$

$\alpha = \frac{FR}{I}$ ✓ $=$

$a = \alpha R$ ✓

Ex:

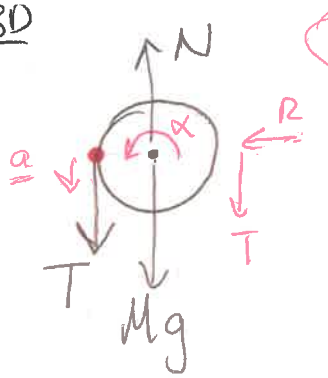
$g \downarrow$



$I = \frac{MR^2}{2}$

FBD

(7)



$\Sigma F = ma_y$

$$\boxed{mg - T = ma}$$

$\tau = I\alpha$

$TR = \frac{MR^2\alpha}{2}$

$T = \frac{MR\alpha}{2}$

$T = m\frac{a}{2}$

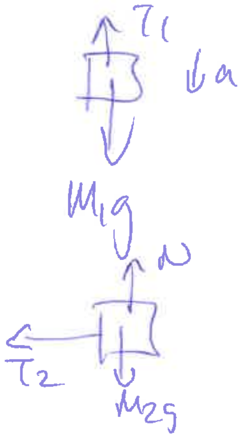
$mg - \frac{ma}{2} = ma$

$mg = \frac{3ma}{2}$

$$\boxed{a = 2g/3}$$

$\alpha = \frac{a}{R}$

$$\boxed{\alpha = \frac{2g}{3R}}$$

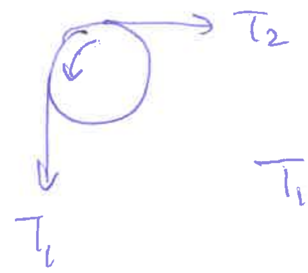


$\Sigma F = ma_y$

$$\boxed{M_1 g - T_1 = m_1 a}$$

$\Sigma F_y = N - M_2 g = 0$

$\Sigma F_x = \boxed{T_2 = M_2 a}$



$\Sigma \tau = I\alpha$

$T_1 R - T_2 R = I\alpha$

$T_1 \cancel{R} - T_2 \cancel{R} = \frac{M_p R^{\cancel{2}} \alpha}{2}$

$\boxed{R\alpha = a}$

$$\boxed{T_1 - T_2 = \frac{M_p a}{2}} \quad (3)$$

$$\boxed{M_1 g - T_1 = M_1 a} \quad (1)$$

$$\boxed{T_2 = M_2 a} \quad (2)$$

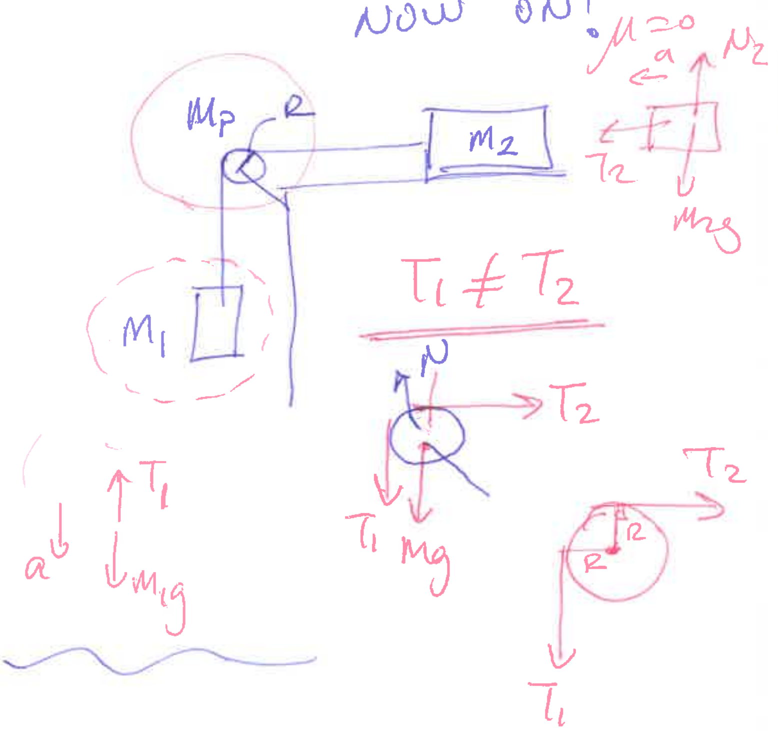
(1) + (2) + (3)

$M_1 g = M_2 a + M_1 a + \frac{M_p a}{2}$

$$\boxed{a = \frac{M_1 g}{\left(M_2 + M_1 + \frac{M_p}{2}\right)}}$$

Remember:
$v = \omega R$
$a = \alpha R$
$x = \theta R$

Pulleys are Rotating From NOW ON!



$T_2 = M_2 a$ ✓

$T_1 = mg - M_1 a$ ✓

$\alpha = \frac{a}{R}$ ✓

PHYS I 3.8.17 (1)

Continue on Rotation

θ (displacement = rotation)

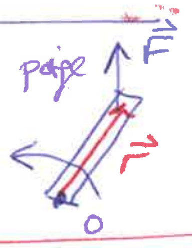
$\omega \rightarrow$ rot. ~~speed~~ / velocity (angular)

$\alpha \rightarrow$ rot. acceleration (angular)

$x = \theta R$	TORQUE
$v = \omega R$	$\vec{\tau} = \vec{r} \times \vec{F}$
$a_t = \alpha R$	capacity to rotate

$\vec{r} \times \vec{F}$ out ~~into~~ the page

$\vec{\tau}$ ⊙



$I \equiv$ moment of inertia (inertia related to rotation)

Different objects have different I values.

$I = C\,MR^2 \longrightarrow 0 \le c \le 1$

sphere $\frac{2}{5} MR^2$;

$$\boxed{I = \sum m_i r_i^2}$$

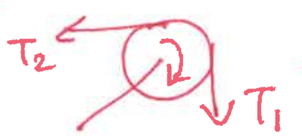
$I \uparrow$; harder to rotate.

$$\boxed{\vec{\tau} = I\vec{\alpha}}$$

torque = (mom. inertia)(alpha)

$$\boxed{\vec{F} = m\vec{a}}$$

when pulleys are rotating

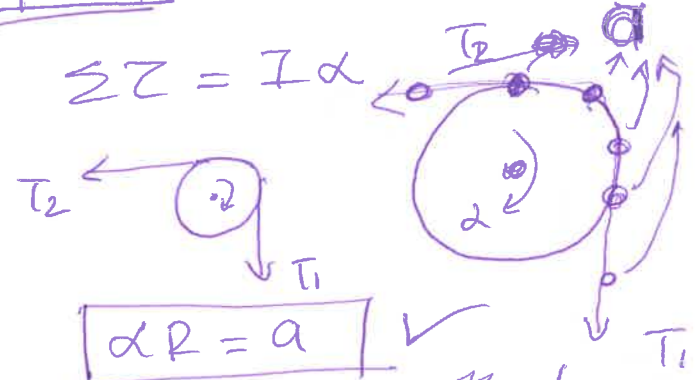


$\underline{\underline{T_1 > T_2}}$

$(T_1 R - T_2 R) = \sum \tau$

pulley (makara)

$\sum \tau = I\alpha$

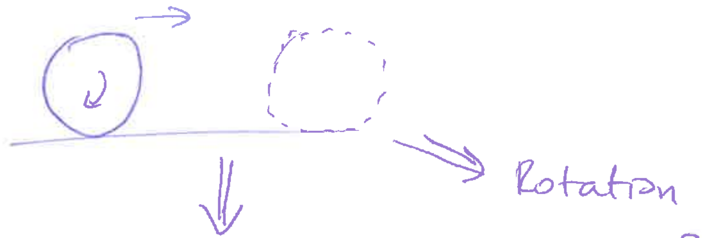


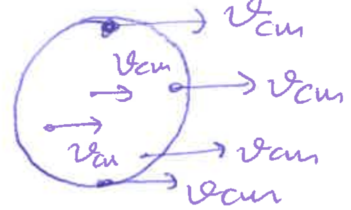
$$\boxed{\alpha R = a}$$ ✓

for rotating without slipping

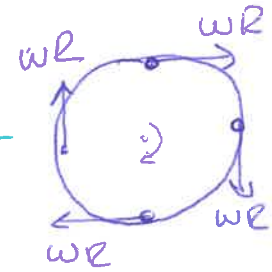
(kaymadan donmesi için $\alpha R = a$ olmalı)

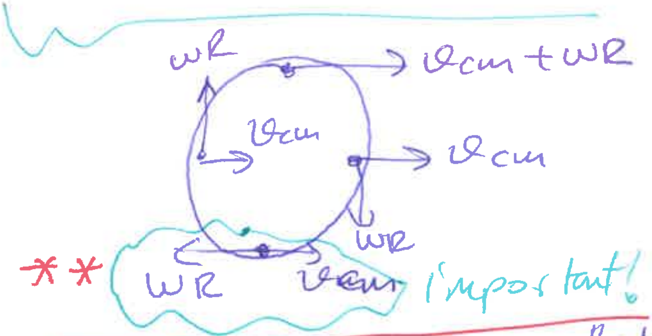
Combined Translation + Rotation





+



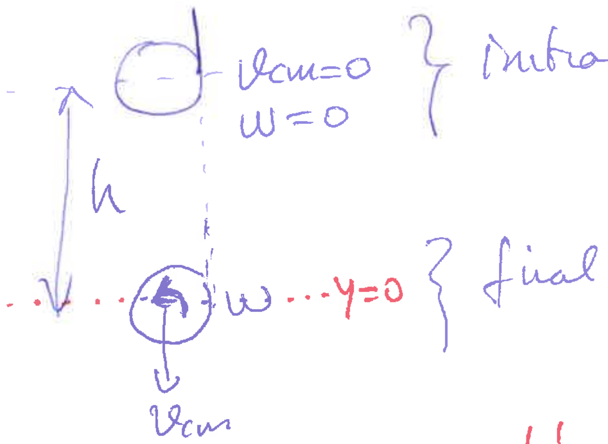


$$\boxed{v_{cm} = \omega R \Rightarrow \text{rotating without slipping}}$$

(kaymadan dönme şartı)

Total Kinetic Energy $K = \frac{1}{2} m v_{cm}^2 + \underbrace{\frac{1}{2} I \omega^2}_{\text{rotational kinetic energy}}$

Ex: <u>YO-YO</u>



Energy conserved!

$E_i = E_f$

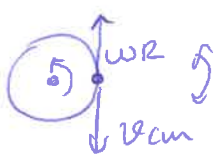
$U_i + K_i = U_f + K_f$

$mgh + 0 = mg0 + \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$

$I = \frac{mR^2}{2}$ (given disc)

$\cancel{m} gh = \frac{\cancel{m} v_{cm}^2}{2} + \frac{1}{2} \frac{\cancel{m} R^2 \omega^2}{2}$

$gh = \frac{v_{cm}^2}{2} + \frac{\omega^2 R^2}{4}$ $(\omega R = v_{cm})$

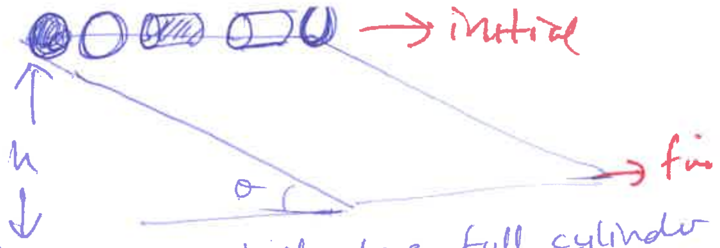


equal for it to roll down

$gh = \frac{v_{cm}^2}{2} + \frac{v_{cm}^2}{4} = \frac{3}{4} v_{cm}^2$

$v_{cm} = \sqrt{\frac{4}{3} gh}$; $\omega = \frac{\sqrt{\frac{4}{3} gh}}{R}$

(8)



full sphere; shell sphere; full cylinder; shell cylinder; Ring

which one should have the maximum speed at the bottom of the inclined plane?

$I = c\, mR^2$

$0 < c \leq 1$

$v_i = 0$

$E_i = E_f$

$\cancel{K_i} + U_i = K_f + \cancel{U_f}$

$mgh = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$

$\cancel{m} gh = \frac{1}{2} \cancel{m} v_{cm}^2 + \frac{1}{2} c \cancel{m} R^2 \omega^2$ ($R^2\omega^2 = v_{cm}^2$)

$gh = \frac{1}{2} v_{cm}^2 + \frac{c}{2} v_{cm}^2$

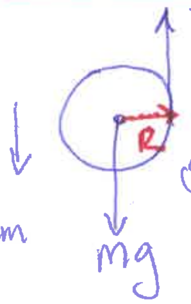
$v_{cm} = \sqrt{\frac{2gh}{1+c}}$ $c =$

		v_{cm}
$c = \frac{1}{2}$	full cylinder	$\rightarrow \left(\frac{2gh}{3/2}\right)^{1/2}$
$c = \frac{2}{5}$	full sphere	$\rightarrow \left(\frac{2gh}{7/5}\right)^{1/2}$
$c = 2/3$	shell sphere	$\rightarrow \left(\frac{2gh}{5/3}\right)^{1/2}$
$c = 1$	Ring	$\rightarrow \sqrt{\frac{2gh}{2}}$
$c = 1$	shell cylinder	

fastest

Ex: Acceleration of Yo-Yo

FBD of yoyo



$\sum \vec{F} = m\vec{a}$

① $\boxed{mg - T = ma}$

translation

rotation:

$\boxed{\alpha R = a_{cm}}$ ③

$\sum \vec{\tau} = \underbrace{\vec{\tau}_{mg}}_{0} + \underbrace{\tau_T}_{TR} = \underbrace{I\alpha}$

② $\boxed{TR = I\alpha}$

$I = \frac{mR^2}{2}$ (given)

$T\cancel{R} = \frac{mR^{\cancel{2}}\alpha}{2}$; $\boxed{R\alpha = a}$

$\boxed{T = \frac{ma}{2}}$

①

$\boxed{mg - T = ma}$

$mg = ma + \frac{mg}{2} = \frac{3ma}{2}$

$g = \frac{3}{2}a \Rightarrow \boxed{a = \frac{2}{3}g}$

$\boxed{\alpha = \frac{2g}{3R}}$

$T = \frac{ma}{2} = \frac{m}{2}\frac{2}{3}g$

$\boxed{T = \frac{mg}{3}}$

Ex:

Previous yo-yo energy conservation example

we have found

$v_{cm} = \sqrt{\frac{4}{3}gh}$

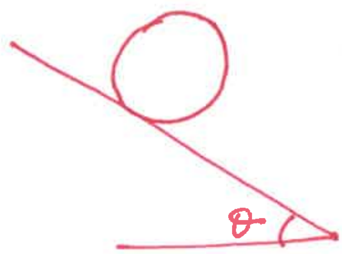
$v_f^2 = v_i^2 + 2ah$

$\Downarrow$ 0

$\frac{4g\cancel{h}}{3} = 2a\cancel{h}$ $\boxed{a = \frac{2g}{3}}$

③ Ex:

Rollip sphere



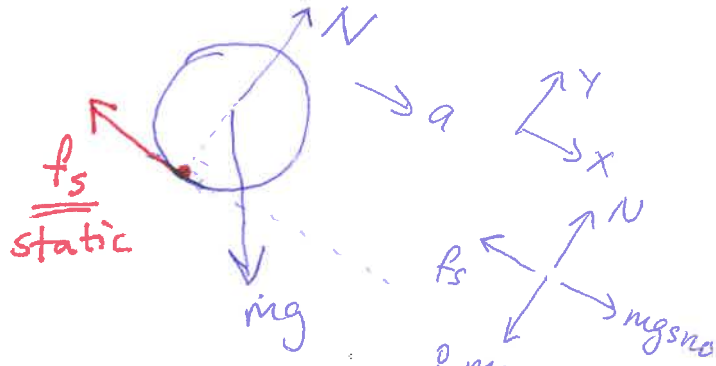
$I = \frac{2}{5}mR^2$

rolls without slipping.

→ FBD

→ a=?

→ friction force?



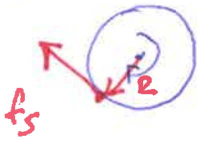
$\sum F_y = N - mg\cos\theta = ma_y$

$N = mg\cos\theta$

$\sum F_x = ma_x$

$\boxed{mg\sin\theta - f_s = ma}$ ①

Torque: $\vec{\tau} = I\vec{\alpha} = \vec{r} \times \vec{F}$



$\boxed{\tau = f_s R = I\alpha}$ ②

$I = \frac{2}{5}mR^2$

$f_s \cancel{R} = \frac{2}{5}mR^{\cancel{2}}\alpha$

$\boxed{\alpha R = a}$

$\boxed{f_s = \frac{2}{5}ma}$ rotating without slipping

into ①

$mg\sin\theta - \frac{2}{5}ma = ma$

$mg\sin\theta = \frac{7}{5}ma$

$\boxed{a = \frac{5}{7}g\sin\theta}$

$f_s = \frac{2}{5}ma = \frac{2}{5}m\frac{5}{7}g\sin\theta$

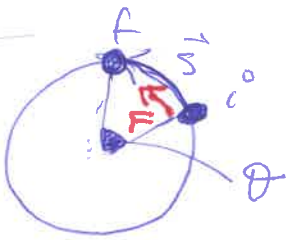
$\mu mg\cos\theta = \mu N = f_s = \frac{2}{7}mg\sin\theta$

$\boxed{\mu = \frac{2}{7}\tan\theta}$

WORK and POWER in Rotational motion (4)

$W = \vec{F} \cdot \vec{S}$

$S = \theta R$



$W = F \theta R$

$\underbrace{F \theta R}_{\text{Tork}}$

$\boxed{W = \tau \theta}$ $\boxed{dW = \tau d\theta}$

$\boxed{\Delta W = \tau \Delta \theta}$

$W_{tot} = \Delta K = \Sigma \tau \Delta \theta$

$= K_F - K_i$

$= \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$

Power: $P = \frac{dW}{dt} = \frac{\tau d\theta}{dt}$

$\boxed{P = \tau \cdot \omega}$ → angular

$\boxed{P = \vec{F} \cdot \vec{v}}$ → linear

Ex: An electric motor exerts 10 N.m torque. $I = 2\ kgm^2$

$\omega_i = 0$

→ Work done by motor in 8s.?

→ $K_F = ?$

→ Paverage delivered by motor?

$W = \tau \Delta\theta (\theta_f - \theta_i)$

$\theta_F = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$

$\theta_F = \frac{1}{2} \alpha t^2$; 8^2

$\tau = I \alpha$

$10 = 2 \alpha$

$\boxed{\alpha = 5}$

$\theta_F = \frac{1}{2} 5 (8^2) = 160\ rad$

$\alpha = 5\ rad/s^2$

$W = \tau \Delta\theta$

$= (10)(160)$

$W_{tot} = 1.6\ kJ$

$K_F = ?$ $W_{tot} = \Delta K$ $\omega_i = 0$

$1600 = K_F - \cancel{K_i}$

$1600 = \frac{1}{2} I \omega_f^2$

$1600 = \frac{1}{2} 2 \omega_f^2$

$\omega_f = \sqrt{1600} = 40\ rad/s$

$\boxed{K_F = 1600 J}$

$P_{av} = \frac{\Delta W}{\Delta t} = \frac{1600 J}{8 s}$

$P_{av} = 200\ Watt$

$P_{av} = \tau \omega$ ↳ avarge ang. velocity

$\omega_f = 40\ rad/s$, $\omega_i = 0$; $\omega_{av} = 20\ rad/s$

$P_{av} = (10)(20)$

$= 200\ watts$

This is the end of midterm 3 !!!

min 3 Q's

→ energy conservation

→ momentum "

→ rotation ⇄ 2 q's

} 4 Q's

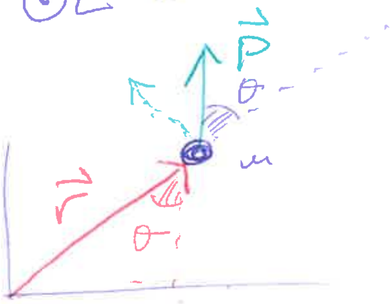
ANGULAR MOMENTUM (5)

$$\vec{L} = \vec{r} \times \vec{p} = m\,\vec{r} \times \vec{v}$$

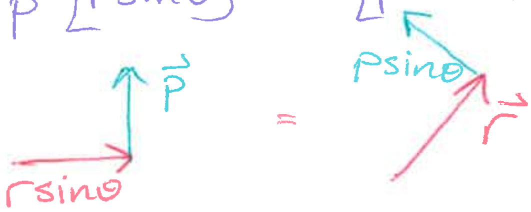
$\vec{p}$: momentum $= m\vec{v}$

$$\left[L = kg \frac{m^2}{s} \right]$$

$\odot \vec{L}$



$$|\vec{L}| = rp\sin\theta$$
$$= p\,[r\sin\theta] = r\,[p\sin\theta]$$



$$\underline{\underline{L}} = |\vec{r} \times m\vec{v}| = \underline{r m v \sin\theta}$$

Langular momentum.

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \vec{v} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$\frac{d\vec{L}}{dt} = \underbrace{\vec{v} \times m\vec{v}}_{\sin 0 = 0,\ 0} + \vec{r} \times \underbrace{m\vec{a}}_{\vec{F}}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

$$\Sigma\vec{\tau} = \frac{d\vec{L}}{dt}$$

rate of angular momentum change equals to the torque applied on it

$$\Sigma\vec{\tau} = \text{net torque} = \frac{d\vec{L}}{dt}$$

$\Downarrow$

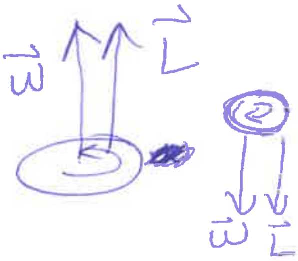
$I\alpha$

$$L = Rmv$$
$v \to wR$

$$L = mR^2\,w$$

I

$$\boxed{\begin{aligned} \vec{L} &= I\vec{w} \\ \vec{L} &= \vec{r} \times \vec{p} \end{aligned}}$$



Ex: A turbine fan in a jet engine has moment of inertia of 2.5 kgm² As it starts up $w = 40 \frac{rad}{s^3} t^2$

a) $L(t) = ?$

$$L = Iw = (2.5)(40)t^2$$
$$L(t) = 100t^2$$

b) $L(t = 3s) = 100(3^2)$
$$= 900\ kgm^2 \frac{rad}{s}$$
$$= 900\ kgm^2/s$$

c) $\Sigma\tau = ?$ as a function of time

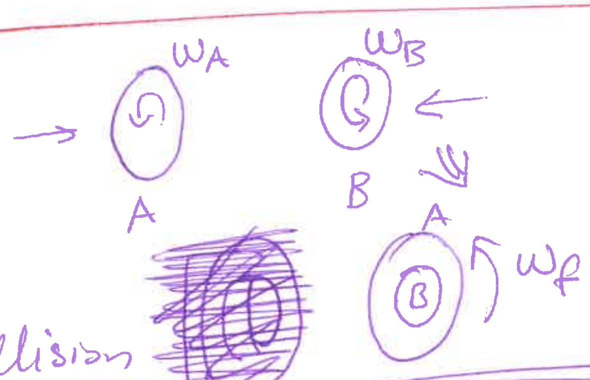
$\Sigma\tau(t = 3s) = ?$

$$\Sigma\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow L = 100t^2$$
$$\frac{dL}{dt} = 200t$$

$$\Sigma \tau = \frac{dL}{dt} = 200t$$

$$\Sigma \tau (t = 3s) = 200(3) = 600 \ N \cdot m$$

Conservation of Angular Momentum



Collision

when the net external torque acting on a system (A+B) is ZERO; total angular momentum of the system is conserved.

$$\vec{\tau}_{A \, on \, B} = \frac{d\vec{L}_B}{dt}$$

$$\vec{\tau}_{B \, on \, A} = \frac{d\vec{L}_A}{dt}$$

action reaction pairs.

$$\frac{d\vec{L}_B}{dt} + \frac{d\vec{L}_A}{dt} = 0 \quad \Big\} \ \Sigma\vec{\tau} = 0$$

$$\Sigma \frac{d\vec{L}}{dt} = 0 \qquad \vec{L} = \vec{L}_A + \vec{L}_B$$

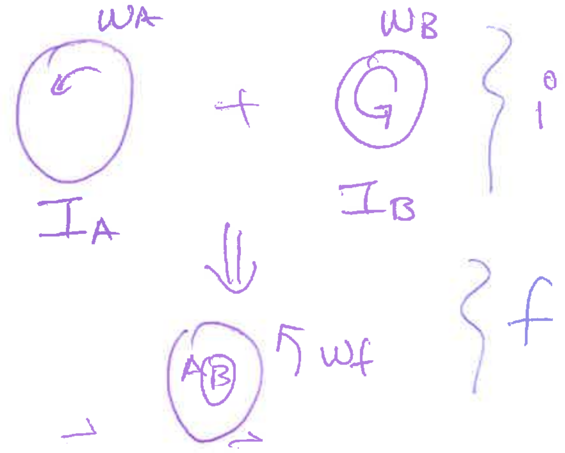
$$\Delta \vec{L} = 0$$

$$\boxed{\Sigma \vec{L}_i = \Sigma \vec{L}_F}$$

angular momentum conservation.

6

Ex:



$$\vec{L}_i = \vec{L}_F$$

$$L_{Ai} + L_{Bi} = (L_{A+B})_F$$

$$I_A \omega_{Ai} + I_B \omega_{Bi} = (I_A + I_B)\omega_f$$

$$\boxed{\omega_f = \frac{I_A \omega_{Ai} + I_B \omega_{Bi}}{I_A + I_B}}$$

Ex: Rotating/spining dancer

$$I_i = I^i_{dancer} + mr^2 + mr^2$$

$$I_f = I^f_{dancer} + \cancel{mr^2}^{\,0} + \cancel{mr^2}^{\,0}$$

$$I_i \text{ dancer} \neq I_f \text{ dancer}$$

I_i dancer $= 3 \, kg \, m^2$

I_f dancer $= 2.2 \, kg \, m^2$

$m = 5 \, kg \qquad \omega_i = 0.5 \, rev/s$

$r = 1 \, m$

$$\Sigma I_i = \Sigma I_f$$

$$(3 + \underbrace{2(5)\,1^2}_{2mr^2})\,\omega_i = I_f \, \omega_f = (2.2)\,\omega_f$$

7

$13\omega_i = (2.2)\,\omega_f$

(0.5)

$\omega_f = \left(\frac{6.5}{2.2}\right)$ rev/s

$\boxed{\omega_f \simeq 3 \text{ rev/s}}$

$a_{rad} = ?$ $\quad a_{rad} = \frac{v^2}{R} = \frac{\omega^2 R^2}{R}$

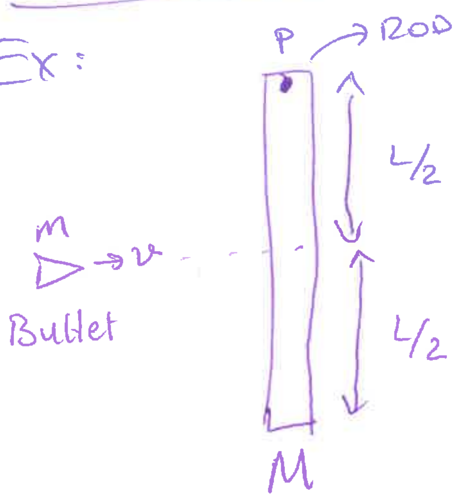
$\boxed{a_{rad} = \omega^2 R}$

$\omega^2 R > a_{critical}$

pass out!

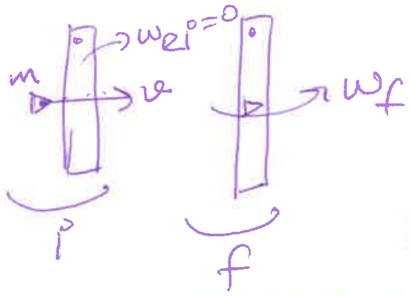
bayılmak.

Ex:



$I_p = \frac{1}{3} M L^2$

initial

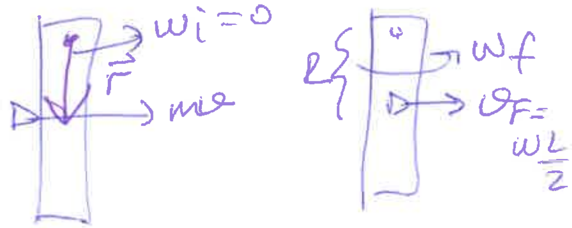


final

$\omega_f = ?$

~~$\Sigma L_i = \Sigma L_f$~~

initially ROD has no angular velocity. bullet m, comes with a velocity v what's the final $\omega_f = ?$



$\Sigma L_i = \Sigma L_F$

$L_{Bi} + I_R \omega_i = I_R \omega_F + L_{BF}$

$\vec{L} = \vec{r} \times \vec{p}$; $L = rp = rmv$

$L_{Bi} = m \frac{L}{2} v_i$

$L_{BF} = m \frac{L}{2} v_F$; $v_F = \omega_F L/2$

$m \frac{L}{2} v_i + 0 = I_R \omega_F \cancel{+ m\omega_F L}$
$+ m \frac{L}{2} v_F$

$v_F \to \omega_F \frac{L}{2}$

$m \frac{L}{2} v_i = \left(I_R + m \frac{L^2}{4}\right) \omega_f$

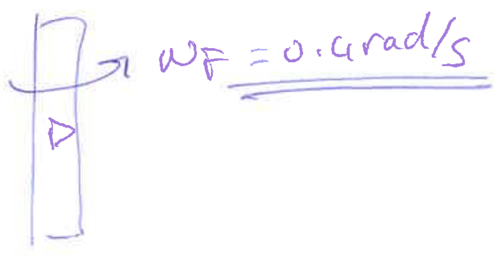
$\omega_F = \dfrac{m \frac{L}{2} v_i}{\left(I_R + m \frac{L^2}{4}\right)}$

$I_R = 5\,kg\,m^2$ $\quad m = 10g$

$L = 1m$ $\quad v_i = 400 m/s$

$\omega_F = \dfrac{(0.01)\frac{1}{2} 400}{5 + (0.01)\frac{1}{4}} \simeq \dfrac{2}{5.002}$

$\omega_F \simeq 0.4$ rad/s



$E_i = E_f$ 8

$\omega = 0.4\ rad/s$ final

initial

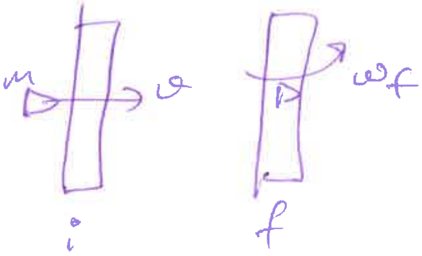
$E_i = E_f$

$K_i + U_i = \cancel{K_f}^{\,0} + U_f$

$\frac{1}{2} I\omega_i^2 + 0 = 0 + (M+m)gh$

$\frac{1}{2}(5.0025)\,0.4^2 = (85.01)\,9.8\,h$

whats the % of energy LOST in the collision



$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2}(0.01)\,400^2$

$= 800J$

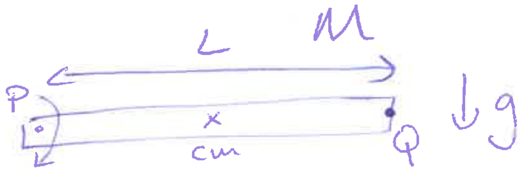
$K_f = \frac{1}{2} I\omega_f^2 = \frac{1}{2}(5.0025)\,0.4^2$

$= 0.4J$

$|K_f - K_i| = \left|\frac{0.4 - 800}{800}\right|$

$= 99\%$ lost

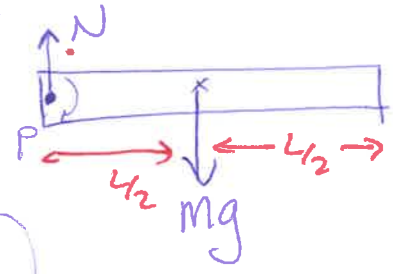
Ex:



$\omega_i = 0$ whats the acceleration of point Q after it starts to rotate?

$I_{cm} = \frac{1}{12} ML^2$

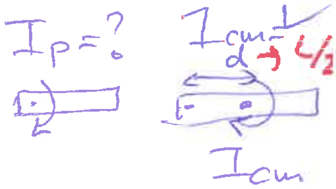
FBD



$\tau = I_p \alpha$

$mg\frac{L}{2} = \frac{ML^2}{3}\alpha$

$\frac{3g}{2L} = \alpha$



parallel axis theorem

$I_p = I_{cm} + Md^2$

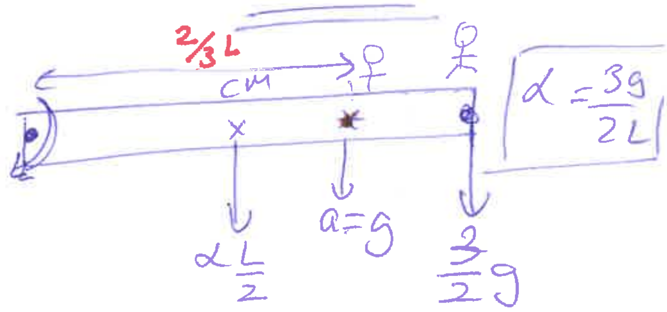
$= \frac{1}{12} ML^2 + M\frac{L^2}{4}$

$a = \alpha L$ $a = ?$ $\boxed{I_p = \frac{ML^2}{3}}$

$= \frac{3g}{2L} L$

$\boxed{a = \frac{3}{2} g}$

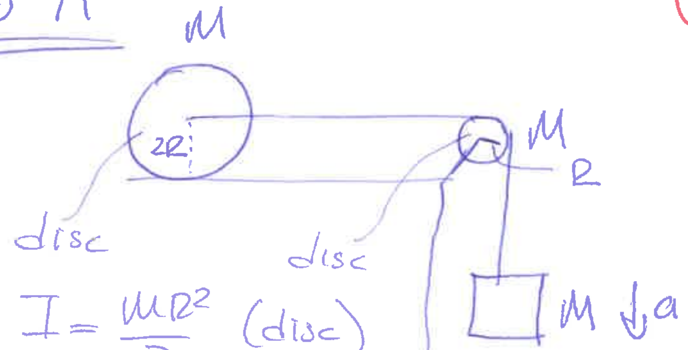
is this right? $a > g$!?



$\frac{3g}{2L}\frac{L}{2}$

$\boxed{\frac{3g}{4}}$

10.75

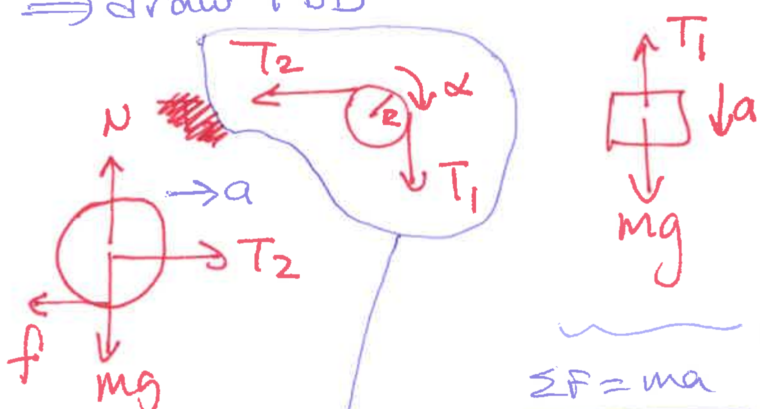


$I = \frac{MR^2}{2}$ (disc)

rolls without slipp.

$a = ?$

$\Rightarrow$ draw FBD



$\Sigma F_y = ma_y^{\,0}$

$N - mg = 0$

$N = mg$

$\Sigma F_x = ma$

$\boxed{T_2 - f = ma}$ ③

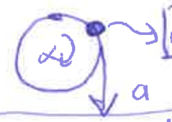
$\Sigma F = ma$

① $\boxed{mg - T_1 = ma}$

$T_1 R - T_2 R = I\alpha$

(given) $I = \frac{MR^2}{2}$

$(T_1 - T_2)\cancel{R} = \frac{mR^{\cancel{2}}\alpha}{2}$

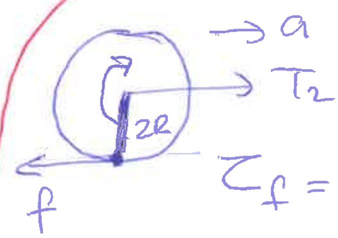


$\boxed{a = \alpha R}$

② $\boxed{T_1 - T_2 = \frac{ma}{2}}$

① + ② + ③

$$mg - \underbrace{f}_{\downarrow} = ma + ma + \frac{ma}{2} = \frac{5}{2} ma$$



$I = \frac{m(2R)^2}{2}$

$\tau_f = f \cdot 2R = I\alpha_1$

$f \cancel{2R} = m\frac{\cancel{4}R^2}{\cancel{2}}\alpha_1$

$\boxed{a = \alpha\, 2R}$

$f = mR\alpha_1$

$\boxed{f = m\frac{a}{2}}$

⑨ $mg - \frac{ma}{2} = \frac{5ma}{2}$

$\cancel{m}g = \frac{6\cancel{m}a}{2}$

$\boxed{a = g/3}$ ✓

$f = \frac{ma}{2} = \frac{mg}{6}$ ✓

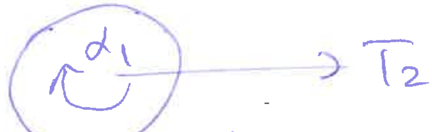
T_1: $mg - T_1 = ma$ ①

$mg - T_1 = \frac{mg}{3}$

$\boxed{T_1 = \frac{2}{3} mg}$

③ $T_2 - \underbrace{f}_{\downarrow\, ma/2} = ma$

✓ $\boxed{T_2 = \frac{3}{2} ma = \frac{mg}{3}}$



$\alpha_1\, 2R = a = g/3$

$\boxed{\alpha_1 = \frac{g}{6R}}$

$(T_1 - T_2)R = I\alpha_2$

$\alpha_2 R = a$

$\boxed{\alpha_2 = \frac{g}{3R}}$

not equal

$f = \frac{ma}{2} = \frac{mg}{6} = \mu mg$

$\mu = 1/6$ //